# TMH Model Solutions

# IIT

## Entrance Exam Papers

## 1984-88

## With Question Bank

Model Solutions to
IIT ENTRANCE EXAM PAPERS
1984-88
With Question Bank

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**Tata McGraw-Hill Publishing Company Limited**
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-Acc no-16855



First Reprint 1988



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Published by Tata McGraw-Hill Publishing Company Limited, 4/12 Asaf Ali Road, New Delhi 110 002, and printed at Rajkamal Electric Press, B-35/9 G T Karnal Road, Delhi 110 033

# INTRODUCTION

This book is meant to prepare candidates for the IIT Joint Entrance Examination. It is obvious that even a thorough preparation for the 10 + 2 pattern of examination is not sufficient for the totally different type of JEE. Therefore, the importance of this book cannot be overemphasized. It is the result of the combined effort of teachers who have had considerable experience and success in preparing students for this highly competitive examination. The book is in two parts. The first part comprises the question papers for the last five years in physics, chemistry and mathematics and their solutions. The English question papers of 1985, 1986 and 1987 have also been provided with their answers. The second part is a 'question bank'.

In the first part a 'model solution' is provided to each question. This is the solution that gets the highest score. Each step of an answer carries marks and the model solution shows the reader the steps that win those valuable marks. The model solutions show how the mind of the paper-setter works and what the evaluator expects. It prepares the candidate for the tricky questions that are always asked in this examination.

The question bank gives about 150 vital questions on each subject which have been carefully prepared by our team of experts. They are of the same standard as those asked in the examination and follow the same pattern. They are meant for practice, after the reader has studied the model solutions section and has understood the method by which the problems are to be tackled. The answer to each question is provided.

EDITOR

# SOME USEFUL HINTS FOR EXAMINEES

1. The students must carefully read the instructions given at the beginning of each question paper and bear them in mind while writing the answer.
2. In the physics and chemistry papers, the students should always use the same numerical value for the physical constants as is given in the question paper.
3. Calculators and log tables are not allowed in these papers and, therefore, the students are required to do the calculations in the answer sheets. It is always advisable to column off about 4 cm on the right-hand side of every answer sheet for rough work.
4. All the papers, whether physics, chemistry and mathematics should be taken with all seriousness.
5. PART A of the question paper comprising 'Objective Questions' should be attempted first as given in the instructions in the question paper, since it covers about 50% of the marks in physics, chemistry and mathematics. PART B should be attempted afterwards. It is advisable not to exceed the time limit while answering PART A.
6. The problems given in these papers are more general and based on concepts. Most of the problems involve more than one concept. The students must carefully read the problems and find the 'catch' before attempting it.
7. Steps should be clearly shown as far as possible while writing any specific answer. Marks are always given for each step and not for the problem as a whole.
8. The textbooks should be studied carefully and the TMH model solutions should be studied only after gaining a thorough knowledge of the respective subjects.

# SYLLABI FOR JEE

## CHEMISTRY

Electron, proton and neutron: constitution of nucleus; properties of alpha, beta and gamma rays; Rutherford's scattering experiment; Bohr atomic model (mathematical details excluded); quantum numbers; Pauli exclusion principle; Hund's rule; Aufbau principle; electronic configuration of elements (up to atomic number 36); shapes of *s*, *p* and *d* orbitals.

Periodic table: periods and groups, classification of elements with respect to *s*, *p* and *d* blocks; periodicity in properties, namely, atomic radius, ionization energy, electron affinity and electronegativity.

Chemical bond: electrovalent (ionic) and covalent (including coordinate) bonds; polarity in molecules; dipole moment (qualitative aspect only); Concept of hybridization involving *s* and *p* orbitals only; hydrogen bond.

Behaviour of gases: Avogadro's law; equation of state for an ideal gas; diffusion of gases; kinetic theory of gases; root mean square velocity and its relation with temperature; average velocity; Gay Lussac' law.

Stoichiometry: balancing equations for chemical reactions (including oxidation-reduction reactions using ion-electron and oxidation number methods): calculations involving oxidation-reduction, neutralization and displacement reactions; use of mole concept. Solutions: expressing concentration in terms of mole fraction, molality, molarity and normality; lowering of vapour pressure; Rauolt's law.

Concepts of chemical equilibrium: law of mass action; equilibrium constant; exothermic and endothermic reactions; Le Chatelier's principle and its applications; ionic equilibria in aqueous solutions; solubility product, common ion effect, acid-base equilibria, hydrolysis of salts; pH; buffer solutions. Simple numerical problems based on above.

Thermochemistry: Heat of formations, heat of combustion and heat of reaction; Hess's law.

Chemical kinetics: rates of reactions, order of reactions, rate constant and half-life period of first-order reactions; elementary concept of catalysis.

Faraday's laws of electrolysis; galvanic cells; cell reactions and emf of cells involving the following electrodes only: $Zn/Zn^{++}$; $Cu/Cu^{-+}$; $Fe/Fe^{++}$; $Sn/Sn^{++}$; Pt, $H_2/H^+$; $Cl^-/Cl_2$, Pt Electrochemical series.

Hydrogen and hydrogenperoxide: preparation, properties and uses.

Sodium, potassium, magnesium and calcium: important minerals, extraction, properties and uses; compounds: oxides, hydroxides, carbonates, bicarbonates and sulphates.

Aluminium: minerals, extraction, properties and uses; compounds: alumina, aluminium chloride and alums.

Carbon: occurrence allotropes and oxides.

Tin and lead: important minerals, extraction, properties and uses; compounds: oxides and chlorides.

Nitrogen and phosphorous: sources; compounds: oxides and oxy-acids, ammonia, fertilizers.

Oxygen and sulphur: properties; allotropes; compounds: hydrogen sulphide, sodium thiosulphate, oxides, sulphurous and sulphuric acids.

Chlorine, bromine and iodine: preparation, properties and uses; compounds: hydrohalic acids, oxy-acids of chlorine, bleaching powder.

Iron, copper, silver and zinc: occurrence, extraction, refining, properties and uses; compounds: ferrous sulphate, ferric oxide, ferric chloride, copper sulphate, silver nitrate, silver bromide, zinc oxide and zinc sulphide.

*Note*: In the case of all the compounds mentioned above, their preparation, properties and uses are included.

Concepts of hybridization of carbon; sigma and pi bonds, resonance; homologous series; structural isomerism; nomenclature.

Alkanes: substitution reactions, alkenes and alkynes addition reactions, ozonolysis.

Benzene: structure, nitration, sulphonation, halogenation and acylation reactions.

Characteristic reactions of the following functional groups: alcohol (esterification, dehydration, oxidation, reaction with sodium and phosphorous halides); phenol (halogenation, nitration, sulphonation); aldehyde and ketone (oxidation, reduction, oxime and hydrazone formation, aldol condensation and Canizzarro's reaction), carboxylic acid (formation of ester, amide and acid chloride).

Penetration, properties and uses of the following: ethanol, ethylene, acetylene, di-ethyl ether, chloroform, carbon tetrachloride, formaldehyde; acetaldehyde, acetone, formic and acetic acids, acetic anhydride, ethyl acetate and aniline.

## PHYSICS

Units and dimensions, SI units, displacement, velocity, kinematics in one and two dimensions with constant acceleration, projectiles, concepts of relative motion, circular motion.

Newton's law of motion, concepts of inertial frames, momentum. Forces: friction, spring force, tension and gravitational force, work, energy and power, conservation of momentum and energy. Universal law of gravitation, variation of the acceleration due to gravity with altitude and latitude, Kepler's laws, motion of planets and satellites assuming circular orbits.

Simple harmonic motion.

Centre of mass of a system of particles, elastic and inelastic collision in one dimension. Rigid bodies: moments of inertia of simple shapes like ring, disk, cylinder and sphere, angular momentum, torque. Conservation of angular momentum.

Hooke's law, Young's modulus, shear and bulk moduli. Principle of buoyancy's, pressure in a fluid.

Wave motion, concepts of amplitude, frequency and phase, longitudinal and transverse waves, progressive and stationary waves, vibration of strings and air columns, resonance, beats, velocity of sound, echoes. Doppler effect, quality and pitch of sound.

Thermal expansion of solids, liquids and gases. Ideal gas laws, absolute temperature, specific heat, $C_p$, $C_v$ and $\gamma$, isothermal and adiabatic processes. Calorimetry, latent heat, equivalence of heat and work, conduction, convection and radiation.

Elements of kinetic theory of gases, pressure of an ideal gas, equipartition of energy temperature.

Coulomb's law, electric field, lines of force, electric potential, capacitance, dielectric constant, parallel plate capacitors, capacitors in series and parallel, energy stored in a capacitor.

Electric current, Ohm's law, series and parallel arrangements of resistances and cells, Kirchhoff's laws. Wheatstone bridge and applications, heating effect of current, magnetic field due to a current, magnetic moment of a current loop, force on a moving charge and on a current carrying wire in a magnetic field, moving coil galvanometer, voltmeter, ammeter electromagnetic induction, Faraday's and Lenz's laws, self and mutual inductances.

Rectilinear propagation and velocity of light, reflection and refraction at plane and curved surfaces, total internal reflection, critical angle, prism, minimum deviation, dispersion, spectrometer and spectra. Curved mirrors and thin lenses, magnification, microscope, telescope.

Wave nature of light, interference phenomena, Young's double slit experiment, fringe width.

Cathode rays, law of radioactivity, nature of $\alpha$, $\beta$ and $\gamma$ rays, law of radioactive decay, half life, photons, photoelectric effect, Bohr's theory of hydrogen-like atoms, X-rays: production and properties. Atomic nucleus, binding energy, nuclear energy by fission and fusion. Diode rectification and triode amplification.

## MATHEMATICS

**Algebra**: Complex number as an ordered pair of real numbers; geometrical representation; real and imaginary parts; absolute value and conjugation, triangle inequality.

Theory of quadratic equations and expressions; relationships between roots and coefficients; linear and quadratic inequations in one variable; arithmetic, geometry and harmonic progressions; permutations and combinations; elementary applications of mathematical induction; binomial theorem for a positive integral index.

Determinants of order two and three and their elementary properties; solutions of simultaneous linear equations in two and three variables.

**Coordinate Geometry:** Rectangular Cartesian coordinates; distance between two points; area of a triangle; straight line; angle between two lines; parallel and perpendicular lines; distance of a point from a line, circle; equation of tangent and normal to a circle, parametric representation.

**Trigonometry:** Trigonometric functions and their graphs; addition and subtraction formulae; multiple and submultiple angles; general solution of

trigonometric equations, solution of triangles; simple applications in finding heights and distances.

**Calculus:** Function, into, onto, and one-one functions; polynomial, rational, trigonometric, logarithmic, and exponential functions.

Notion of limit and continuity of a function; derivative of a function at a point; derivatives of sum, difference, product and quotients of functions; derivatives of composite and implicit functions, derivatives of polynomial, rational trigonometric, inverse trigonometric, logarithmic, and exponential functions; geometrical interpretation of derivative; tangents and normals, sign of the derivative and monotonicity; maximum and minimum values of function.

Integration as the inverse process of differentiation; integration by parts and by substitution; definite integrals and its application for the determination of areas (simple cases).

**Vectors:** Addition of vectors; multiplication by a scalar; scalar product, cross product and scalar triple product, with geometrical applications.

**Probability:** Probability, sum and product laws, conditional probability.

# 1984

# QUESTION PAPERS

## PHYSICS

TIME—3 HOURS MAXIMUM MARKS—100

*Note:*—(1) You are advised not to spend more than sixty minutes in answering Part A.

(2) Answers to Part A must be given only on the first six pages of the answer-book in the order in which the questions appear in the question paper.

**Useful Physical Data**

Speed of light in vacuum = $3 \times 10^8$ m/s; Planck's Constant = $6.6 \times 10^{-34}$ J s Mass of the proton = $1.67 \times 10^{-27}$ kg; 1 rydberg = $2.2 \times 10^{-18}$ J; Bohr radius of hydrogen atom = $5 \times 10^{-11}$ m; Boltzmann's Constant = $1.38 \times 10^{-23}$ J/K; Acceleration due to gravity = 9.8 m/s$^2$; Radius of the earth = 6400 km; Speed of sound in air = 330 m/s; Speed of sound in water = 1400 m/s; Young's modulus of steel = $2 \times 10^{11}$ N/m$^2$; Coefficient of linear expansion of steel = $1.21 \times 10^{-5}$/°C.

## PART A

1. In each of the statements below, several alternatives are given. In some cases, more than one alternative may be correct. Select all the correct alternative(s) in each case, and write down the corresponding letter(s) [*A*], [*B*], [*C*] or [*D*] in your answer-book. For each Part, marks will be given only if all the correct alternatives are selected:

$2 \times 9 = 18$

(i) $L$, $C$ and $R$ represent the physical quantities, inductance, capacitance and resistance respectively. The combinations which have the dimensions of frequency are

[*A*] $1/RC$ [*B*] $R/L$ [*C*] $\sqrt{LC}$ [*D*] $C/L$

(ii) A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time $t$ is proportional to

[*A*] $t^{1/2}$ [*B*] $t^{3/4}$ [*C*] $t^{3/2}$ [*D*] $t^2$

(iii) A transverse wave is described by the equation $Y=Y_0 \sin 2\pi (ft-x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if

[*A*] $\lambda=\pi y_0/4$ [*B*] $\lambda=\pi y_0/2$ [*C*] $\lambda=\pi y_0$ [*D*] $\lambda=2\pi y_0$

(iv) White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is $b$ and the screen is at a distance $d(\gg b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are

[*A*] $\lambda=b^2/d$ [*B*] $\lambda=2b^2/d$ [*C*] $\lambda=b^2/3d$ [*D*] $\lambda=2b^2/3d$

(v) Two equal negative charge $-q$ are fixed at the points $(0, a)$ and $(0, -a)$ on the $y$-axis. A positive charge $Q$ is released from rest at the point $(2a, 0)$ on the $x$-axis. The charge $Q$ will

[*A*] execute simple harmonic motion about the origin;
[*B*] move to the origin and remain at rest;
[*C*] move to infinity;
[*D*] execute oscillatory but *not* simple harmonic motion.

(vi) From the following equations pick out the possible nuclear fusion reactions:

[*A*] ${}_5C^{13}+{}_1H^1 \rightarrow {}_6C^{14}+4.3$ MeV
[*B*] ${}_6C^{12}+{}_1H^1 \rightarrow {}_7N^{13}+2$ MeV
[*C*] ${}_7N^{14}+{}_1H^1 \rightarrow {}_8O^{15}+7.3$ MeV
[*D*] ${}_{92}U^{235}+{}_0n^1 \rightarrow {}_{34}Xe^{140}+{}_{38}Sr^{94}+{}_0n^1+{}_0n^1+\gamma+200$ MeV

(vii) In the Bohr model of the hydrogen atom

[*A*] the radius of the $n$th orbit is proportional to $n^2$;
[*B*] the total energy of the electron in the $n$th orbit is inversely proportional to $n$;
[*C*] the angular momentum of the electron in an orbit is an integral multiple of $h/2\pi$;
[*D*] the *magnitude* of the potential energy of the electron in any orbit is greater than its kinetic energy.

(viii) Select the correct statements from the following:

[*A*] A diode can be used as a rectifier
[*B*] A triode cannot be used as a rectifier
[*C*] The current in a diode is always proportional to the applied voltage
[*D*] The linear portion of the $I-V$ characteristic of a triode is used for amplification without distortion.

(ix) At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is

[*A*] $H_2$ [*B*] $F_2$ [*C*] $O_2$ [*D*] $Cl_2$.

2. Read the following statements carefully. Indicate the sub-division number and write down the matter corresponding to each blank strictly in the order in which it appears in the statement:

$2\times10=20$

(i) The numerical value of the angular velocity of rotation of the earth should be ... rad/s in order to make the effective acceleration due to gravity equal to zero at the equator.

(ii) A uniform cube of side $a$ and mass $m$ rests on a rough horizontal table. A horizontal force $F$ is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of $F$ for which the cube begins to tip about an edge is .... (Assume that the cube does not slide.)

(iii) A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s$^2$, the frictional force acting on the block is ... N.

(iv) Four persons $K$, $L$, $M$, $N$ are initially at the four corners of a square of side $d$. Each person now moves with a uniform speed $v$ in such a way that $K$ always moves directly towards $L$, $L$ directly towards $M$, $M$ directly towards $N$ and $N$ directly towards $K$. The four persons will meet at a time....

(v) One mole of a monatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is ....

(vi) Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have the maximum amplitude of vibration is ... m.

(vii) Five identical capacitor plates, each of area $A$, are arranged such that adjacent plates are at a distance $d$ apart. The plates are connected to a source of emf $V$ as shown in Fig. 1. The charge on plate 1 ... and that on plate 4 is ....

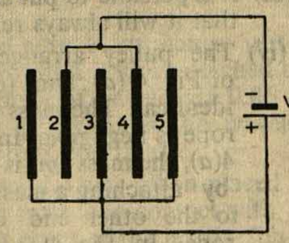



Fig. 1

(viii) Figure 2 shows lines of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points $A$, $B$ and $C$, the magnitude of the electric field is greatest at the point ....

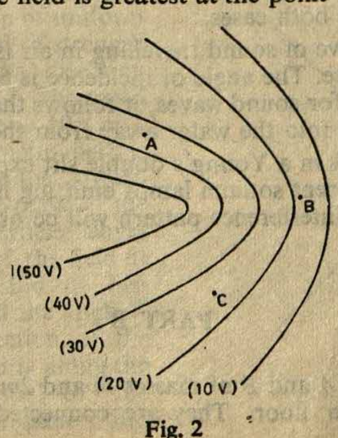



Fig. 2

(ix) The maximum kinetic energy of electrons emitted in the photoelectric effect is lineárly dependent on the ... of the incident radiation.

(x) A neutron, a proton, an electron and an alpha particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper.. The tracks of the particles are labelled in Fig. 3. The electron follows track ... and the alpha particle follows track ....

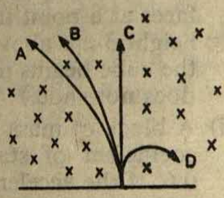



Fig. 3

3. State whether the following are TRUE or FALSE. Give reasons in brief in support of your answers. Marks will be awarded only if the correct reosons are given: $2 \times 6 = 12$

(i) A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path.

(ii) A simple pendulum with a bob of mass $m$ swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is greater than $mg$ cos 20°.

(iii) It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi.

(iv) The pulley arrangements of Fig. 4 (*a*) and (*b*) are identical. The mass of the rope is negligible. In Fig. 4(*a*), the mass $m$ is lifted by attaching a mass $2\,m$ to the other end of the rope. In Fig. 4(b), $m$ is lifted up by pulling the other end of the rope with a constant downward force $F = 2\,mg$.

The acceleration of $m$ is the same in both cases.

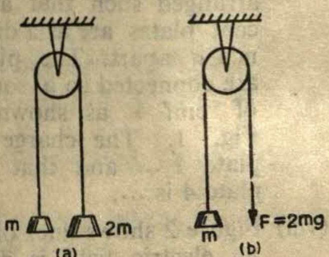



Fig. 4

(v) A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into the water away from the normal.

(vi) The two slits in a Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. No interference pattern will be observed on the screen.

## PART B

4. (*a*) Two bodies $A$ and $B$ of masses $m$ and $2m$ respectively are placed on a smooth floor. They are connected by a spring. A third

body $C$ of mass $m$ moves with a velocity $v_0$ along the line joining $A$ and $B$ and collides elastically with $A$, as shown in Fig. 5. At a certain instant of time $t_0$ after collision, it is found that the instantaneous velocities of $A$ and $B$ are the same. Further, at this instant the compression of the spring is found to be $x_0$. Determine (i) the common velocity of $A$ and $B$ at time $t_0$; and (ii) the spring constant. 6

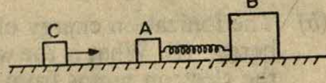


Fig. 5

(*b*) In the circuit shown in Fig. 6, $E$, $F$, $G$ and $H$ are cells of emf 2, 1, 3 and 1 volts respectively, and their internal resistances are 2, 1, 3 and 1 $\Omega$ repectively.

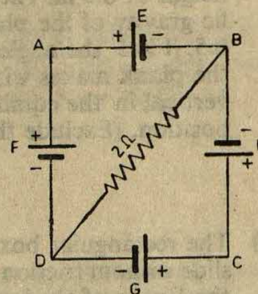


Fig. 6

Calculate

(i) the potential difference between $B$ and $D$; and

(ii) the potential difference across the terminals of each of the cells $G$ and $H$. 6

5. (*a*) A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? 6

(*b*) A plano-convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face of the lens is found to be 25/8 cm. Find the focal length of the lens. 6

6. (*a*) A particle of mass $m = 1.6 \times 10^{-27}$ kg and charge $q = 1.6 \times 10^{-19}$ C enters a region of uniform magnetic field of strength 1 T along the direction shown in Fig. 7. The speed of the particle is $10^7$ m/s. (i) The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region of the field at the point $F$. Find the distance $EF$ and the angle $\theta$. (ii) If the direction of the magnetic field is along the outward normal to the plane of the paper, find

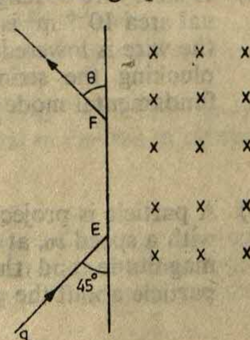


Fig. 7

the time spent by the particle in the region of the magnetic field after entering it at $E$. 8

(*b*) The ionization energy of a hydrogen-like Bohr atom is 4 rydbergs. (i) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state? (ii) What is the radius of the first orbit for this atom? 5

7. (*a*) A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in Fig. 8. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle $\theta$ that the plank makes with the vertical in the equilibrium position. [Exclude the case $\theta = 0°$.] 7

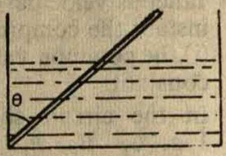


**Fig. 8**

OR

(*a*) The rectangular box shown in Fig. 9 has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a monatomic ideal gas ($\gamma = 5/3$) at a pressure $P_0$ volume $V_0$ and temperature $T_0$. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands, pushing the partition until the final pressure in both chambers becomes $243\ P_0/32$. Determine (i) the final temperature of the gas in each chamber and (ii) the work done by the gas in the right chamber. 7

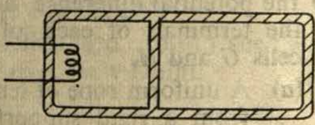


**Fig. 9**

(*b*) A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area $10^{-6}\ m^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. 6

OR

(*b*) A particle is projected at time $t = 0$ from a point $P$ on the ground with a speed $v_0$, at an angle of 45° to the horizontal. Find the magnitude and the direction of the angular momentum of the particle about the point $P$, at time $t = v_0/g$. 6

# CHEMISTRY

TIME—3 HOURS MAXIMUM MARKS—100

*Note*—(1) Attempt All questions.

(2) You are advised not to spend more than sixty minutes in answering Part A.

(3) Answers to Part A must be given only on the first six pages of the answer-book in the order in which the questions appear in the question paper.

**Some Useful Data**

Planck's constant $= 6.627 \times 10^{-27}$ erg s; Velocity of light $= 3.0 \times 10^{10}$ cm s$^{-1}$; Gas constant $= 2.0$ cal deg$^{-1}$ mol$^{-1}$ or 0.083 litre-atm deg$^{-1}$ mol$^{-1}$; Atomic weights: C = 12; H = 1.00, Br = 80; Na = 23; O = 16.

## PART A

1. Amongst the four alternate answers given for the sub-questions below *one* or *more* are correct. Write down all the correct answers.

[*Example*—Solid sodium chloride is (*A*) ionic (*B*) brittle (*C*) non-conductor (*D*) covalent. Correct answer: *A*, *B*, *C*]

(i) An isotone of $^{76}_{32}$Ge is:

(*A*) $^{77}_{32}$Ge (*B*) $^{77}_{33}$As (*C*) $^{77}_{3}$Se (*D*) $^{78}_{34}$Se

(ii) Many elements have non-integral atomic masses because:
(*A*) they have isotopes (*B*) their isotopes have non-integral masses (*C*) their isotopes have different masses (*D*) the constituents, neutrons, protons and electrons, combine to give fractional masses

(iii) When alpha particles are sent through a thin metal foil, most of them go straight through the foil because:
(*A*) alpha particles are much heavier than electrons (*B*) alpha particles are positively charged (*C*) most part of the atom is empty space (*D*) alpha particles move with high velocity.

(iv) When an ideal gas undergoes unrestrained expansion, no cooling occurs because the molecules:
(*A*) are above the inversion temperatures (*B*) exert no attractive forces on each other (*C*) do work equal to the loss in kinetic energy (*D*) collide without loss of energy.

(v) A catalyst:
(*A*) increases the average kinetic energy of reacting molecules (*B*) decreases the activation energy (*C*) alters the reaction mechanism (*D*) increases the frequency of collisions of reacting species

(vi) For the gas phase reaction,

$$[C_2H_4 + H_2 \rightleftharpoons C_2H_6 (\Delta H = -32.7 \text{ kcal})]$$

carried out in a vessel, the equilibrium concentration of $C_2H_4$ can be increased by:
(*A*) increasing the temperature (*B*) decreasing the pressure
(*C*) removing some $H_2$ (*D*) adding some $C_2H_6$

(vii) Resonance structures of a molecule should have:
(*A*) identical arrangements of atoms (*B*) nearly the same energy content (*C*) the same number of paired electrons (*D*) identical bonding

(viii) Base catalysed aldol condensation occurs with:
(*A*) propionaldehyde (*B*) benzaldehyde (*C*) 2-Methyl propionaldehyde (*D*) 2, 2-dimethyl propionaldehyde

(ix) Which of the following compounds will give a yellow precipitate with iodine and alkali:
(*A*) 2-hydroxy propane (*B*) acetophenone (*C*) methyl acetate (*D*) acetamide?

(x) Which of the following compounds will react with ethanolic KCN:
(*A*) ethyl chloride (*B*) acetyl chloride (*C*) chlorobenzene (*D*) benzaldehyde. $(1 \times 10 = 10)$

2. Amongst the four alternate answers given for the sub-questions below *only one* answer is correct. Indicate the correct answer:

(i) The radiations from a naturally occurring radioactive substance, as seen after deflection by a magnetic field in one direction, are:
(*A*) definitely alpha rays (*B*) definitely beta rays (*C*) both alpha and beta rays (*D*) either alpha or beta rays.

(ii) The increasing order (lowest first) for the values of $e/m$ (charge/mass) for electron $e$, proton $p$, neutron $n$ and alpha particle $a$ is:
(*A*) $e, p, n, \alpha$ (*B*) $n, p, e, \alpha$ (*C*) $n, p, \alpha, e$ (*D*) $n, \alpha, p, e$

(iii) On hybridization of one $s$ and one $p$ orbitals we get:
(*A*) two mutually perpendicular orbitals (*B*) two orbitals at 180° (*C*) four orbitals directed tetrahedrally by (*D*) three orbitals in a plane

(iv) Correct set of four quantum numbers for the valence (outermost) electron of rubidium ($Z = 37$) is:
(*A*) 5, 0, 0, $+\frac{1}{2}$ (*B*) 5, 1, 0, $+\frac{1}{2}$ (*C*) 5, 1, 1, $+\frac{1}{2}$ (*D*) 6, 0, 0, $+\frac{1}{2}$.

(v) Which electronic level would allow the hydrogen atom to absorb a photon but not to emit a photon?

(*A*) 3*s* (*B*) 2*p* (*C*) 2*s* (*D*) 1*s*

(vi) Equal weights of methane and hydrogen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by hydrogen is:

(*A*) 1/2 (*B*) 8/19 (*C*) 1/9 (*D*) 16/17

(vii) A liquid is in equilibrium with its vapour at its boiling point. On the average, the molecules in the two phases have equal:
(*A*) inter-molecular forces (*B*) potential energy (*D*) kinetic energy

(viii) Pure ammonia is placed in a vessel at a temperature where its dissociation constant ($\alpha$) is appreciable. At equilibrium:
(*A*) $K_p$ does not change significantly with pressure (*B*) $\alpha$ does not change with pressure (*C*) concentration of $NH_3$ does not change

with pressure (*D*) concentration of hydrogen is less than that of nitrogen

(ix) Zinc-copper couple that can be used as a reducing agent is obtained by:
(*A*) mixing zinc dust and copper gauze (*B*) coated zinc with copper (*C*) coating copper with zinc zinc (*D*) welding together zinc and copper wires

(x) A solution containing one mole per litre each of $Cu(NO_3)_2$, $AgNO_3$, $Hg_2(NO_3)_2$ and $Mg(ND_3)_2$is electrolysed using inert electrodes. standard electrode potentials in volts (reduction potentials) are:
$Ag/Ag^+ = +0.80$, $2Hg/Hg_2^{2+} = +0.79$, $Cu/Cu^{2+} = +0.34$,
$Mg/Mg^{2+} = -2.37$. With increasing voltage, the sequence of deposition of metals on the cathode will be:
(*A*) Ag, Hg, Cu, Mg (*B*) Mg, Cu, Hg, Ag (*C*) Ag, Hg, Cu
(*D*) Cu, Hg, Ag

(xi) The electric charge for electrode deposition of one gram equivalent of a substance is:
(*A*) One ampere for one second (*B*) 96,500 coulombs per second (*C*) one ampere for one hour (*D*) charge on one mole of electrons

(xii) The hydration energy of $Mg^{2+}$ is greater than that of:
(*A*) $Al^{3+}$ (*B*) $Na^+$ (*C*) $Be^{2+}$ (*D*) $Mg^{3+}$

(xiii) A certain buffer solution contains equal concentration of $X^-$ and HX. The $K_b$ for $X^-$ is $10^{-10}$. The pH of the buffer is:
(*A*) 4 (*B*) 7 (*C*) 10 (*D*) 14

(xiv) A certain weak acid has a dissociation constant of $1.0 \times 10^{-4}$. The equilibrium constant for its reaction with a strong base is:
(*A*) $1.0 \times 10^{-4}$ (*B*) $1.0 \times 10^{-10}$ (*C*) $1.0 \times 10^{10}$ (*D*) $1.0 \times 10^{14}$

(xv) The IUPAC name of the compound having the formula

$$\begin{array}{c} CH_3 \\ | \\ H_3C{-}C{-}CH{=}CH_2 \\ | \\ CH_2 \end{array}$$

is:
(*A*) 3,3,3-trimethyl-1-propene (*B*) 1,1,1-trimethyl-2-propene
(*C*) 3,3-dimethyl-1-butene (*D*) 2,2-dimethyl-3-butene

(xvi) Baeyer's reagent is:
(*A*) alkaline permanganate solution (*B*) acidified permanganate solution (*C*) neutral permanganate solution (*D*) aqueous bromine solution

(xvii) An industrial method of preparation of methanol is:
(*A*) catalytic reduction of carbon monoxide in presence of $ZnO\text{-}Cr_2O_3$ (*B*) by reacting methane with steam at 900°C with a nickel catalyst (*C*) by reducing formaldehyde with lithium aluminium hydride (*D*) by reacting formaldehyde with aqueous sodium hydroxide solution

(xviii) When phenol is treated with excess bromine water, it gives: (*A*) *m*-bromophenol (*B*) *o*- and *p*-bromophenol (*B*) 2,4-dibromophenol (*D*) 2,4,6-tribromophenol.

(xix) Carbylamine test is performed in alcoholic KOH by heating a mixture of: (*A*) chloroform and silver powder (*B*) trihalogenated methane and a primary amine (*C*) an alkyl halide and a primary amine (*D*) an alkyl cyanide and a primary amine.

(xx) Chlorobenzene can be prepared by reacting aniline with: (*A*) hydrochloric acid (*B*) cuprous chloride (*C*) chlorine in the presence of anhydrous aluminium chloride (*D*) nitrous acid followed by heating with cuprous chloride. (1 × 20 = 20)

3. (*a*) Write the matched set (*of three*) for each entry in column *A*:

(*Example*: Answer for nitric oxide is: nitric oxide-paramagnetic-air pollutant.)

| *A* | *B* | *C* |
|---|---|---|
| Asbestos | molecular sieve | air pollutant |
| Fluorocarbons | paramagnetic | carcinogen |
| Lithium metal | refrigeration | fluorescent paint |
| Nitric oxide | reducing agent | electron donor |
| Zeolites | semiconductor | ion exchanger |
| Zinc oxide | silicates of (Ca + Mg) | propellent |

(1 × 5 = 5)

(*b*) Indicate if the following statements are true or false by writing T or F. Support your answer with reasons (in *one* or *two* sentences only):

(i) In aqueous solution chlorine is a stronger oxidizing agent than fluorine.

(ii) Silver chloride is more soluble in very concentrated sodium chloride solution than in pure water.

(iii) If equilibrium constant for the reaction $A_2 + B_2 \rightleftharpoons 2AB$, is $K$, then for the backward reaction $AB \rightleftharpoons \frac{1}{2}A_2 + \frac{1}{2}B_2$, the equilibrium constant is $1/K$.

(iv) The H—N—H bond angle in $NH_3$ is greater than the H—As—H bond angle in $AsH_3$.

(v) When a liquid and its vapour are in equilibrium and the pressure is suddenly decreased, cooling occurs. (1 × 5 = 5)

4. (*a*) One (or *two*) words are needed in the blank space to complete the following statements. Write those words only for each sub-question. [*Example*: Answer for (v) : R]:

(i) The total energy of one mole of an ideal monatomic gas at 27°C is...calories.

(ii) An element that can exist as a positive ion in acidic solution and as a negative ion in basic solution is said to be....

(iii) The acidity of phenol is due to the...of its anion.

(iv) If metal ions of group III are precipitated by $NH_4Cl$ and $NH_4OH$ without prior oxidation by conc. $HNO_3$...is not completely precipitated.

(v) $C_p$—$C_v$ for an ideal gas is.... (1 × 4 = 4)

(*b*) Each of the following statements is true only under same specific conditions. Write the condition for each sub-question in *not more than two* sentences. (*Example*: Answer to (vii) is "gases are assumed ideal and $P$ and $T$ are constant"):

(i) Two volatile and miscible liquids can be separated by fractional distillation into pure components.

(ii) The heat energy $k$ absorbed by a gas is $\Delta H$.

(iii) Metals can be recovered from their ores by chemical methods.

(iv) High purity metals can be obtained by zone refining method.

(v) 2-Methyl propene can be converted into isobutyl bromide by hydrogen bromide.

(vi) Ethyne and its derivatives will give white precipitate with ammoniacal silver nitrate solution.

(vii) Equal volumes of gases contain equal number of moles. $(1 \times 6 = 6)$

## PART B

5. (*a*) The electron energy in hydrogen atom is given by $E = (-21.7 \times 10^{-12})/n^2$ ergs. Calculate the energy required to remove an electron completely from the $n=2$ orbit. What is the longest wavelength (in cm) of light that can be used to cause this transition? 3

(*b*) Radioactive decay is a first order process. Radioactive carbon in wood sample decays with a half life of 5770 years. What is the rate constant (in years$^{-1}$) for the decay? What fraction would remain after 11540 years? 3

(*c*) A solution contains a mixture of $Ag^+$ (0.10 M) and $Hg_2^{2+}$ (0.10 M) which are to be separated by selective precipitation. Calculate the maximum concentration of iodide ion at which one of them gets precipitated almost completely. What percentage of that metal ion is precipitated? [$K_{SP}$: $AgI = 8.5 \times 10^{-17}$; $Hg_2I_2 = 2.5 \times 10^{-26}$] 4

6. (*a*) Show with balanced equations what happens when the following are mixed:

(i) aqueous solution of ferric sulphate and potassium iodide.

(ii) aqueous solution of potassium manganate and acid.

(iii) aqueous solution of potassium chromate and acid. $(2 \times 3 = 6)$

(*b*) State with equations, what happens when:

(i) Chloral is heated with aqueous sodium hydroxide

(ii) *p*-xylene is reacted with concentrated sulphuric acid and the resultant product is fused with KOH $(2 \times 2 = 4)$

7. (*a*) Write down the reactions involved in the preparation of the following using the reagents indicated against each in parentheses.

(i) Ethyl benzene from benzene [$C_2H_5OH$, $PCl_5$, anhydrous $AlCl_3$]

(ii) Propionic anhydride from propionaldehyde [$AgNO_3/NH_4OH$, $P_2O_5$]

(iii) Acetoxime from acetaldehyde [$K_2Cr_2O_7/H^+$, $Ca(OH)_2$ and $NH_2OH \cdot HCl$] $(3 \times 2 = 6)$

(*b*) Write structural formulae for all the isomeric alcohols having the molecular formula $C_4H_{10}O$. 1

(*c*) $2.68 \times 10^{-3}$ moles of a solution containing an ion $A^{+n}$ require $1.61 \times 10^{-3}$ moles of $MnO_4^-$ for the oxidation of $A^{+n}$ to $AO_3^-$ in acid medium. What is the value of *n*? 3

8. (*a*) A certain hydrocarbon A was found to contain 85.7 per cent carbon and 14.3 per cent hydrogen. This compound consumes 1 molar equivalent of hydrogen to give a saturated hydrocarbon B. 1.00 g of hydrocarbon A just decolourized 38.05 g of a 5 per cent solution (by weight) of $Br_2$ in $CCl_4$. Compound A, on oxidation with concentrated $KMnO_4$, gave compound C (molecular formula $C_4H_8O$) and acetic acid. Compound C could easily be prepared by the action of acidic aqueous mercuric sulphate on 2-butyne. Determine the molecular formula of A and deduce the structures of A, B and C. 5

(*b*) When 16.8 g of white solid X were heated, 4.4 g of acid gas *A* that turned lime water milky was driven off together with 1.8 g of a gas *B* which condensed to a colourless liquid. The solid that remained, *Y*, dissolved in water to give an alkaline solution, which with excess barium chloride solution gave a white precipitate *Z*. The precipitate effervesced with acid giving off cabondioxide. Identify *A*, *B* and *Y* and write down the equation for the thermal decomposition of *X*.

9. (*a*) Given the following standard heats of reactions:

(i) heat of formation of water $= -68.3$ kcal,

(ii) heat of combustion of acetylene $= -310.6$ kcal,

(iii) heat of combustion of ethylene $= -337.2$ kcal, calculate the heat of reaction for the hydrogenation of acetylene at constant volume (25°C).

(*b*) One mole of $N_2$ and 3 moles of $PCl_5$ are placed in a 100 litre vessel heated to 227°C. The equilibrium pressure is 2.05 atmospheres. Assuming ideal behaviour, calculate the degree of dissociation for $PCl_5$ and for the reaction: $PCl_5(g) \rightleftharpoons PCl_4(g) + Cl_2(g)$. 6

# MATHEMATICS

TIME—3 HOURS MAXIMUM MARKS—100

*Note*—(1) You are advised not to spend more than sixty minutes in answering Part A.

(2) Answers to part A must be given only on the first four pages of the answer-book in the order in which the questions and their parts appear in the question paper.

(3) Use of Logarithmic and Trigonometric tables and Graph paper is NOT allowed.

## PART A

**(Attempt ALL questions in this Part)**

1. There are five parts in this question. Four choices are given for each part and one of them is correct. Indicate your choice of the correct answer for each part in your answer-book by writing the letter, (*A*) (*B*), (*C*) or (*D*) whichever is appropriate. (10)

(i) The locus of the mid-point of a chord of the circle $x^2+y^2=4$ which subtends a right angle at the origin is

(*A*) $x+y=2$ (*B*) $x^2+y^2=1$ (*C*) $x^2+y^2=2$ (*D*) $x+y=1$

(ii) The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has

(*A*) no root (*B*) one root (*C*) two equal roots (*D*) infinitely many roots

(iii) $\lim_{n\to\infty}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+\ldots+\frac{n}{1-n^2}\right\}$ is equal to

(*A*) 0 (*B*) $-\frac{1}{2}$ (*C*) $\frac{1}{2}$ (*D*) none of these.

(iv) Three identical dice are rolled. The probability that the same number will appear on each of them is

(*A*) $\frac{1}{6}$ (*B*) $\frac{1}{36}$ (*C*) $\frac{1}{18}$ (*D*) $\frac{3}{28}$

(v) If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval

(*A*) $[\frac{1}{2}, 2]$ (*B*) $[-1, 2]$ (*C*) $[-\frac{1}{2}, 1]$ (*D*) $[-1, \frac{1}{2}]$

2. There are five parts in this question. Each part has one or more than one correct answers. Indicate all correct answers for each part by writing the corresponding letters from (*A*), (*B*), (*C*), (*D*) in the answer-book:

(i) If $y=f(x)=\frac{x+2}{x-1}$ then

(*A*) $x=f(y)$ (*B*) $f(1)=3$ (*C*) $y$ increases with $x$ for $x<1$ (*D*) $f$ is a rational function of $x$.

(ii) $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

(*A*) $\frac{1}{2}$ (*B*) $\cos\frac{\pi}{8}$ (*C*) $\frac{1}{8}$ (*D*) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

(iii) If $x+|y|=2y$, then $y$ as a function of $x$ is

(*A*) defined for all real $x$ (*B*) continuous at $x=0$ (*C*) differentiable for all $x$ (*D*) such that $\frac{dy}{dx}=\frac{1}{3}$ for $x<0$

(iv) If $M$ and $N$ are any two events, the probability that exactly one of them occurs is

(*A*) $P(M)+P(N)-2P(M\cap N)$ (*B*) $P(M)+P(N)-P(M\cap N)$
(*C*) $P(M^c)+P(N^c)-2P(M^c\cap N^c)$ (*D*) $P(M\cap N^c)+P(M^c\cap N)$

(v) For real $x$, the function $\dfrac{(x-a)(x-b)}{x-c}$ will assume all real values provided

(*A*) $a > b > c$ (*b*) $a < b < c$ (*C*) $a > c > b$ (*D*) $a < c < b$

3. This question contains five statements, each of which is either true or false. Indicate your choice of the answer in the answer-book by writing TRUE or FALSE for each statement. 10

(i) If $a < b < c < d$,

then the roots of the equation

$$(x-a)(x-c)+2(x-b)(x-d)=0$$

are real and distinct.

(ii) If the complex numbers $Z_1$, $Z_2$, and $Z_3$ represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1+Z_2+Z_3 \neq 0$.

(iii) There exists a value of $\theta$ between 0 and $2\pi$ that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$.

(iv) For $0 < a < x$, the minimum value of the function $\log_a x + \log_a x$ is 2.

(v) The points with position vectors $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, and $\mathbf{a}+k\mathbf{b}$ are collinear for all real values of $k$.

4. This question contains ten incomplete statements. Determine your answers to be inserted in the blanks so that the statements are complete. Write these answers only in your answer-book strictly in order in which the statements appear below: 20

(i) The sum of integers from 1 to 100 that are divisible by 2 or 5 is…

(ii) If the product of the roots of the equation $x^3 - 3kx + 2e^{2\ln k} - 1 = 0$ is 7, then the roots are real for $k=$…

(iii) The system of equations

$$\lambda x + y + z = 0$$
$$-x + \lambda y + z = 0$$
$$-x - y + \lambda z = 0$$

will have a non-zero solution if real values of $\lambda$ are given by…

(iv) The numerical value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to…

(v) If $a$, $b$ and $c$ are in A.P., then the straight line $ax+by+c=0$ will always pass through a fixed point whose coordinates are (…, …)

(vi) $A$, $B$, $C$ and $D$, are four points in a plane with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively such that $(\mathbf{a}-\mathbf{d})\cdot(\mathbf{b}-\mathbf{c}) = (\mathbf{b}-\mathbf{d})\cdot(\mathbf{c}-\mathbf{a}) = 0$. The point $D$, then, is the…of the triangle $ABC$.

(vii) The sides $AB$, $BC$ and $CA$ of a triangle $ABC$ have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is…

(viii) The domain of the function $f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is given by…

(ix) $\lim_{x\to 1} (1-x) \tan\frac{\pi x}{2} = \ldots$

(x) The lines $3x-4y+4=0$ and $6x-8y-7=0$ are tangents to the same circle. The radius of this circle is....

## PART B

**(Question No. 5 in this Part is compulsory. Attempt any TEN questions from the remaining FOURTEEN)**

5. (i) If $1, a_1, a_2, \ldots, a_{n-1}$ are the $n$ roots of unity, then show that

$$(1-a_1)(1-a_2)(1-a_3)\ldots(1-a_{n-1})=n \qquad 2$$

(ii) If $a > 0$, $b > 0$ and $c > 0$, prove that

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geqslant 9 \qquad 2$$

(iii) If $n$ is a natural number such that

$$n = p_1^{a_1}\cdot p_2^{a_2}\cdot p_3^{a_3}, \ldots p_k^{a_k}$$

and $p_1, p_2, \ldots, p_k$ are distinct primes, then show that $\ln n \geqslant k \ln 2$.

(iv) Find the values of $x$ in $(-\pi, \pi)$ which satisfy the equation

$$8^{(1+|\cos x|+\cos^2 x+\cos^3 x| \ldots} = 4^3 \qquad 2$$

(v) $A$ and $B$ are two independent events. The probability that both $A$ and $B$ occur is $1/6$ and the probability that neither of them occurs is $1/3$. Find the probability of the occurrence of $A$. 2

6. For a triangle $ABC$ it is given that $\cos A+\cos B+\cos C=3/2$, prove that the triangle is equilateral. 4

7. If $p$ be a natural number then prove that $p^{n+1}+(p+1)^{2n-1}$ is divisible by $p^2+p+1$ for every positive integer $n$. 4

8. Given $s_n = 1+q+q^2+\ldots+q^n$

$$S_n = 1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+\ldots+\left(\frac{q+1}{2}\right)^n q \neq 7$$

prove that

$$^{n+1}C_1+{}^{n+1}C_2 s_1+{}^{n+1}C_3 s_2+\ldots+{}^{n+1}C_{n+1} s_n = 2^n S_n \qquad 4$$

9. With usual notation, if in a triangle $ABC$,

$$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}; \text{ then prove that}$$

$$\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25} \qquad 4$$

10. Two equal sides of an isosceles triangle are given by the equations $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side.

11. The abscissae of the two points $A$ and $B$ are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $x^2+2px-q^2=0$. Find the equation and the radius of the circle with $AB$ as the diameter.

12. If $a$ be a repeated root of a quadratic equation $f(x)=0$ and $A(x)$, $B(x)$, and $C(x)$ be polynomials of degrees 3, 4 and 5 respectively, then show that

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix}$$

is divisible by $f(x)$, where prime denotes the derivative. 4

13. Find the derivative with respect to $x$ of the function

$$(\log_{\cos x} \sin x)\left(\log_{\sin x} \cos x\right)^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ at } x=\frac{\pi}{4}$$ 4

14. Find the equation of the normal to the curve $x^2=4y$ which passes through the point (1, 2).

15. Find the coordinates of the point on the curve $y=\dfrac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.

16. Evaluate the following:

(i) $\displaystyle\int_0^{1/2} \frac{x \sin^{-1}x}{\sqrt{1-x^2}}\,dx$ ; (ii) $\displaystyle\int \frac{dx}{x^2(x^4+1)^{3/4}}$ 4

17. Find the area of the region bounded by the $x$-axis and the curves defined by

$$y=\tan x, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

$$p=\cot x, \quad \frac{\pi}{6} \leqslant x \leqslant \frac{3\pi}{2}$$ 4

18. Give a function $f(x)$ such that

(i) it is integrable over every interval on the real line, and

(ii) $f(t+x)=f(x)$, for every $x$ and a real $t$ then show that the integral $\displaystyle\int_a^{a+t} f(x)\,dx$ is independent of $a$.

19. In a certain city only two newspapers $A$ and $B$ are published. It is known that 25% of the city population reads $A$ and 20% reads $B$ while 8% reads both $A$ and $B$. It is also known that 30% of those who read $A$ but not $B$ look into advertisements and 40% of those who read $B$ but not $A$ look into advertisements while 50% of those who read both $A$ and $B$ look into advertisements. What percentage of the population reads an advertisement? 4

# MODEL SOLUTIONS

## PHYSICS

### PART A

**Ans. 1.**

(i) [A] $1/RC$, [B] $R/L$, [C] $\frac{1}{\sqrt{LC}}$

(ii) [C] $t^{3/2}$

(iii) [B] $\lambda = \pi y_0/2$

(iv) [A] $\lambda = b^2/d$, [C] $\lambda = \frac{b^2}{3d}$

(v) [D] Execute oscillatory but not simple harmonic motion.

(vi) [B] ${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + 2\text{MeV}$

[C] ${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + 7.3$ MeV

(vii) [A] The radius of the $n$th orbit is proportional to $n^2$

[C] The angular momentum of the electron in an orbit is an integral multiple of $\frac{h}{2\pi}$

[D] The magnitude of the potential energy of the electron in any orbit is greater than its kinetic energy

(viii) [A] A diode can be used as a rectifier.

[D] The linear portion of the $I$-$V$ characteristic of a triode is used for amplification without distortion

(ix) [A] $H_2$

**Ans. 2.**

(i) $1.2 \times 10^{-3}$

(ii) $\frac{2}{3}mg$

(iii) 5

(iv) $d/v$

(v) 4 cal deg$^{-1}$ mol$^{-1}$ or $2R$ or 16.6 J deg$^{-1}$ mol$^{-1}$

(vi) 1/8

(vii) $\frac{\epsilon_0 AV}{d}$, $\frac{-2\epsilon_0 AV}{d}$ or $\frac{K\,AV}{4\pi d}$, $\frac{-KAV}{2\pi d}$

(viii) $B$

(ix) frequency

(x) $D$, $B$

**Ans. 3.**

(i) True

The sum of kinetic and potential energies is conserved throughout the parabolic path of the projectile. The vertical component of velocity is zero and potential energy is maximum at the top of its path. Its kinetic energy and, therefore, its velocity is minimum at the top of its path.

(ii) True

The tension $T$ in the string of length $r$ moving with velocity $v$ at angle $\theta$ is given by:

$$T = mg\cos\theta + \frac{mv^2}{r}$$

Referring to Fig. 10 for $\theta = 40°$, $v = 0$; For $\theta = 20°$,

Let $T(20°)$ be tension at 20°

$\surd\, v \neq 0$

$$T(20°) - mg\cos 20° = \frac{mv^2}{r} > 0$$

$\therefore \quad T(20°) > mg\cos 20°$

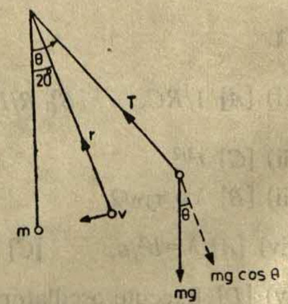



Fig. 10

(iii) False

In order to put an artificial satellite into an orbit such that it will always remain directly over a location, it should relate with the same time-period as that of the earth in the equilateral plane. As New Delhi does not lie on the equatorial plane, a geostationary satellite cannot be used for New Delhi.

(iv) False

Let $f$ be the acceleration in the case in Fig. 11 (a)

$$T - mg = mf \quad \text{(i)}$$

$$2mg - T = 2mf \quad \text{(ii)}$$

Adding Eqs. (i) and (ii),

$$mg = 3mf$$

or $\quad f = g/3$

Let $f'$ be the acceleration in the case in Fig. 11 (*b*)

$\therefore \quad T' - 2mg = 0$

or $\quad T' = 2mg$

$$T' - mg = mf'$$

o. $\quad 2mg - mg = mf'$

or $\quad mg = mf'$, i.e. $f' = g$

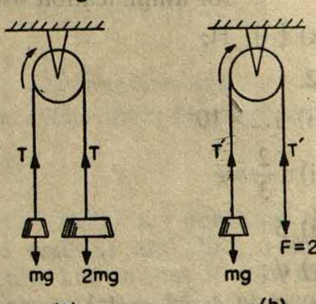



Fig. 11

$\therefore$ Acceleration in case (b) is greater than that in case (a).

(v) False

The refractive index $\eta$ of any medium is given by:

$$\eta = \frac{\text{Velocity of sound in air}}{\text{Velocity of sound in medium}}$$

$$\eta \text{ (for water)} = \frac{330}{1400} = 0.235$$

Let $C$ be the critical angle such that the angle of refraction $r$ is equal to 90°. Then

$$\frac{\sin C}{\sin 90^\circ} = \eta$$

or $$C = \sin^{-1}(\eta) = \sin^{-1}(0.235) = 13.5^\circ$$

As the angle of incidence is 60° and exceeds the critical angle, the sound waves will be internally reflected in air,

(vi) True

No interference pattern will be observed on the screen as two sodium lamps emitting light of the same wavelength will not be coherent and will not have constant phase difference.

## PART B

**Ans. 4(a)**

(i) As body $C$ of mass $m$ collides elastically with body $A$ of mass $m$, with velocity $v_0$, velocity of $A$ is equal to $v_0$.

Let $v$ be common velocity of $A$ as well as $B$ (Fig. 12) after collision. Then conservation of momentum requires that

$$mv_0 = mv + 2mv$$

or $$v_0 = 3v$$

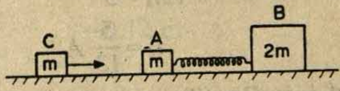



Fig. 12

$\therefore$ Common velocity of $A$ and $B$ at time $t_0 = v = \dfrac{v_0}{3}$

(ii) Initial energy $= \frac{1}{2}mv_0^2$

Let $k$ be the spring constant.

Final energy $= \frac{1}{2}mv^2 + \frac{1}{2}kx_0^2 + \frac{1}{2}(2m)v^2$

As energy must be conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx_0^2 + \frac{1}{2}(2m)v^2$

or $$mv_0^2 = mv^2 + kx_0^2 + 2mv^2$$

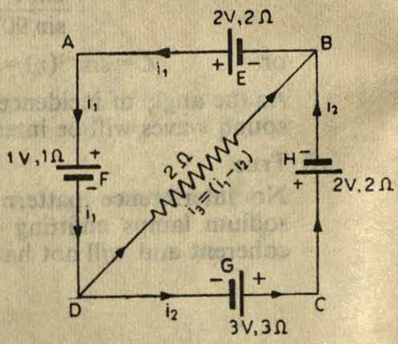
$$\therefore \quad kx_0^2 = mv_0^2 - 3mv^2 = mv_0^2 - \frac{3mv_0^2}{9}$$

$$= \frac{2}{3}mv_0^2$$

or $\quad$ Spring constant $k = \dfrac{2mv_0^2}{3kx_0^2}$

**Ans. 4(b)**

Let $i_1$ be the current flowing from $B$ to $A$ and $A$ to $D$ in the anti-clockwise direction (Fig. 13). Let $i_2$ be the current flowing from $D$ to $C$ and $C$ to $B$ in the anti-clockwise direction. Let $i_3$ be current flowing from $D$ to $B$. Therefore, according to Kirchhoff's law, at point $D$




ig. 13

$$i_1 = i_2 + i_3$$

or $\quad i_1 - i_2 = i_3$

Applying Kirchhoff's law to mesh $ABD$:

$$2i_1 + 1i_1 + 2(i_1 - i_2) = 2 - 1 = 1$$

or $\quad 5i_1 - 2i_2 = 1 \quad$ (i)

For the mesh $DCB$:

$$3i_2 + 1i_2 - 2(i_1 - i_2) = 3 - 1 = 2$$

or $\quad 6i_2 - 2i_1 = 2 \quad$ (ii)

Multiplying Eq. (i) by 3,

$$15i_1 - 6i_2 = 3 \quad \text{(iii)}$$

Adding Eqs. (ii) and (iii),

$$13i_1 = 5$$

or $\quad i_1 = \dfrac{5}{13}$ A

Substituting the value of $i_1$ in Eq. (ii),

$$6i_2 - 2 \times \frac{5}{13} = 2$$

or $\quad 6i_2 = 2 + \dfrac{10}{13} = \dfrac{36}{13}$

or $\quad i_2 = \dfrac{6}{13}$ A

The current through $DB = i_1 - i_2 = \dfrac{5}{13} - \dfrac{6}{13} = -\dfrac{1}{13}$ A

$\therefore$ Potential difference between $D$ and $B = 2 \times -\dfrac{1}{13} = -\dfrac{2}{13}$ V

or Potential difference between $B$ and $D = \dfrac{2}{13}$ V

Potential difference across the terminals of $G = 3 - \frac{6}{13} \times 3$

$$= \frac{21}{13}\ \text{V}$$

Potential difference across the terminals of $H = 1 + \frac{6}{13} \times 1 = \frac{19}{13}\ \text{V}$

**Ans. 5(a)**

If $m$ is the mass per unit length of the rope and $T$ the tension at any point, the velocity $v$ of the wave is given by

$$v = \sqrt{\frac{T}{m}}$$

The tension at the lower end of the rope $= 2g$ N.

Let $\nu$ be the frequency of the transverse pulse,

$$v = \nu\lambda$$

$$= \nu \times 0.06 = \sqrt{\frac{2g}{m}}$$

or
$$\nu = \frac{1}{0.06}\sqrt{\frac{2g}{m}}$$

The tension at the upper end of the rope is equal to $(2+6)g = 8g$ N.

Let $\lambda'$ be the new wavelength and $v'$ the new velocity of the wave when it reaches the top of the rope.

$$v' = \nu\lambda' = \sqrt{\frac{8g}{m}}$$

$$\therefore \quad \lambda' = \frac{1}{\nu}\sqrt{\frac{8g}{m}}$$

$$= 0.06\sqrt{\frac{m}{2g}} \times \sqrt{\frac{8g}{m}}$$

$$= 0.12\ \text{m}$$

**Ans. 5(b)**

Let (Fig. 14) $O$, $I$ and $C$ be the respective positions of object, image and centre of plane face of the lens. When the plano-convex lens is placed with its curved surface in contact with horizontal table, refractive index,

$$\eta = \frac{OC}{IC} = \frac{\text{Real thickness}}{\text{Apparent thickness}} = \frac{4}{3}$$

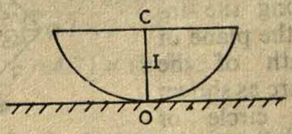



Fig. 14

When the plano-convex lens is placed with its plane face in contact with the table (Fig. 15), the distance of the object $O'$ from middle point $C'$, $O'C' = u = +4$ cm.

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$$\therefore \quad kx_0^2 = mv_0^2 - 3mv^2 = mv_0^2 - \frac{3mv_0^2}{9}$$

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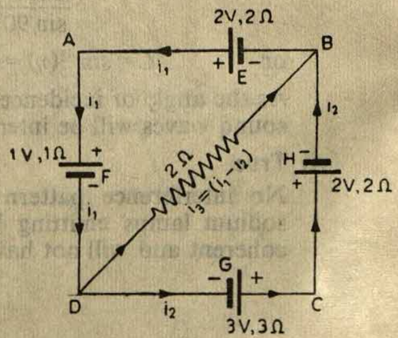


ig. 13

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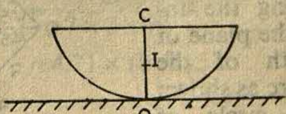



**Fig. 14**

When the plano-convex lens is placed with its plane face in contact with the table (Fig. 15), the distance of the object $O'$ from middle point $C'$, $O'C' = u = +4$ cm.

The distance of image, $I'$ from $C'$, $I'C' = v = -\frac{25}{8}$ cm

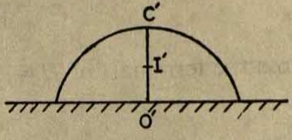



Fig. 15

Let $R$ he the radius of curvature of the curved surface. Therefore,

$$\frac{1}{v} + \frac{\eta}{u} = \frac{\eta - 1}{R}$$

$$-\frac{8}{25} + \frac{4}{3 \times 4} = \frac{\frac{4}{3} - 1}{R}$$

or
$$\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$$

i.e.
$$R = 25 \text{ cm}.$$

Let $f$ be the focal length of the lens.

$$\therefore \quad \frac{1}{f} = (\eta - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1 = \infty, \; R_2 = 25 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{25}\right) = \frac{1}{75}$$

or
$$f = 75 \text{ cm}$$

**Ans. 6(a)**

(i) $m = 1.6 \times 10^{-27}$ kg

$q = 1.6 \times 10^{-19}$ C

$B = 1$ T

$v = 10^7$ ms$^{-1}$

If the direction of the magnetic field is directed along the inward normal to the plane of the paper, the path of the particle will be an arc as shown in Fig. 16, of a circle of radius $OE = R$.

$\therefore$ $AE$ is tangential to the arc and $\angle OEA = 90°$

$\therefore \quad \angle COE = 45°$

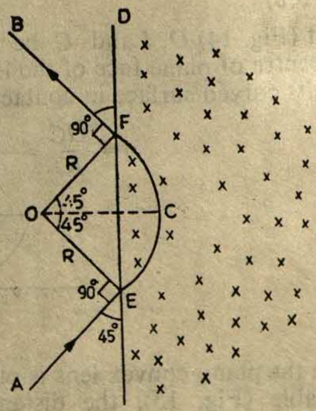



Fig.16

Also $BF$ is a tangent to the arc $CF$ at point $F$.

$\therefore$ $\angle BFD = \theta = 45°$

Also $\angle FOC = 45°$

(As the angle between lines $DF$ and $FB$ is equal to angle between then perpendiculars)

$\therefore$ $EF = 2R \cos 45°$

where
$$R = \frac{mv}{Bq}$$
$$= \frac{1.6 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19} \times 1}$$
$$= 10^{-1} = 0.1 \text{ m}$$

or
$$EF = 2 \times 0.1 \times \frac{1}{\sqrt{2}}$$
$$= 0.141 \text{ m}$$

(ii) If the direction of the magnetic field is directed along the outward normal to the paper the path $EGH$ of the particle in the region of the magnetic field will be a circular one in the clockwise direction with radius $R$ as shown in Fig. 17.

$\therefore$ $\angle EO'H = 90°$

as radii $O'E$ and $O'H$ are perpendicular to the respective tangents drawn at points $E$ and $H$.

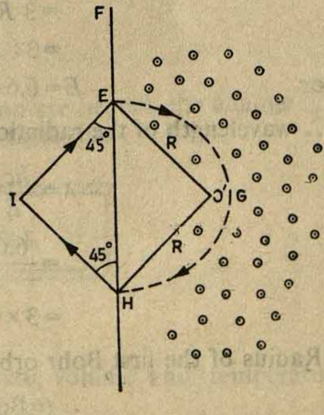


Fig. 17

Length of arc
$$EGH = 2\pi R - \frac{\pi R}{2}$$
$$= \frac{3\pi R}{2}$$

Time spent by particle along arc $EGH$ in the magnetic field
$$= \frac{3\pi R}{2v}$$
$$= \frac{3 \times 22 \times 0.1}{7 \times 2 \times 10^7}$$
$$= 4.71 \times 10^{-8} \text{ s.}$$

**Ans. 6(b)**

The ionization energy $E$ of a hydrogen-like Bohr atom of charge $Z$ is given by:
$$E = -RZ^2 = -\frac{2\pi^2 k^2 Z^2 m e^4}{n^2 h^2}$$

where Rydberg constant

$$R=\frac{2\pi^2k^2me^4}{n^2h^2}$$

$$=2.2\times10^{-18}\ \text{J}$$

As ionization energy

$$E=4\ \text{Rydberg}$$

$$=4\ R$$

$\therefore$ $$4R=RZ^2$$

or $$Z=2.$$

(i) Energy of radiation emitted $E$ when the electron jumps from the first excited state to the ground state is given by:

$$E=RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$=4R\left(1-\frac{1}{4}\right)$$

$$=3\ R$$

$$=3\times2.2\times10^{-18}\ \text{J}$$

or $$E=6.6-10^{-18}\ \text{J}$$

$\therefore$ wavelength of the radiation emitted,

$$\lambda=\frac{hc}{E}$$

$$=\frac{6.6\times10^{-34}\times3\times10^{8}}{6.6\times10^{-18}}$$

$$=3\times10^{-8}\ \text{cm}.$$

(ii) Radius of the first Bohr orbit

$$=\frac{\text{Bohr radius of hydrogen atom}}{Z}$$

$$=\frac{5\times10^{-11}}{2}\ \text{m}$$

$$=2.5\times10^{-11}\ \text{m}$$

**Ans. 7(a)**

Let $L$ be the length, $d$ the specific-gravity and $a$ the area of cross-section of the plank. The gravitational force $F_1$ acting downwards at a distance $AG=L/2$ from the hinge is given by (Fig. 18):

$$F_1=aL\,dg$$

Let $l$ be the length of the plank in water. The buoyant force $F_2$ acting upwards at a distance $AC=l/2$ from the hinge is given by:

$$F_2=al\,d_1g$$

where $d_1$ is the density of water.

Torque on the plank due to gravitational force $= aL\,dg\,L/2 \sin\theta$

Torque on the plank due to buoyant force $= al\,d_1 g\,l/2 \sin\theta$.

Since the plank is in equilibrium:

$$aL\,dg\,\frac{L}{2}\sin\theta = al\,d_1 g\frac{l}{2}\sin\theta$$

or $$L^2 d = l^2 d_1$$

or $$l = L\sqrt{\frac{d}{d_1}} = 1\times\sqrt{0.5}$$

$$=\sqrt{0.5}$$

$\therefore$ $$\cos\theta = \frac{h}{l} = \frac{0.5}{\sqrt{0.5}} = \sqrt{0.5}$$

$$=\frac{1}{\sqrt{2}}$$

i.e. $$\theta = 45^\circ.$$

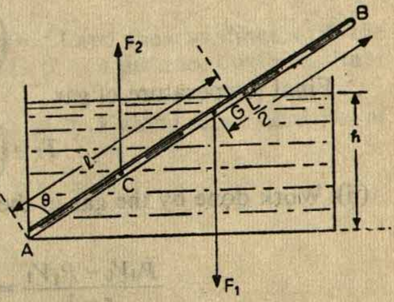



Fig. 18

*Alternative*

**Ans. 7(a)**

As the walls of the box and the partition are heated, the change is adiabatic when the pressure increases on the right side.

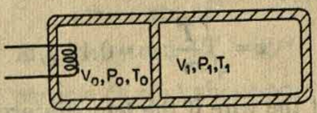



Fig. 19

(i) Let $P_1$, $V_1$ and $T_1$ be the new pressure, volume and temperature respectively of the right chamber (Fig. 19).

$\therefore$ Under adiabatic conditions of change

$$P_0 V_0^r = P_1 V_1^r$$

$$P_1 = \frac{243}{32} P_0,\quad r = \frac{5}{3}$$

$\therefore$ $$P_0 V_0^r = \frac{243}{32} P_0 V_1^r$$

or $$V_1^r = \frac{32}{243} V_0^r$$

or $$V_1 = \left(\frac{32}{243}\right)^{1/r} V_0$$

$\therefore$ $$V_1 = \left(\frac{32}{243}\right)^{3/5} V_0 = \left(\frac{2}{3}\right)^3 V_0 = \frac{8}{27} V_0$$

Also $$T_0 V_0^{r-1} = T_1 V_1^{r-1}$$

or
$$T_1=\left(\frac{V_0}{V_1}\right)^{r-1} T_0$$
$$=\left(\frac{V_0}{8/27\ V_0}\right)^{5/3-1} T_0$$

$\therefore$ Final temperature of gas
$$T_1=\left(\frac{27}{8}\right)^{2/3} T_0=\frac{9}{4}\ T_0$$

(ii) Work done by the gas in the right chamber
$$\frac{P_0V_0-P_1V_1}{r-1}=\frac{P_0V_0-\left(\frac{243}{32}P_0\times\frac{8}{27}V_0\right)}{\frac{5}{3}-1}$$
$$=\frac{P_0V_0\left(1-\frac{9}{4}\right)}{\frac{2}{3}}=-P_0V_0\times\frac{5}{4}\times\frac{3}{2}$$
$$=-\frac{15}{8}P_0V_0$$

The negative sign indicates that the work is done on the gas.

**Ans. 7(b)**

If $T$ is the tension in the wire of mass $m$ per unit length, the velocity $v$ of transverse waves is given by:
$$v=\sqrt{\frac{T}{m}},\ m=0.1\ \text{kg/m}$$

The decrease in length of the wire if the temperature decreases by
$$20^\circ\text{C}=\Delta L=\alpha(T_1-T_2)$$
$$=1.21\times10^{-5}\times20$$
$$=2.42\times10^{-4}\ \text{m}$$

As
$$Y=\frac{\text{stress}}{\text{strain}}=\frac{\text{Tension}}{\text{area}\times\frac{\Delta L}{L}}$$

$\therefore$ Tension,
$$T=Y\times\text{area}\times\frac{\Delta L}{L}$$
$$=2\times10^{11}\times10^{-6}\times2.42\times10^{-4}$$
$$=48.4\ \text{N}$$

$\therefore$
$$v=\sqrt{\frac{48.4}{0.1}}=\sqrt{484}=22\ \text{m/s}$$

For fundamental mode of vibration $L=\lambda/2$

$\therefore$
$$f=\frac{v}{\lambda}=\frac{v}{2L}$$
$$=\frac{22}{2\times1}$$
$$=11\ \text{cycles/s}$$

*Alternative*

**Ans. 7(b)**

Referring to Fig. 20 let $P$ be the origin (0, 0) and the coordinates of the particle after time $t$ be $(x, y)$ when it reaches $Q$ at a distance $r$ with resultant velocity $V$.

The rectangular components $V_x$, $V_y$ of velocity $V$ along the $x$- and $y$-axes are:

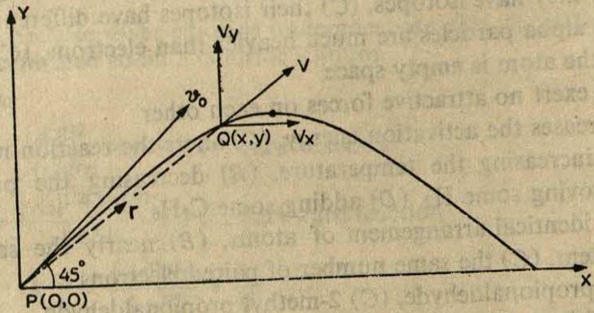



Fig. 20

$$V_x = v_0 \cos 45°$$
$$V_y = v_0 \sin 45° - gt$$

The coordinates $(x, y)$ of point $Q$ after time $t$ are given by:

$$x = v_0 t \cos 45°$$
$$y = v_0 t \sin 45° - \tfrac{1}{2}gt^2$$

Angular momentum of particle $Q(x, y)$ about point $P(0, 0)$

$$= mV_y x - mV_x y$$
$$= m(v_0 \sin 45° - gt)\, v_0 t \cos 45° - m v_0 \cos 45° \,(v_0 t \sin 45° - \tfrac{1}{2}gt^2)$$
$$= m[v_0^2 t \sin 45° \cos 45° - v_0 g t^2 \cos 45° - v_0^2 t \sin 45° \cos 45°$$
$$+ \tfrac{1}{2} v_0 g t^2 \cos 45°]$$
$$= m\left[-\frac{1}{2} v_0 g t^2 \cos 45°\right] = -\frac{m}{2\sqrt{2}} v_0 g t^2 \left(\because\ t = \frac{v_0}{g}\right)$$
$$= -\frac{m}{2\sqrt{2}} v_0 g \frac{v_0^2}{g^2}$$
$$= -\frac{1}{2\sqrt{2}} \frac{m v_0^3}{g}$$

Magnitude of angular momentum of the particle about point $P$

$$= \frac{1}{2\sqrt{2}} \frac{m v_0^3}{g}$$

The direction of angular momentum will be mutually perpendicular to distance **r** and momentum vector. The direction will be along the inward normal to the plane (⊙) of the paper.

# CHEMISTRY

## PART A

**Ans. 1**

(i) (*B*) $^{77}_{33}As$, (*D*) $^{78}_{34}Se$

(ii) (*A*) they have isotopes, (*C*) their isotopes have different masses

(iii) (*A*) alpha particles are much heavier than electrons, (*C*) most part of the atom is empty space

(iv) (*B*) exert no attractive forces on each other

(v) decreases the activation energy, (*C*) alters the reaction mechanism

(vi) (*A*) increasing the temperature, (*B*) decreasing the pressure, (*C*) removing some $H_2$, (*D*) adding some $C_2H_6$

(vii) (*A*) identical arrangement of atoms, (*B*) nearly the same energy content, (*C*) the same number of paired electrons

(viii) (*A*) propionaldehyde, (*C*) 2-methyl propionaldehyde

(ix) (*A*) 2-hydroxy propane, (*B*) acetophenone

(x) (*A*) ethyl chloride, (*D*) benzaldehyde.

**Ans. 2**

(i) (*D*) either alpha or beta rays

(ii) (*D*) $n$, $\alpha$, $p$, $e$

(iii) (*B*) two orbitals at 180°

(iv) (*A*) 5, 0, 0, $+\frac{1}{2}$

(v) (*D*) 1*s*

(vi) (*B*) 8/9

(vii) (*D*) Kinetic energy

(viii) (*A*) $K_p$ does not change significantly with pressure

(ix) (*B*) Coating zinc with copper

(x) (*C*) Ag, Hg, Cu

(xi) (*D*) charge on one mole of electrons

(xii) (*B*) $Na^+$

(xiii) (*A*) 4

(xiv) (*C*) $1.0 \times 10^{10}$

(xv) (*C*) 3, 3-dimethyl-1-butene

(xvi) (C) neutral permanganate solution

(xvii) (*A*) catalytic reduction of carbon monoxide in the presence of $ZnO$-$Cr_2O_3$

(xviii) (*D*) 2, 4, 6-tribromophenol

(xix) (*B*) trihalogenated methane and a primary amine

(xx) (*D*) nitrous acid followed by heating with cuprous chloride.

**Ans. 3(a)**

Asbestos—silicates of (Ca + Mg)—carcinogen
Fluorocarbons—refrigeration—propellent
Nitric oxide—paramagnetic—air pollutant

Lithium metal—reducing agent—electron donor
Zeolites—molecular sieve—ion exchanger
Zinc oxide—semiconductor—fluorescent paint

**Ans. 3(b)**

(i) *False*

Because the hydration energy of $F^-$ is more.

(ii) *True*

It is because silver chloride undergoes complex formation in very concentrated sodium chloride solution.

(iii) *False*

$$K=\frac{[AB]^2}{[A_2][B_2]} \text{ for the forward reaction}$$

$$K'=\frac{[A_2]^{1/2}[B_2]^{1/2}}{[AB]} \text{ for the backward reaction}$$

$$\text{or} \quad (K')^2=\frac{[A_2][B_2]}{[AB]}=\frac{1}{K}$$

$$\text{Clearly } K'\neq\frac{1}{K} \quad \text{but} \quad (K')^2=\frac{1}{K}$$

(iv) *True*

It is because in $NH_3$, there is $sp^3$ hybridization whereas no hybridization occurs in $AsH_3$.

(v) *True*

Because the liquid absorbs internal energy when it evaporates, therefore, the temperature of the liquid falls.

**Ans. 4(a)**

(i) 900 (ii) amphoteric
(iii) resonance stabilization (iv) $Fe^{2+}$ ion
(v) *R*

**Ans. 4(b)**

(i) provided they do not form azeotropes
(ii) when pressure is kept constant
(iii) provided a reducing agent is used
(iv) if the impurity has lower melting point
(v) if any peroxide is present
(vi) provided some replaceable acidic hydrogen can be used

## PART B

**Ans. 5(a)**

Energy of an electron in the $n^{th}$ orbit of hydrogen atom is given by:

$$E=-21.7\times10^{-12}\times\frac{1}{n^2}\text{ ergs}$$

$$\therefore \quad \Delta E = -21.7\times10^{-12}\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)$$

$$= -21.7\times10^{-12}\times\frac{1}{4}$$

$$= -5.42\times10^{-12}\text{ ergs}$$

But $\Delta E = h\nu$, where $h$ is Planck's constant and $\nu$ frequency of the radiation

$$\text{or} \quad |\Delta E| = h\frac{c}{\lambda}$$

$$\therefore \quad \lambda = \frac{hc}{|\Delta E|}$$

$$= \frac{6.627\times10^{-27}\times3\times10^{10}}{5.42\times10^{-12}}$$

$$= 3.67\times10^{-5}\text{ cm}$$

**Ans. 5(b)**

Half-life of the wood sample, $t_{1/2}=5770$ years. Since the radioactive decay is a first order process, half-life is treated with the radioactive (rate) constant given by:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5770}$$

$$= 1.2\times10^{-4}\text{ year}^{-1}$$

After 5770 years, only half of the given sample will be left and after another 5770 years $\frac{1}{2}\times\frac{1}{2}$ of the given sample is left, i.e. 25% of the original sample will be left.

**Ans. 5(c)**

$$K_{SP}\text{ for AgI} = 8.5\times10^{-17}$$

$$K_{SP}\text{ for }Hg_2I_2 = 2.5\times10^{-26}$$

Since the mixture contains $Ag^+$ (0.1 M) and $Hg_2^{2+}$(0.1 M), therefore, $[I^-]$ required to fully precipitate AgI

$$= \frac{8.5\times10^{-17}}{0.1} = 8.5\times10^{-16}\text{ M}$$

Similarly, $[I^-]$ required to precipitate $Hg_2I_2$

$$= \sqrt{\frac{2.5\times10^{-26}}{0.1}} = 5.0\times10^{-13}\text{ M}$$

First of all, AgI will be precipitated and larger precipitation of AgI can be obtained if more amount of $I^-$ is added. But along with AgI, $Hg_2I_2$ will also be precipitated from the mixture only when the molar concentration of iodide ion, $[I^-]$ approaches $5.0\times10^{-13}$ M. The molar concentration of $Ag^+$, $[Ag^+]$, left when $HgI_2$ begins to precipitate, is given by:

$$\frac{K_{SP}\text{ of AgI}}{[I^-]_{Hg_2I_2}} = \frac{8.5\times10^{-17}}{5.0\times10^{-13}}$$

$$= 1.7\times10^{-4}\text{ M}$$

$$\text{Percentage of } [Ag^+] \text{ which is left} = \frac{1.7 \times 10^{-4}}{0.1} \times 100$$
$$= 0.17\%$$

$$\therefore \text{ Percentage of } Ag^+ \text{ precipitated} = (100 - 0.17)$$
$$= 99.83\%$$

**Ans. 6(a)**

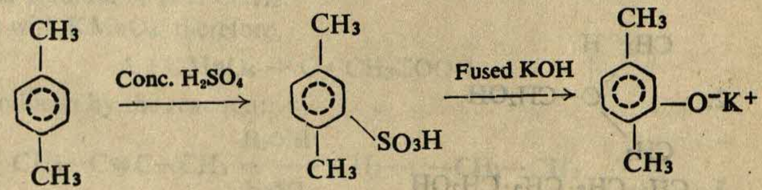
The correct balanced equations are given by:

(i) $Fe_2(SO_4)_3\ (aq.) + 2KI\ (aq.) \rightarrow 2FeSO_4 + K_2SO_4 + I_2$

(ii) $3K_2MnO_4\ (aq.) + 4H^+ \rightarrow 2KMnO_4 + MnO_2 + 2H_2O + 4K^+$

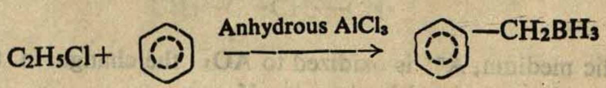
(iii) $2K_2CrO_4\ (aq.) + 2H^+ \rightarrow K_2Cr_2O_7 + H_2O + 2K^+$

(*A*) $CCl_3CHO \xrightarrow{\text{aq. NaOH}} CHCl_3 + H-\overset{\overset{O}{\|}}{C}-\overset{\ominus\oplus}{ONa}$

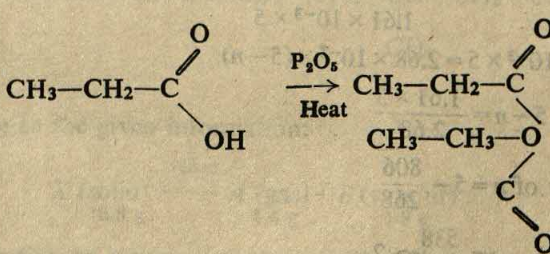
(*B*) 

**Ans. (7a)**

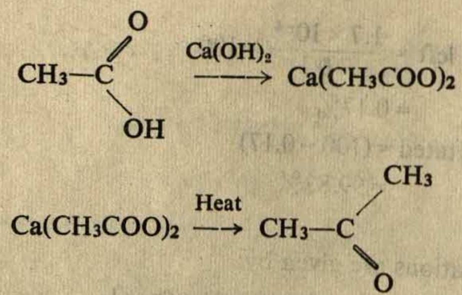
(i) $C_2H_5OH + PCl_5 \rightarrow C_2H_5Cl$



(ii) $CH_3CH_2CHO \xrightarrow{AgNO_3/NH_4OH} CH_3-CH_2-C(=O)-OH$

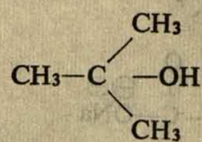


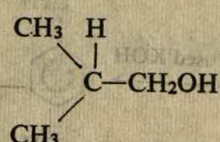
(iii) $CH_3-C(=O)-H \xrightarrow{K_2Cr_2O_7/H^{\oplus}} CH_3-C(=O)-OH$



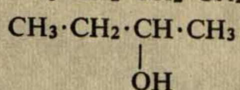
$$NH_2OH \cdot HCl + (CH_3)_2C{=}O \rightarrow (CH_3)_2C{=}N{-}OH$$

**Ans. 7(b)**

1. 

2. 

3. $CH_3 \cdot CH_2 \cdot CH_2 \cdot CH_2OH$

4. 

**Ans. 7(c)**

As in acidic medium, $A^{n+}$ is oxidized to $AO_3^-$, the change in the oxidation state from $(+5)$ to $(-n)$ is given by: $(5-n)$

∴ The total number of electrons that have been given out during the oxidation of $2.68 \times 10^{-3}$ moles of $A^{n+}$ is given by: $2.68 \times 10^{-3} \times (5-n)$

Thus the number of electrons added to reduce $1.61 \times 10^{-3}$ moles of $MnO_4^-$ to $Mn^{2+}$, i.e. $(+7)$ to $(+2)$ is given by:

$$1.61 \times 10^{-3} \times 5$$

∴ $$1.61 \times 10^{-3} \times 5 = 2.68 \times 10^{-3} \times (5-n)$$

or $$5-n = \frac{1.61 \times 5}{2.68}$$

∴, $$\text{Valency of } n = 5 - \frac{806}{268}$$

$$= \frac{538}{268} \simeq 2$$

**Ans. 8(a)**

$$\% \text{ of C} = 85.7$$

∴ $$\text{number of C atoms} = \frac{85.7}{12} = 7.14$$

Similarly, number of H atoms $= \frac{14.3}{1} = 14.3$

Atomic ratio C : H is 7.14 : 14.3

or C : H is 1 : 2

$\therefore$ Empirical formula is $CH_2$.

As the hydrocarbon consumes 1 molar equivalent of hydrogen, it comprises one double bond.

$$1 \text{ g of A consumes } \frac{38.05 \times 5}{100} \text{ g of } Br_2$$

or 160 g of $Br_2$ (1 mole of $Br_2$) is consumed by

$$\frac{160 \times 100}{38.05 \times 5} \text{ g}$$

$$= 84.0 \text{ g of A}$$

$\therefore$ Molecular weight of A = 84.0 g

or Molecular formula of A is $C_6H_{12}$.

As A reacts with $KMnO_4$, therefore,

$$A + KMnO_4 \rightarrow C + CH_3COOH$$

and C is produced by the reaction:

$$CH_3{-}C{\equiv}C{-}CH_3 \xrightarrow[HgSO_4]{H_2O/H^+} CH_3{-}\underset{\underset{O}{\|}}{C}{-}CH_2{-}CH_3$$

Since the molecular formula of C is $C_4H_8O$, C must be

$$CH_3{-}\underset{\underset{O}{\|}}{C}{-}CH_2{-}CH_3$$

$\therefore$ Molecular formula for A is given by:

$$CH_3{-}CH_2{-}\underset{\underset{CH_3}{|}}{C}{=}CH{-}CH_3$$

As $A + H_2 \rightarrow B$, therefore, $B$ is $CH_3{-}CH_2{-}\underset{\underset{CH_3}{|}}{CH}{-}CH_2{-}CH_3$

**Ans. 8(b)**

According to the given information:

$$\underset{16.8\text{ g}}{X\text{ (solid)}} \xrightarrow{\text{Heat}} \underset{4.4\text{ g}}{A\text{ (gas)}} + \underset{1.8\text{ g}}{B\text{ (vapour)}} \qquad \text{(i)}$$

*A* must be $CO_2$ as it turns lime water milky. Since the dissolved residue gives the test for $CO_3^{2-}$, therefore, the given solid is either $MCO_3$ or $M_2CO_3$. Moreover, when *X* is heated it yields a carbonate and $CO_2$. Therefore, it must be a bicarbonate. Since it is quite consistent with the relative masses of *A* and *B*, *B* must be water.

$$2MHCO_3 \rightarrow CO_2 + H_2O + M_2CO_3 \qquad \text{(ii)}$$

Since $2 \times 1/10$ moles of the bicarbonate yields $1/10$ moles of $H_2O$ as is clear from Eq. (i), the molecular weight of $X$ will be equal to:

$$16.8 \times \frac{10}{2} = 84$$

$$\therefore \qquad M + 61 = 84,$$

where $M$ is atomic weight of the metal

$$\text{or} \qquad M = 84 - 61 = 23$$

Thus $M$ is Na and the given salt $X$ is $NaHCO_3$.

**Ans. 9(a)**

The equations involving heats of reactions can be written as:

(*a*) $H_2(g) + 1/2O_2(g) \rightarrow H_2O(l),\ \Delta H = -68.3$ kcal (i)

(*b*) $C_2H_2(g) + 5/2O_2(g) \rightarrow H_2O(l) + 2CO_2(g),\ \Delta H = -310.6$ kcal (ii)

(*c*) $C_2H_4(g) + 3O_2(g) \rightarrow 2H_2O(l) + 2CO_2(g),\ \Delta H = -337.2$ kcal (iii)

By Subtracting Eq. (iii) from sum of Eqs. (i) and (ii), we get:

$$C_2H_2(g) + H_2(g) \rightarrow C_2H_4(g),\ \Delta H = -(310.6 + 68.3 - 337.2) = -41.7 \text{ kcal}$$

$\Delta E$ the heat of reaction for the hydrogenation of acetylene at constant volume is given by:

$$\Delta E = \Delta H - \Delta nRT$$

$\therefore$ moles of the product − moles of the reactant

$$= \Delta n = 1 - (1 + 1) = -1$$

$$\text{or} \qquad \Delta E = -41.7 + 2 \times (273 + 25) \times 10^{-3}$$
$$= -41.7 + 2 \times 298 \times 10^{-3}$$
$$= -41.7 + 0.596$$
$$= -41.104 \text{ kcal}$$

**Ans. 9(b)**

Dissociation of $PCl_5$ can be written as:

$$PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$$

If $x$ is the degree of dissociation, we have

| | | | |
|---|---|---|---|
| Initially : | 3 | 0 | 0 |
| At equilibrium: | $3(1-x)$ | $3x$ | $3x$ |

$\therefore$ Total number of moles at equilibrium $= 3(1-x) + 3x + 3x = 3(1+x)$

Using the gas equation:

$$PV = nRT$$

$$\text{or} \qquad n = \frac{PV}{RT} = \frac{2.05 \times 100}{0.082 \times 500} = 5$$

$$\therefore \qquad 3(1+x)+1=n=5$$
$$\Rightarrow x=0.333$$

Thus the percentage of dissociation of $PCl_5$ is 33.3%.

(ii) $K_p$ for the above reaction can be written as:

$$K_p=\frac{[PCl_3][Cl_2]}{[PCl_5]}$$
$$=\frac{\left[\frac{3xP}{3(1+x)+1}\right]\left[\frac{3xP}{3+1\,(1+x)}\right]}{\left[\frac{3(1-x)}{3(1+x)+1}P\right]}$$

or
$$K_p=\frac{9x^2}{4+3x}\times\frac{1\cdot P}{3(1-x)}$$

Substituting $x=1/3$ and $P=2.05$ atm, we get:

$$K_p=\frac{3x^2\times 2.05}{(4+3x)(1-x)}$$
$$=\frac{3\times\frac{1}{9}\times 2.05}{\left(4+3\times\frac{1}{3}\right)\left(1-\frac{1}{3}\right)}$$
$$=\frac{2.05}{10}=0.205$$

# MATHEMATICS

## PART A

**Ans. 1**

(i) $(C)$ $x^2+y^2=2$

(ii) $(A)$ no root

(iii) $(B)$ $-\frac{1}{2}$

(iv) $(B)$ $\frac{1}{36}$

(v) $(B)$ $[-1, 2]$; $(C)$ $\left[-\frac{1}{2}, 1\right]$

**Ans. 2**

(i) $(A)$ $x=f(y)$; $(D)$ $f$ is a rational function of $x$

(ii) $(C)$ $\frac{1}{8}$

(iii) $(A)$ defined for all real $x$; $(B)$ continuous at $x=0$; $(D)$ differentiable for all $x$

(iv) (*A*) $P(M)+P(N)-2P(M\cap N)$
(*C*) $P(M^c)+P(N^c)-2P(M^c\cap N^c)$
(*D*) $P(M\cap N^c)+P(M^c\cap N)$
(v) (*C*) $a>c>b$
(*D*) $a<c<b$

**Ans. 3**

(i) True (ii) False
(iii) False (iv) False
(v) True

**Ans. 4**

(i) 3050 (ii) $K=2$
(iii) $\lambda=0$ (iv) $\left(\frac{-7}{17}\right)$
(v) (1, −2) (vi) orthocentre
(vii) 205 (viii) $[-2,-1]\cup[1,2]$
(ix) $\frac{2}{\pi}$ (x) $\frac{3}{4}$

## PART B

**Ans. 5 (i)**

As 1, $a_1, a_2, \dots, a_{n-1}$ are the $n$ roots of unity, we get:

$$x^{n-1} \equiv (x-1)(x-a_1)(x-a_2)\cdots(x-a_{n-1})=0$$

or

$$(x-a_1)(x-a_2)\cdots(x-a_{n-1})=\frac{x^n-1}{x-1}$$
$$=x^{n-1}+x^{n-2}+\dots+1$$

Putting $x=1$ on both sides, we get:

$$(1-a_1)(1-a_2)\cdots(1-a_{n-1})=n$$

**Ans. 5 (ii)**

As $a>0$, $b>0$ and $c>0$ and arithmetic mean is greater than or equal to geometric mean, we get:

$$\frac{a+b+c}{3}\geqslant(abc)^{1/3} \qquad \text{(i)}$$

As $1/a$, $1/b$ and $1/c$ are positive integers, their arithmetic mean is greater than or equal to their geometric mean. Therefore,

$$\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}\geqslant\left(\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}\right)^{1/3}$$

$$\Rightarrow\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geqslant3\left(\frac{1}{abc}\right)^{1/3} \qquad \text{(ii)}$$

Multiplying Equations (i) and (ii), we get:

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geqslant9$$

**Ans. 5 (iii)**

It is given that for each value of $i$,

$$a_i \geqslant 1,\ p_i \geqslant 2$$

$$n = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot \ \dots \ \cdot p_k^{a_k}$$

Taking log of both sides, we get:

$$\begin{aligned} \log n &= a_1 \log p_1 + a_2 \log p_2 + a_3 \log p_3 + \dots + a_k \log p_k \\ &= \sum_{i=1}^{k} a_i \log p_i \\ &\geqslant (a_1 + a_2 + a_3 + \dots + a_k) \log_2 \quad (\because\ a_i \geqslant 1 \text{ and } p_i \geqslant 2) \\ &\geqslant K \log_2 \end{aligned}$$

$$\therefore \quad \log n \geqslant K \log_2$$

**Ans. 5 (iv)**

Consider the equation:

$$8^{(1+|\cos x|+\cos^2 x+|\cos^3 x|+\dots)} = 4^3 \qquad \text{(i)}$$

$$\Rightarrow 2^{3(1+|\cos x|+\cos^2 x+|\cos^3 x|+\dots)} = 2^6$$

$$\Rightarrow 1 + |\cos x| + \cos^2 x + |\cos^3 x| + \dots = 2$$

$$\Rightarrow \frac{1}{1-|\cos x|} = 2$$

$$\Rightarrow 1 - |\cos x| = \frac{1}{2}$$

$$\Rightarrow |\cos x| = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, -\frac{2\pi}{3}$$

Thus the values of $x$ in $(-\pi, \pi)$ which satisfy the given equation (i) are $\pm\frac{\pi}{3}$ and $\pm\frac{2\pi}{3}$.

**Ans. 5 (v)**

$$P(AB) = \frac{1}{6} \text{ and } P(A^c B^c) = \frac{1}{3}$$

$$\Rightarrow P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$$

But

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{6} + \frac{2}{3} = \frac{5}{6}$$

As $A$ and $B$ are independent events,

$$P(AB) = P(A)\, P(B) = \frac{1}{6}$$

Thus we have two equations:

$$P(A) + P(B) = \frac{5}{6} \qquad \text{(i)}$$

$$P(A)P(B)=\frac{1}{6} \qquad \text{(ii)}$$

Solving these equations we get:

$$P(A)-P(B)=[P(A)+P(B)]^2-4\,P(A)\,P(B)$$

$$=\frac{25}{36}-4\times\frac{1}{6}$$

$$=\frac{1}{36}$$

or $$P(A)-P(B)=\pm\frac{1}{6} \qquad \text{(iii)}$$

$$\therefore \quad P(A)+P(B)=\frac{5}{6}$$

$$P(A)-P(B)=\frac{1}{6}$$

or $$P(A)=\frac{1}{2}$$

Again,

$$P(A)+P(B)=\frac{5}{6}$$

$$P(A)-P(B)=-\frac{1}{6}$$

or $$P(A)=\frac{1}{3}$$

Thus the values of $P(A)$ are $\frac{1}{2}$ or $\frac{1}{3}$.

**Ans. 6**

As $A$, $B$ and $C$ are the angles of a triangle,

$$A+B+C=\pi$$

$$\cos A+\cos B+\cos C=3/2$$

$$\Rightarrow 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}=\frac{3}{2}-\cos C$$

$$\Rightarrow \quad 2\sin\frac{C}{2}\cos\frac{A-B}{2}=\frac{3-2\cos C}{2}$$

$$\left[\because \cos\frac{A+B}{2}=\sin\frac{C}{2}\right]$$

$$=\frac{1+2(1-\cos C)}{2}$$

$$=\frac{1+2\times 2\sin^2\frac{C}{2}}{2}$$

$$=\frac{1+4\sin^2\frac{C}{2}}{2}$$

or $$4\sin\frac{C}{2}\cos\frac{A-B}{2}=1+4\sin^2\frac{C}{2}$$

$$\Rightarrow \cos\frac{A-B}{2}=\frac{1+4\sin^2\frac{C}{2}}{4\sin\frac{C}{2}}$$

$$=\frac{\left(1+4\sin^2\frac{C}{2}+4\sin\frac{C}{2}\right)-4\sin\frac{C}{2}}{4\sin\frac{C}{2}}$$

$$=\frac{\left(1-2\sin\frac{C}{2}\right)^2}{4\sin\frac{C}{2}}+1$$

$$\therefore\ \cos\frac{A-B}{2}=\frac{\left(1-2\sin\frac{C}{2}\right)^2}{4\sin\frac{C}{2}}+1$$

Since half of any angle in a triangle is always acute, the R.H.S. is greater than one only when

$$1-2\sin\frac{C}{2}=0 \qquad \text{(i)}$$

$$\Rightarrow \cos\frac{A-B}{2}=1 \qquad \text{(ii)}$$

From Eq. (i) $\sin\frac{C}{2}=\frac{1}{2}=\sin 30^\circ$

or $C=60^\circ$. Therefore, $A+B=120^\circ$

From Eq. (ii) $\frac{A-B}{2}=0$ or $A=B=60^\circ$

Thus the given triangle is an equilateral triangle.

**Ans. 7**

We shall prove it by the method of induction.

Let $$f(n)=p^{n+1}+(p+1)^{2n-1} \qquad \text{(i)}$$

For $$n=1$$

$$f(1)=p^2+(p+1)^{2n-1}$$

or $$f(1)=p^2+p+1 \qquad \text{(ii)}$$

It is divisible by $p^2+p+1$.

Let us assume the given expression to be true for any arbitrary value of $(n+1)$.

$$\therefore\ f(n+1)=p^{n+2}+(p+1)^{2n+1}$$

$$=p\cdot p^{n+1}+(p+1)^{2n-1}\cdot(p+1)^2$$

$$=p\cdot p^{n+1}+(p+1)^{2n-1}\cdot(p^2+2p+1)$$
$$=p[p^{n+1}+(p+1)^{2n-1}]+(p^2+p+1)(p+1)^{2n-1}$$
$$=p\cdot f(n)+(p^2+p+1)\cdot(p+1)^{2n-1} \qquad \text{(iii)}$$

It shows that if $f(n)$ is divisible by $(p^2+p+1)$, $f(n+1)$ is also divisible by $(p^2+p+1)$.

Equation (ii) shows that $f(1)$ is divisible by $(p^2+p+1)$.

$\therefore$ Equation (iii) shows that $f(2)=f(1+1)$ is also divisible by $(p^2+p+1)$. Thus $f(n)$ is divisible by $(p^2+p+1)$ for every positive integer $n$.

**Ans. 8**

Given that $\qquad s_n=1+q+q^2+\ldots+q^n$

$$\therefore \quad s_n=\frac{q^{n+1}-1}{q-1}$$

$$\sum_{j=0}^{n} {}^{n+1}C_{j+1}\, s_j=\frac{1}{q-1}\sum_{j=0}^{n} {}^{n+1}C_{j+1}\,(q^{j+1}-1)$$
$$=\frac{1}{q-1}\left[\sum_{j=0}^{n} {}^{n+1}C_{j+1}\, q^{j+1}-\sum_{j=0}^{n} {}^{n+1}C_{j+1}\right]$$
$$=\frac{1}{q-1}[\{(1+q)^{n+1}-1\}-\{2^{n+1}-1\}]$$
$$=\frac{1}{q-1}[(1+q)^{n+1}-2^{n+1}]$$

$$\therefore \qquad \sum_{j=0}^{n} {}^{n+1}C_{j+1}\, s_j=\frac{(1+q)^{n+1}-2^{n+1}}{q-1}$$
$$=2^n\left[\frac{\left(\frac{1+q}{2}\right)^{n+1}-1}{\left(\frac{1+q}{2}\right)-1}\right]$$

Thus, $\qquad \sum_{j=0}^{n} {}^{n+1}C_{j+1}\, s_j=2^n S_n$

$$\left[\because \quad S_n=1+\left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+\ldots+\left(\frac{q+1}{2}\right)^n, q\neq 1\right]$$

**Ans. 9**

Since $a$, $b$ and $c$ are the sides of $\Delta ABC$, and

$$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}, \text{ let}$$
$$\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=x$$
$$\therefore \quad b+c=11x$$
$$c+a=12x$$
$$a+b=13x$$
or $\qquad 2(a+b+c)=36x$

or $$a+b+c=18\,x$$

$$\therefore\quad a=7\,x,\ b=6x\,,\ c=5\,x$$

Consider:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$=\frac{(bx)^2+(5x)^2-(7x)^2}{2(6x)(5x)}$$

$$=\frac{36x^2+25x^2-49x^2}{60x^2}$$

$$=\frac{12x^2}{60x^2}=\frac{1}{5}$$

Similarly,

$$\cos B=\frac{19}{35}\text{ and }\cos C=\frac{5}{7}$$

$$\Rightarrow\ \frac{\cos A}{1/5}=\frac{\cos B}{19/35}=\frac{\cos C}{5/7}$$

$$\Rightarrow\ \frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{35}$$

**Ans. 10**

Equations of equal sides of an isosceles triangle are given by:

$$7x-y+3=0 \tag{i}$$

$$x+y-3=0 \tag{ii}$$

Therefore equations of two bisectors of the angle formed by eqs. (i) and (ii) are given by:

$$\frac{7x-y+3}{\sqrt{(7)^2+(-1)^2}}=\pm\frac{x+y-3}{\sqrt{(1)^2+(1)^2}}$$

$$\Rightarrow\ \frac{7x-y+3}{\sqrt{50}}=\pm\frac{x+y-3}{\sqrt{2}}$$

$$\Rightarrow\ \frac{7x-y+3}{\sqrt{25}}=\pm\frac{x+y-3}{1}$$

$$\Rightarrow\ \frac{7x-3y}{5}=\pm(x+y-3)$$

$$\Rightarrow\ x-3y+9=0\text{ and }3x+y-3=0 \tag{iii}$$

Let $m$ be the slope of line passing through the point $(1,\,-10)$. Therefore, the equation of the line will be:

$$y+10=m\,(x-1) \tag{iv}$$

This the line with slope $m$ is parallel to the bisectors whose equations are given by (iii).

$$\therefore\ m=1/3\text{ and }m=-3$$

Thus the possible equations of the third side are given by:

$$y+10=1/3\,(x-1)$$

or $$3y-x+31=0$$
and $$y+10=-3(x-1)$$
or $$y+3x+7=0$$

**Ans. 11**

Let the two points be $A=(x_1, y_1)$ and $B=(x_2, y_2)$. Therefore, $(x_1, x_2)$ are the roots of $x^2+2ax-b^2=0$ and $(y_1, y_2)$ are the roots of $y^2+2py-q^2=0$

$$\therefore \quad x_1+x_2=-2a \quad \text{and} \quad y_1+y_2=-2p$$
$$x_1x_2=-b^2 \quad \text{and} \quad y_1y_2=-q^2$$

The equation of the circle with the line joining $(x_1, x_2)$ and $(y_1, y_2)$ as diameter is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
i.e. $$\{x^2-(x_1+x_2)\,x+x_1x_2\}+\{y^2-(y_1+y_2)\,y+y_1y_2\}=0$$
or $$x^2-(-2a)\,x-b^2+y^2-(-2p)\,y-q^2=0$$
or $$x^2+2ax-b^2+y^2+2py+q^2=0$$

Thus the required equation of the circle is given by:

$$x^2+y^2+2ax+2py-b^2-q^2=0$$

By adding $a^2+p^2$ to both sides, it can also be written as

$$(x+a)^2+(y+p)^2=b^2+q^2+a^2+p^2$$
or $$\{x-(-a)\}^2+\{y-(-p)^2\}=(\sqrt{b^2+q^2+a^2+p^2})^2$$

Thus the radius of the circle is $\sqrt{b^2+q^2+a^2+p^2}$

**Ans. 12**

Let $$\phi(x)=\begin{vmatrix} A(x) & B(x) & C(x) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix} \qquad \text{(i)}$$

Put $x=a$ on both sides.

$$\therefore \quad \phi(a)=\begin{vmatrix} A(a) & B(a) & C(a) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix}$$

Since the first and second rows are identical,

$$\phi(a)=0$$

Differentiating Equation (i) w.r.t. $x$, we get:

$$\phi'(x)=\begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix}$$

Put $x=a$ on both sides.

$$\phi'(a)=\begin{vmatrix} A'(a) & B'(a) & C'(a) \\ A(a) & B(a) & C(a) \\ A'(a) & B'(a) & C'(a) \end{vmatrix}$$

Since the first and third rows are identical,

$$\phi'(a)=0$$

$$\phi(a)=0 \text{ and } \phi'(a)=0$$

Thus $\phi(x)$, and hence the determinant given by Eq. (i), is divisible by $f(x)$. [$\because$ $A(x)$, $B(x)$ and $C(x)$ are polynomials of degrees 3, 4 and 5 respectively.]

**Ans. 13**

Let $\quad y=(\log_{\cos x}^{\sin x} \log_{\sin x}^{\cos x})^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$=\frac{(\log_{\cos x} \sin x)}{(\log_{\sin x} \cos x)}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$=(\log_{\cos x} \sin x)^2+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

or $$y=\left(\frac{\ln \sin x}{\ln \cos x}\right)^2+\sin^{-1}\left(\frac{2x}{1+x^2}\right) \quad \text{(i)}$$

Differentiate Eq. (i) w.r.t. $x$ we get

$$\frac{dy}{dx}=2\left(\frac{\ln \sin x}{\ln \cos x}\right)\cdot\left\{\frac{\cot x \ln \cos x+\tan x \ln \sin x}{(\ln \cos x)^2}\right\}$$

$$+\frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$\left[\text{For } \frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)\text{, put } x=\tan\theta\text{, we get:}\right.$

$$\left.\frac{d}{dx}\sin^{-1}\sin 2\theta=\frac{d}{dx}2\tan^{-1}x=\frac{2}{1+x^2}\right]$$

$$\therefore \quad \frac{dy}{dx}=2\left(\frac{\ln \sin x}{\ln \cos x}\right)\cdot\left\{\frac{\cot x \ln \cos x+\tan x \ln \operatorname{six} x}{(\ln \cos x)^2}\right\}+\left(\frac{2}{1+x^2}\right)$$

$$\left(\frac{dy}{dx}\right)_{x=\pi/4}=-\frac{8}{\ln 2}+\frac{2}{1+\left(\frac{\pi}{4}\right)^2}$$

$$=-\frac{8}{\ln 2}+\frac{32}{\pi^2+16}$$

**Ans. 14**

Let $(x_1, y_1)$ be the point of intersection of the normal and the curve $x^2=4y$.

$$y=\frac{x^2}{4} \quad \text{(i)}$$

Slope of the tangent to the curve (i) is: $\frac{dy}{dx}=\frac{x}{2}$

$\therefore$ Slope of the normal to the curve (i) at $(x_1, y_1)$ is $\frac{-2}{x_1}$

As the normal passes through (1, 2), its equation can be written as:

$$y-2=\frac{-2}{x_1}(x-1), \quad \text{where } x_1^2=4y_1$$

Since the normal passes through $((x_1, y_1)$, therefore,

$$y_1-2=\frac{-2}{x_1}(x_1-1)$$

or

$$\frac{x_1^2}{4}-2=-2+\frac{2}{x_1}$$

$$\Rightarrow x_1^3=8$$

$$\Rightarrow x_1=2 \text{ and } y_1=1$$

Thus the equation of the normal is:

$$y-2=\frac{-2}{2}(x-1)$$

or

$$x+y-3=0$$

**Ans. 15**

Equation of the curve is given by

$$y=\frac{x}{1+x^2} \tag{i}$$

Slope of tangent $\frac{dy}{dx}=\frac{1+x^2-x(2x)}{(1+x^2)^2}$

$$=\frac{1-x^2}{(1+x^2)^2}$$

Let

$$f(x)=\frac{dy}{dx}=\frac{1-x^2}{(1+x^2)^2}$$

$\therefore$

$$f'(x)=\frac{(1+x^2)^2(-2x)-(1-x^2)\cdot 2(1+x^2)(2x)}{(1+x^2)^4}$$

$$=\frac{\{-(1+x^2)(2x)-4x(1-x^2)\}(1+x^2)}{(1+x^2)^4}$$

$$=\frac{-2x(1+x^2+2-2x^2)}{(1+x^2)^3}$$

or

$$f'(x)=\frac{-2x(3-x^2)}{(1+x^2)^3} \tag{ii}$$

$$f'(x)=0 \Rightarrow x=0, \pm\sqrt{3}$$

Differentiating Eq. (ii) again w.r.t. $x$ we get:

$$f''(x)=\frac{12x^2(3-x^2)}{(1+x^2)^4}-\frac{6(1-x^2)}{(1+x^2)^3}$$

$$f''(0)=-6 \quad \text{and} \quad f''(\pm\sqrt{3})=3/16$$

$$\Rightarrow f''(0)<0 \text{ and } f''(\pm\sqrt{3})>0$$

Putting $x=0$ in Eq. (i) we get $y=0$

Thus the tangent to the curve has the maximum slope at (0, 0).

**Ans. 16(i)**

Let $$I=\int_0^{1/2}\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

Put $x=\sin\theta$

$\therefore$ $dx=\cos\theta\,d\theta$, Limits for $x=0$, $\theta=0$

and for $x=\frac{1}{2}$, $\theta=\pi/6$

Thus,
$$I=\int_0^{\pi/6}\frac{\sin\theta\sin^{-1}(\sin\theta)}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta$$

$$=\int_0^{\pi/6}\theta\sin\theta\,d\theta$$

$$=\Big[\theta(-\cos\theta)\Big]_0^{\pi/6}+\int_0^{\pi/6}1\cdot\cos\theta\,d\theta$$

$$=\Big[-\theta\cos\theta+\sin\theta\Big]_0^{\pi/6}$$

$$=\frac{-\pi}{6}\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}$$

Thus, $$I=\frac{6-\pi\sqrt{3}}{12}$$

**Ans. 16(ii)**

Let $$I=\int\frac{dx}{x^2(x^4+1)^{3/4}}$$

$$=\int\frac{dx}{x^2\cdot x^3(1+x^{-4})^{3/4}}$$

$$=\int\frac{dx}{x^5(1+x^{-4})^{3/4}}$$

Put $1+x^{-4}=t$

$\therefore$ $-4x^{-5}\,dx=dt$

$\therefore$ $$I=\int\frac{-dt}{4t^{3/4}}$$

$$=-\frac{1}{4}\int\frac{dt}{t^{3/4}}$$

$$=-\frac{1}{4}\,\frac{t^{-3/4+1}}{-3/4+1}+C=-t^{1/4}+C$$

Thus, $I=-(1+x^{-4})^{1/4}+C$, where $C$ is a constant of integration.

**Ans. 17**

Since $\tan x$ lies between $-\pi/3\leqslant x\leqslant\pi/3$ and $\cot x$ between $\pi/6\leqslant x\leqslant 3\pi/2$, the required area is given by:

$$A=\int_0^{\pi/4}\tan x\,dx+\int_{\pi/4}^{\pi/2}\cot x\,dx$$

$$= \left| \ln \sec x \right|_0^{\pi/4} + \left| \ln \sin x \right|_{\pi/4}^{\pi/2}$$

$$= \ln \sqrt{2} + 0 - \ln\left(\frac{1}{\sqrt{2}}\right)$$

$$= \ln\left(\frac{\sqrt{2}}{1/\sqrt{2}}\right)$$

$$= \ln 2$$

Thus the required area is ln 2.

**Ans. 18**

The given function $f(x)$ is

(i) integrable over every interval on the real line and (ii) $f(x+t) = f(x)$, for all $x$ and a real $t$. The given integral $\int_a^{a+t} f(x)\,dx$ can be written as:

$$\int_a^{a+t} f(x)\,dx = \int_a^0 f(x)\,dx + \int_0^t f(x)\,dx + \int_t^{a+t} f(x)\,dx$$

Let $\quad I = \int_t^{a+t} f(x)\,dx$

Substitute $\quad y = x - t \quad$ or $\quad dy = dx$

For $\quad x = t, y = 0$

and at $\quad x = a + t, y = a$

$\therefore$

$$I = \int_t^{a+t} f(x)\,dx = \int_0^a f(y+t)\,dy$$

$$= \int_0^a f(y)\,dy$$

$$= \int_0^a f(x)\,dx$$

$$= -\int_a^0 f(x)\,dx$$

$\therefore$

$$\int_a^{a+t} f(x)\,dx = \int_a^0 f(x)\,dx + \int_0^t f(x)\,dx - \int_a^0 f(x)\,dx$$

$$= \int_0^t f(x)\,dx$$

Thus $\int_a^{a+t} f(x)\,dx$ is independent of $a$.

**Ans. 19**

It is given that

$$P(A) = \frac{25}{100} = 0.25$$

$$P(B) = \frac{20}{100} = 0.20$$

$$P(AB) = \frac{8}{100} = 0.08$$

$\therefore$
$$\begin{aligned} P(AB^C) &= P(A) - P(AB) \\ &= 0.25 - 0.08 \\ &= 0.17 \end{aligned}$$

and
$$\begin{aligned} P(A^C B) &= P(B) - P(AB) \\ &= 0.20 - 0.08 \\ &= 0.12 \end{aligned}$$

Let $y$ be the event that a person reads an advertisement. Therefore,

$$\begin{aligned} P(y) &= P(yAB^C) + P(yA^CB) + P(yAB) \\ &= 0.17 \times \frac{30}{100} + 0.12 \times \frac{40}{100} + 0.04 \\ &= 0.051 + 0.048 + 0.04 \\ &= 0.139 \end{aligned}$$

Thus the population that reads advertisements is 13.9%.

# 1985

# QUESTION PAPERS

## PHYSICS

TIME ALLOWED : 3 HOURS

MAXIMUM MARKS : 100

*Notes*: (1) Answers must be written in English.

(2) Figures in brackets on the right-hand margin indicate marks for the questions.

(3) Answers to Part 'A' *must* be given only on the *first six pages* of the answer-book *in the order in which the questions appear in the question paper*.

(4) Answer to each question of Part 'B' should begin on a fresh page.

(5) Answer all parts of a question at one place.

(6) You are advised not to spend more than sixty minutes in answering Part 'A'.

(7) Useful physical data:

Velocity of sound in air = 330 m/sec

Gas constant R = 2 cal/mole K

Velocity of light in vacuum = $3 \times 10^8$ m/sec

Acceleration due to gravity = 9.8 m/sec$^2$

Planck's constant = $6.6 \times 10^{-34}$ joule sec

1 ev = $1.6 \times 10^{-19}$ joule

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

### PART A

1. In each of the statements below, several alternatives are given. In some cases more than one alternative is correct. Select the correct alternative(s) in each case and write down the corresponding letter(s) (A) (B) (C) or (D) in your answer-book. For each part, marks will be awarded only if ALL the correct alternatives are selected.

(i) A uniform chain of length $L$ and mass $M$ is lying on a smooth table and one-third of its length is hanging vertically down over

the edge of the table. If $g$ is acceleration due to gravity, the work required to pull the hanging part on to the table is:

(A) $MgL$ (B) $MgL/3$ (C) $MgL/9$ (D) $MgL/18$

(ii) A mass $M$ is moving with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin:

(A) is zero (B) remains constant
(C) goes on increasing (D) goes on decreasing

(iii) An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is:

(A) 31.25 (B) 62.50 (C) 93.75 (D) 125

(iv) 70 calories of heat are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is:

(A) 30 (B) 50 (C) 70 (D) 90

(v) A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $E$ and $B$ represent the electric and magnetic fields respectively, this region of space may have:

(A) $E=O$ $B=O$ (B) $E=O$ $B\neq O$
(C) $E\neq O$ $B=O$ (D) $E\neq O$ $B\neq O$

(vi) A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with this capacitor are given by $Q_0$, $V_0$, $E_0$ and $U_0$, respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by $Q$, $V$, $E$ and $U$ are related to the previous ones as:

(A) $Q > Q_0$ (B) $V > V_0$ (C) $E > E_0$ (D) $U > U_0$

(vii) For a given plate voltage, the plate current in a triode valve is maximum when the potential of:

(A) the grid is positive and plate is negative
(B) the grid is zero and plate is positive
(C) the grid is negative and plate is positive
(D) the grid is positive and plate is positive

(viii) A rectangular loop carrying a current $i$ is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current $I$ is established in the wire as shown in the figure, the loop will:

(A) rotate about an axis parallel to the wire
(B) move away from the wire
(C) move towards the wire
(D) remain stationary

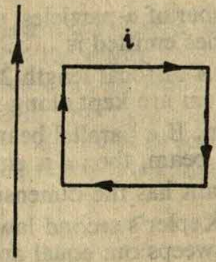



Fig. 1

(ix) The X-ray beam coming from an X-ray tube will be:

(A) monochromatic

(B) having all wavelengths smaller than a certain maximum wavelength

(C) having all wavelengths larger than a certain minimum wavelength

(D) having all wavelengths lying between a minimum and a maximum wavelength

(x) The spring balance *A* reads 2 kg with a block *m* suspended from it. A balance *B* reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation:

(A) the balance *A* will read more than 2 kg

(B) the balance *B* will read more than 5 kg

(C) the balance *A* will read less than 2 kg and *B* will read more than 5 kg

(D) the balances *A* and *B* will read 2 kg and 5 kg respectively

**(2 × 10)**

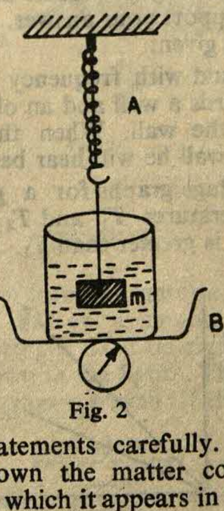



Fig. 2

2. Read the following statements carefully. Indicate the sub-division number and write down the matter corresponding to each blank strictly in the order in which it appears in the statement:

(i) In the Uranium radioactive series the initial nucleus is $^{238}_{92}U$ and the final nucleus is $^{206}_{82}Pb$. When the Uranium nucleus decays to

Lead, the number of $\alpha$-particles emitted is ...... and the number of $\beta$-particles emitted is ......

(ii) A convex lens $A$ of focal length 20 cm and a concave lens $B$ of focal length 5 cm are kept along the same axis with a distance $d$ between them. If a parallel beam of light falling on $A$ leaves $B$ as a parallel beam, then $d$ is equal to ......... cm.

(iii) Planck's constant has the dimensions .........

(iv) According to Kepler's second law, the radius vector to a planet from the sun sweeps out equal areas in equal intervals of time. This law is a consequence of the conservation of .........

(v) The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in solid state at the point $O$. The state of the material at the point $P$ is .........

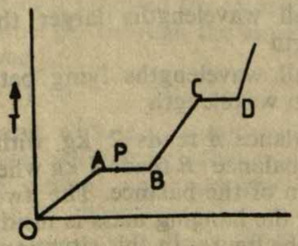



Fig. 3

(vi) A monochromatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ........., its frequency is ......... **(2 × 6)**

3. State whether the following statements are TRUE or FALSE, giving reasons in brief to support your answer. Marks will be awarded only if correct reasons are given:

(i) A source of sound with frequency 256 Hz is moving with a velocity $V$ towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats.

(ii) The current-voltage graphs for a given metallic wire at two different temperatures $T_1$ and $T_2$ are shown in the figure. The temperature $T_2$ is greater than $T_1$.

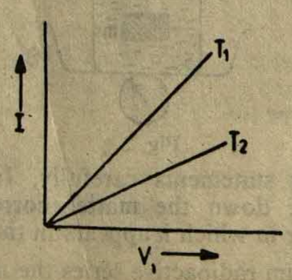



Fig. 4

(iii) Two identical trains are moving on rails along the equator on the earth in opposite directions with the same speed. They will exert the same pressure on the rails.

(iv) The curves $A$ and $B$ in the figure show $P-V$ graphs for an isothermal and an adiabatic process for an ideal gas. The isothermal process is represented by the curve $A$.

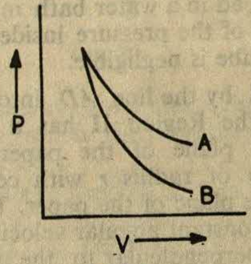



Fig. 5

(v) An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular paths of the same radius.

(vi) A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through $A$ and (b) passing through $B$, by the application of the same force, $F$, at $C$ (midpoint of $AB$) as shown in the figure. The angular acceleration in both the cases will be the same. **(3 × 6)**

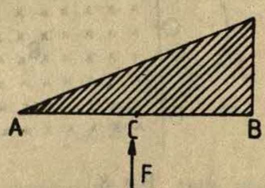



Fig. 6

## PART B

4. (*a*) Two light springs of force constants $k_1$ and $k_2$ and a block of mass $m$ are in one line $AB$ on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance $CD$ between the free ends of the springs is 60 cm. If the block moves along $AB$

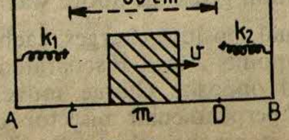



Fig. 7

with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block ($k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m = 200$ g). **(6)**

(*b*) Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. **(6)**

5. (*a*) Space is divided by the line *AD* into two regions. Region I is field free and the Region II has a uniform magnetic field *B* directed into the plane of the paper. *ACD* is a semicircular conducting loop of radius *r* with center at *O*, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity $\omega$ about an axis passing through *O* and perpendicular to the plane of the paper. The effective resistance of the loop is *R*.

(i) Obtain an expression for the magnitude of the induced current in the loop.

(ii) Show the direction of the current when the loop is entering into the Region II.

(iii) Plot a graph between the induced e.m.f. and the time of rotation for two periods of rotation. **(6)**

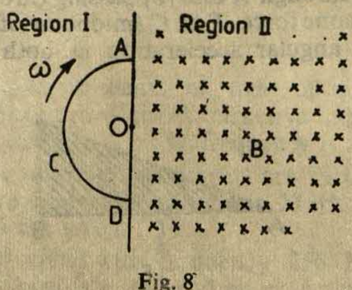



Fig. 8

(*b*) The vibrations of a string of length 60 cm fixed at both ends are represented by the equation:

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\,\pi t)$$

where $x$ and $y$ are in centimeter and $t$ in seconds.

(i) What is the maximum displacement of a point at $x = 5$ cm?

(ii) Where are the nodes located along the string?

(iii) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ sec?

(iv) Write down the equations of the component waves whose superposition gives the above wave. **(6)**

6. (*a*) Two fixed, equal, positive charges, each of magnitude $5 \times 10^{-5}$ C, are located at points *A* and *B* separated by a distance of 6 m. An equal and opposite charge moves towards them along the line *COD*, the perpendicular bisector of the *AB*. The moving charge, when it reaches the point *C* at a distance of 4 m from

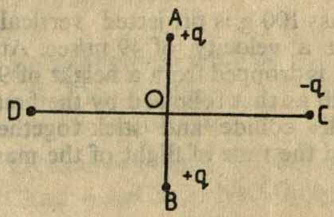



Fig. 9

$O$, has a kinetic energy of 4 joules. Calculate the distance of the farthest point $D$ which the negative charge will reach before returning towards $C$. **(6)**

(*b*) A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass M = 2 kg attached to it at a distance of 1 m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which thc mass M will hit the wall when the mass $m$ is released? **(6)**

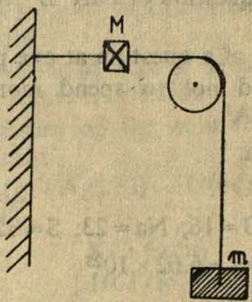



Fig. 10

7. (*a*) Å beam of light consisting of two wavelengths, 6500Å and 5200Å, is used to obtain interference fringes in a Young's double slit experiment:

(i) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500Å.

(ii) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm. **(6)**

*Or*

(*a*) A doubly ionised lithium atom is hydrogen-like with atomic number 3:

(i) Find the wavelength of the radiation required to excite the electron in $Li^{++}$ from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 ev.)

(ii) How many spectral lines are observed in the emission spectrum of the above excited system? **(6)**

(*b*) A ball of mass 100 g is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. (8)

# CHEMISTRY

TIME ALLOWED : 3 HOURS MAXIMUM MARKS : 100

*Notes*: (1) Answers must be written in English.

(2) Figures in brackets on the right-hand margin indicate marks for the question.

(3) Answers to Part 'A' *must* be given only on the *first six pages* of the answer-book *in the order in which the questions appear in the question paper.*

(4) Answer to each question of Part 'B' should begin on a fresh page.

(5) Answer all parts of a question at one place.

(6) You are advised not to spend more than sixty minutes in answering Part 'A'.

(7) Some useful data:

Atomic masses

H = 1; C = 12; O = 16; Na = 23; S = 32; Cl = 35.5; Ag = 108.

Avogadro number = $6.02 \times 10^{23}$

1 Faraday = 96,500 C

$g = 981$ cm/Sec$^2$

Density of mercury = 13.6 $g/cm^3$

## PART A

1. Four alternative answers are given for each sub-question of which only one is correct. Indicate the correct answer.

*Example*: In the nitration of benzene with a mixture of concentrated nitric acid and concentrated sulphuric acid, the active species involved is:

(A) nitrite ion, (B) nitrate ion,
(C) nitronium ion (D) nitric oxide

The correct answer is C.

(i) Bohr model can explain:

(A) the spectrum of hydrogen atom only

(B) spectrum of an atom or ion containing one electron only

(C) the spectrum of hydrogen molecule
(D) the solar spectrum

(ii) The molecule having one unpaired electron is:
(A) NO (B) CO (C) $CN^-$ (D) $O_2$

(iii) An example of a reversible reaction is:
(A) $Pb(NO_3)_2(aq) + 2NaI(aq) = PbI_2(s) + 2NaNO_3(aq)$
(B) $AgNO_3(aq) + HCl(aq) = AgCl(s) + NaNO_3(aq)$
(C) $2Na(s) + H_2O(l) = 2NaOH(aq) + H_2(g)$
(D) $KNO_3(aq) + NaCl(aq) = KCl(aq) + NaNO_3(aq)$

(iv) Rate of diffusion of a gas is:
(A) directly proportional to its density
(B) directly proportional to its molecular weight
(C) directly proportional to the square root of its molecular weight
(D) inversely proportional to the square root of its molecular weight

(v) For a dilute solution, Raoult's law states that:
(A) the lowering of vapour pressure is equal to the mole fraction of solute
(B) the relative lowering of vapour pressure is equal to the mole fraction of solute
(C) the relative lowering of vapour pressure is proportional to the amount of solute in solution
(D) the vapour pressure of the solution is equal to the mole fraction of solvent

(vi) The reaction $\frac{1}{2}H_2(g) + AgCl(s) = H^+(aq) + Cl^-(aq) + Ag(s)$ occurs in the galvanic cell:
(A) Ag | AgCl(s) | KCl (soln) | $AgNO_3$ (soln) | Ag
(B) Pt | $H_2(g)$ | HCl (soln) | $AgNO_3$ (soln) | Ag
(C) Pt | $H_2(g)$ | HCl (soln) | AgCl(s) | Ag
(D) Pt | $H_2(g)$ | KCl (soln) | AgCl(s) | Ag

(vii) The radius of an atomic nucleus is of the order of:
(A) $10^{-10}$ cm (B) $10^{-13}$ cm (C) $10^{-15}$ cm (D) $10^{-8}$ cm

(viii) The best indicator for detection of end point in titration of a weak acid and a strong base is:
(A) methyl orange (3 to 4) (B) methyl red (5 to 6)
(C) bromothymol blue (6 to 7.5) (D) phenolphthalein (8 to 9.6)

*Note*: Figures in the brackets show the pH range of the indicator.

(ix) Nitrogen dioxide cannot be obtained by heating:
(A) $KNO_3$ (B) $Pb(NO_3)_2$ (C) $Cu(NO_3)_2$ (D) $AgNO_3$

(x) The oxide that gives hydrogen peroxide on treatment with a dilute acid is:
(A) $PbO_2$ (B) $Na_2O_2$ (C) $MnO_2$ (D) $TiO_2$

(xi) The molecular formula of Glauber's salt is:
(A) $MgSO_4 \cdot 7H_2O$ (B) $CuSO_4 \cdot 5H_2O$
(C) $FeSO_4 \cdot 7H_2O$ (D) $Na_2SO_4 \cdot 10H_2O$

(xii) A gas that cannot be collected over water is:
(A) $N_2$ (B) $O_2$ (C) $SO_2$ (D) $PH_3$

(xiii) The conjugate acid of $NH_2^-$ is:
(A) $NH_3$ (B) $NH_2OH$ (C) $NH_4^+$ (D) $N_2H_4$

(xiv) Acidic hydrogen is present in:
(A) ethyne (B) ethene (C) benzene (D) ethane

(xv) The compound that will not give iodoform on treatment with alkali and iodine is:
(A) acetone (B) ethanol (C) diethyl ketone (D) isopropyl alcohol

(xvi) Anti-Markownikoff addition of HBr is not observed in:
(A) propene (B) butene (C) but-2-ene (D) pent-2-ene

(xvii) The compound that is not a Lewis acid is:
(A) $BF_3$ (B) $AlCl_3$ (C) $BeCl_2$ (D) $SnCl_4$

(xviii) The compound that is most reactive towards electrophilic nitration is:
(A) toluene (B) benzene (C) benzoic acid (D) nitrobenzene

(xix) Hydrogen gas will not reduce:
(A) heated cupric oxide (B) heated ferric oxide
(C) heated stannic oxide (D) heated aluminium oxide

(xx) Electromagnetic radiation with maximum wavelength is:
(A) ultraviolet (B) radio wave (C) X-ray (D) infrared

**(1 × 20)**

2. (*a*) Give reasons in one or two sentences for each of the following:
   (i) Anhydrous HCl is a bad conductor of electricity but aqueous HCl is a good conductor.
   (ii) o-Nitrophenol is steam volatile whereas p-nitrophenol is not.
   (iii) Formic acid is a stronger acid than acetic acid.
   (iv) Graphite is used as a solid lubricant.
   (v) Fluorine cannot be prepared from fluorides by chemical oxidation.

   **(1 × 5)**

   (*b*) Complete the following sentences:
   (i) The rate of chemical change is directly proportional to.........
   (ii) The terminal carbon atom in butane is.........hybridised.
   (iii) On Mulliken scale, the average of ionization potential and electron affinity is known as.........
   (iv) The number of neutrons in the parent nucleus which gives $^{14}N$ on beta emission is.........
   (v) Sodium dissolved in liquid ammonia conducts electricity because.........

   **(1 × 5)**

3. (*a*) State if the following statements are true or false:
   (i) Kinetic energy of a molecule is zero at 0°C.
   (ii) Carbon tetrachloride is inflammable.
   (iii) All molecules with polar bonds have dipole moment.
   (iv) A gas in a closed container will exert much higher pressure due to gravity at the bottom than at the top.

(v) m-Chlorobromobenzene is an isomer of m-bromochloro-benzene.

(vi) Iodide is a better nucleophile than bromide.

(vii) $SnCl_2$ is a nonlinear molecule.

(viii) Heat capacity of a diatomic gas is higher than that of a monoatomic gas.

(ix) Addition of ammonium chloride to a solution containing ferric and magnesium ions is essential for selective precipitation of ferric hydroxide by aqueous ammonia.

(x) Solubility of sodium hydroxide increases with increase in temperature. **($\frac{1}{2} \times 10$)**

(*b*) Arrange the following in order of their:

(i) Decreasing ionic size

$Mg^{2+}$, $O^{2-}$, $Na^+$, $F^-$

(ii) Increasing acidic property

$ZnO$, $Na_2O_2$, $P_2O_5$, $MgO$

(iii) Increasing first ionization potential

Mg, Al, Si, Na

(iv) Increasing reactivity towards HCN

$CH_3CHO$, $CH_3COCH_3$, $HCHO$, $C_2H_5COCH_3$

(v) Increasing bond length

$F_2$, $N_2$, $Cl_2$, $O_2$ **($1 \times 5$)**

4. Match each item of the right-hand column with an appropriate item in the left hand column for each of the following section:

(a) (A) spinel — (i) $MgAl_2O_4$
(B) felspar — (ii) $PbCO_3$
(C) cerussite — (iii) $KAlSi_3O_8$
(D) malachite — (iv) $MgSO_4 \cdot H_2O$
(E) kisserite — (v) $Cu(OH)_2 \cdot CuCO_3$ **($\frac{1}{2} \times 5$)**

(b) (F) liquid air — (vi) Deacon process
(G) $Na_2CO_3$ — (vii) Parke process
(H) nitric oxide — (viii) Claude process
(I) silver — (ix) Ostwald process
(J) chlorine — (x) Solvay process **($\frac{1}{2} \times 5$)**

(c) (K) phenol — (xi) coloured glass
(L) $Na_2S_2O_3$ — (xii) antichlor
(M) salicylic acid — (xiii) refractory material
(N) quick lime — (xiv) antiseptic
(O) CuO — (xv) analgesic **($\frac{1}{2} \times 5$)**

(d) (P) Aston — (xvi) radium
(Q) Priestley — (xvii) radioactivity
(R) Ramsay — (xviii) oxygen
(S) Marie Curie — (xix) inert gas
(T) Becquerel — (xx) mass spectrum **($\frac{1}{2} \times 5$)**

## PART B

5. (a) Give reason why the ground state outermost electronic configuration of silicon is:

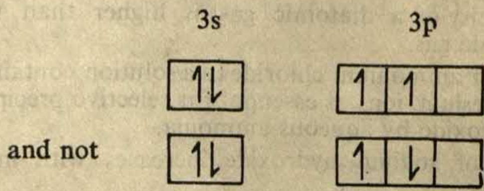



Fig. 11

(b) What is the maximum number of electrons that may be presen in all the atomic orbitals with principal quantum number 3 and azimuthal quantum number 2?

(c) How many sigma bonds and pi bonds are present in a benzene molecule?

(d) Write down the resonance structures of nitrous oxide.

(e) Suggest a reason for the large difference between the boiling points of butanol and butanal, although they have almost the same solubility in water. **(1 × 5)**

6. (a) A compound of molecular formula $C_7H_8O$ is insoluble in water and dilute sodium bicarbonate but dissolves in dilute aqueous sodium hydroxide. On treatment with bromine water, it readily gives a precipitate of $C_7H_5OBr_3$. Write down the structure of the compound. **(1)**

(b) Give a chemical test to distinguish between the following pairs of compounds: **(2)**

(i) 2-butyne and 1-butyne (ii) methanol and ethanol

(c) Write the structure of all the possible isomers of dichloroethene. Which of them will have zero dipole moment? **(2)**

(d) Write down the main product of each of the following reactions:

(i) benzene $\xrightarrow{CH_3CH_2COCl/AlCl_3}$

(ii) propanal $\xrightarrow[\text{heat}]{NaOH}$

(iii) ethanol $\xrightarrow[NaOH]{I_2}$ **(3)**

(e) How would you convert:

(i) aniline to chlorobenzene (ii) acetylene to acetone **(2)**

7. (a) Write down the balanced equations for the reactions when:

(i) a mixture of potassium chlorate, oxalic acid and sulphuric acid is heated

(ii) potassium permanganate interacts with manganese dioxide in presence of potassium hydroxide

(iii) calcium phosphate is heated with a mixture of sand and carbon

(iv) potassium ferrocyanide is heated with concentrated sulphuric acid

(v) ammonium sulphate is heated with a mixture of nitric oxide and nitrogen dioxide. (1 × 5)

(b) What happens when:

(i) hydrogen sulphide is bubbled through an aqueous solution of sulphur dioxide

(ii) aqueous ammonia is added drop-wise to a solution of copper sulphate till it is in excess

(iii) tin is treated with concentrated nitric acid

(iv) $CrCl_3$ solution is treated with sodium hydroxide and then with hydrogen peroxide

(v) $Pb_3O_4$ is treated with nitric acid (1 × 5)

8\. (a) How long a current of 3 ampere has to be passed through a solution of silver nitrate to coat a metal surface of 80 $cm^2$ with a 0.005 mm thick layer? Density of silver is 10.5 $g/cm^3$. (3)

(b) The concentration of hydrogen ions in a 0.2 molar solution of formic acid is $6.4 \times 10^{-3}$ mole/litre. To this solution sodium formate is added so as to adjust the concentration of sodium formate to one mole per litre. What will be the pH of this solution? The dissociation constant of formic acid is $2.4 \times 10^{-4}$ and the degree of dissociation of sodium formate is 0.75. (4)

9\. (a) Calculate the root mean square velocity of ozone kept in a closed vessel at 20°C and 82 cm mercury pressure. (4)

(b) The bond dissociation energies of gaseous $H_2$, $Cl_2$ and HCl are 104, 58 and 103 k cal/mole respectively. Calculate the enthalpy of formation of HCl gas. (2)

(c) The equilibrium constant of the reaction $A_2(g) + B_2(g) \rightleftharpoons 2AB(g)$ at 100°C is 50. If a one litre flask containing one mole of $A_2$ is connected to a two litre flask containing two mole of $B_2$, how many moles of AB will be formed at 373 K? (5)

10\. (a) Five ml of 8 N nitric acid, 4.8 ml of 5 N hydrochloric acid and a certain volume of 17 M sulphuric acid are mixed together and made upto 2 litre. Thirty ml of this acid mixture exactly neutralise 42.9 ml of sodium carbonate solution containing one gram of $Na_2CO_3 \cdot 10H_2O$ in 100 ml of water. Calculate the amount in gram of the sulphate ions in solution. (5)

(b) While studying the decomposition of gaseous $N_2O_5$ it is observed that a plot of logarithm of its partial pressure versus time is linear. What kinetic parameters can be obtained from this observation? (2)

# MATHEMATICS

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

*Notes:* (1) Answers must be written in English.

(2) Figures in brackets on the right-hand margin indicate marks for the questions.

(3) Answers to Part 'A' *must* be given only on the *first six pages* of the answer-book *in the order in which the questions apppear in the question paper*.

(4) Answer to each question of Part 'B' should begin on a fresh page.

(5) You are advised not to spend more than sixty minutes in answering Part 'A'.

(6) Answer all parts of a question at one place.

(7) Use of calculator, logarithmic and trigonometric tables and graph paper is not allowed.

## PART A

(Attempt all questions in this part)

1. There are five parts in this question. Four choices are given for each part and one of them is correct. Indicate your choice of correct answer for each part in your answer-book by writing one of the letters (A), (B), (C), (D) whichever is appropriate:

(i) If $f(x) = \frac{\sin [x]}{[x]}$ , $[x] \neq 0$

$= 0$ , $[x] = 0$

where $[x]$ denotes the greatest integer less than or equal to $x$, then $\lim_{x \to 0} f(x)$ equals:

(A) 1 (B) 0 (C) $-1$ (D) none of these

(ii) If $a, b, c$ are in GP, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in:

(A) AP (B) GP (C) HP (D) none of these

(iii) For any integer $n$, the integral

$$\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n+1)x \, dx$$

has the value:

(A) $\pi$ (B) 1 (C) 0 (D) none of these

(iv) If $a, b, c$ and $u, v, w$ are complex numbers representing the vertices of two triangles such that $c = (1-r)a + rb$ and $w = (1-r)u + rv$, where $r$ is a complex number, then the two triangles:

(A) have the same area (B) are similar
(C) are congruent (D) none of these

(v) If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then $x$ lies in the interval:

(A) $(2, \infty)$ (B) $(1, 2)$
(C) $(-2, -1)$ (D) none of these

**(5 × 2)**

2. There are three parts in this question. Each part has one or more than one correct answer. Indicate all correct answers for each part by writing the corresponding letters from (A), (B), (C), (D) in the answer-book.

(i) Three lines $px+qy+r=0$, $qx+ry+p=0$ and $rx+py+q=0$ are concurrent if:

(A) $p+q+r=0$ (B) $p^2+q^2+r^2=pq+qr+rp$

(C) $p^3+q^3+r^3=3pqr$ (D) none of these

(ii) If $f(x)=x(\sqrt{x}-\sqrt{x+1})$, then

(A) $f(x)$ is continuous but not differentiable at $x=0$

(B) $f(x)$ is differentiable at $x=0$

(C) $f(x)$ is not differentiable at $x=0$

(D) none of these

(iii) If $z_1=a+ib$ and $z_2=c+id$ are complex numbers such that $|z_1|=|z_2|=1$ and $\text{Re}(z_1\bar{z}_2)=0$, then the pair of complex numbers $w_1=a+ic$ and $w_2=b+id$ satisfies:

(A) $|w_1|=1$ (B) $|w_2|=1$

(C) $\text{Re}(w_1\bar{w}_2)=0$ (D) none of these **(3 × 2)**

3. This question contains six statements, each of which is either true or false. Indicate your choice of the answer in the answer-book by writing true or false for each statement.

(i) If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

then the two triangles with vertices $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ and $(a_1, b_1)$, $(a_2, b_2)$, $(a_3, b_3)$ must be congruent.

(ii) The product of any $r$ consecutive natural numbers is always divisible by $r!$

(iii) If three complex numbers are in A.P. tnen they lie on a circle in the complex plane.

(iv) If $n_1, n_2, \ldots\ldots\ldots, n_p$ are $p$ positive integers, whose sum is an even number, then the number of odd integers among them is odd.

(v) No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices

$(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(3, -\sqrt{3})$

(vi) If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+dx+c$, where $ac \neq 0$, then $P(x)Q(x)=0$ has at least two real roots. **(6 × 1)**

4. This question contains fourteen incomplete statements. Determine your answers to be inserted in the blanks so that the statements are complete. Write these answers only in your answer-book, strictly in order in which the statements appear below.

(i) If $f_r(x)$, $g_r(x)$, $h_r(x)$, $r=1, 2, 3$, are polynomials in $x$ such that $f_r(a)=g_r(a)=h_r(a)$, $r=1, 2, 3$

and $F(x)=\begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$

then $F'(x)$ at $x=a$ is.........

(ii) If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

and the vectors $\vec{A}=(1, a, a^2)$, $\vec{B}=(1, b, b^2)$, $\vec{C}=(1, c, c^2)$ are non-coplanar, then the product $abc=$ .........

(iii) $P(A \cup B)=P(A \cap B)$ if and only if the relation between $P(A)$ and $P(B)$ is.........

(iv) If $\vec{A}, \vec{B}, \vec{C}$ are three non-coplanar vectors, then:

$$\frac{\vec{A}\cdot\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}\cdot\vec{B}}+\frac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}}=\text{.........}$$

(v) Let $A$ be a set of $n$ distinct elements. Then the total number of distinct functions from $A$ to $A$ is.........and out of these......... are onto functions.

(vi) The set of all real numbers $a$ such that $a^2+2a$, $2a+3$ and $a^2+3a+8$ are the sides of a triangle is.........

(vii) In a triangle $ABC$, if $\cot A$, $\cot B$ and $\cot C$ are in AP, then $a^2, b^2, c^2$ are in ......... progression.

(viii) Let $x^2+y^2-4x-2y-11=0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilateral of area.........

(ix) If $\vec{A}=(1, 1, 1)$, $\vec{C}=(0, 1, -1)$ are given vectors, then a vector $\vec{B}$ satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}\cdot\vec{B}=3$ is.........

(x) The orthocentre of the triangle formed by the lines $x+y=1$, $2x+3y=6$ and $4x-y+4=0$ lies in quadrant number .........

(xi) If $f(x)=\sin \ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then the domain of $f(x)$ is ......... and its range is .........

(xii) If $f(x)=\log_x(\ln x)$, then $f'(x)$ at $x=e$ is.........

(xiii) A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability.........

(xiv) From the origin chords are drawn to the circle $(x-1)^2+y^2=1$. The equation of the locus of the mid-points of these chords is .........

**(14 × 2)**

## PART B

*Notes*: (1) There are *fourteen* questions in this part. Attempt any *ten* questions.

(2) Justify your answers with mathematical arguments.

5. Use mathematical induction to prove that $2\cdot 7^n+3\cdot 5^n-5$ is divisible by 24 for all $n > 0$. **(5)**

6. Evaluate the following:

(i) $\displaystyle\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x+\sin^4 x}\,dx$ **(2)**

(ii) $\displaystyle\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ **(3)**

7. A ladder rests against a wall at an angle $\alpha$ to the horizontal. Its foot is pulled away from the wall through a distance $a$, so that it slides a distance $b$ down the wall making an angle $\beta$ with the horizontal.

Show that $a=b\tan\frac{1}{2}(\alpha+\beta)$. **(5)**

8. (a) Show that: **(2)**

$$\begin{vmatrix} {}^{x}c_{r} & {}^{x}c_{r+1} & {}^{x}c_{r+2} \\ {}^{y}c_{r} & {}^{y}c_{r+1} & {}^{y}c_{r+2} \\ {}^{z}c_{r} & {}^{z}c_{r+1} & {}^{z}c_{r+2} \end{vmatrix} = \begin{vmatrix} {}^{x}c_{r} & {}^{x+1}c_{r+1} & {}^{x+2}c_{r+2} \\ {}^{y}c_{r} & {}^{y+1}c_{r+1} & {}^{y+2}c_{r+2} \\ {}^{z}c_{r} & {}^{z+1}c_{r+1} & {}^{z+2}c_{r+2} \end{vmatrix}$$

(b) One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If $A$ and $B$ are the points $(-3, 4)$ and $(5, 4)$ respectively, then find the area of the rectangle. **(3)**

9. Sketch the region bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ and find its area. **(5)**

10. Solve for $x$:

$$(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10 \quad \textbf{(5)}$$

11. Two sides of a rhombus $ABCD$ are parallel to the lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at the point $(1, 2)$ and the vertex $A$ is on the $y$-axis, find possible coordinates of $A$. **(5)**

12. A man has 7 relatives, 4 of them are ladies and 3 gentlemen; his wife has also 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of the man's relative and 3 of the wife's relatives? **(5)**

13. In a triangle $ABC$, the median to the side $BC$ is of length

$$\frac{1}{\sqrt{11-6\sqrt{3}}}$$

and it divides the angle $A$ into angles of 30° and 45°. Find the length of the side $BC$. **(5)**

14. Find all the tangents to the curve $y=\cos(x+y)$, $-2\pi \leqslant x \leqslant 2\pi$, that are parallel to the line $x+2y=0$. **(5)**

15. Let $f(x)=\sin^3 x+\lambda \sin^2 x$, $-\pi/2 < x < \pi/2$. Find the intervals in which $\lambda$ should lie in order that $f(x)$ has exactly one minimum and exactly one maximum. **(5)**

16. Find the sum of the series: **(5)**

$$\sum_{r=0}^{n}(-1)^r\,{}^{n}c_{r}\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\ \dots \text{ up to } m \text{ terms}\right]$$

17. Let $f(x) = x^3 - x^2 + x + 1$

and $g(x) = \max\{f(t): 0 \leqslant t \leqslant x\}, \quad 0 \leqslant x \leqslant 1$

$= 3 - x, \quad 1 < x \leqslant 2$

Discuss the continuity and differentiability of the function $g(x)$ in the interval (0, 2). **(5)**

18. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks all the correct answers. The candidate decides to tick answers at random. If he is allowed upto three chances to answer the question, find the probability that he will get marks in the question. **(5)**

# ENGLISH

TIME ALLOWED : 3 HOURS MAXIMUM MARKS : 100

*Notes:* (1) Answers must be written in English.
(2) Figures in brackets on the right-hand side indicate full marks for the questions.
(3) Answers to Part 'A' *must* be written only on the *first six pages* of the answer-book *in the order in which the questions appear on the question paper.*
(4) Answer to each question of Part 'B' should begin on a fresh page.
(5) Answer all parts of a question at one place.
(6) You are advised not to spend more than sixty minutes in answering 'Part 'A'.
(7) You are advised not to exceed the length of the answer, wherever prescribed.

## PART A

1. Fill the gaps with the words whose first letters and meanings (in brackets) are given. Write the number of the sentence, and the word. *Do not rewrite the sentences.*

(i) Dark clouds are gathering. Rain seems im.........(likely to come or happen very soon).

(ii) The spectacular performance of the circus art......... (a skilled performer) on the bar won applause all round.

(iii) The department does not have its com.........(the full number needed) of the staff.

(iv) The journey through tor.........(full of twists and bends) roads left them weak and tired.

(v) It may appear strange, but I had an int.........(hunch) that the plane would crash.

(vi) During elections every political party promises to provide a government that is res.........(which answers) to people's needs.

(vii) Bar.........(with the exception of) a few, all spoke highly of the show.

(viii) The book reveals some of the unknown fac.........(many sides) of the great man's personality.

(ix) Steps have been taken to avoid the rec.........(repetition) of such industrial accidents.

(x) The zamindar was bru.........(in a savage, cruel manner) done to death under mysterious circumstances.

(xi) In def.........(out of respect for) to the minister's wishes, expenditure on public functions was cut down.

(xii) Nobody likes a co.........(not brave) person. **(13)**

(xiii) The cadets thought that the drill sergeant was really the devil inc.........(having a body in human form).

2. Each of the following sentences has a verb in brackets. Write the number of the sentence and the correct form of the verb. *Do not rewrite the sentences.*

(i) I know him well. He (be) my classmate since we were at school.

(ii) I sincerely wish you (enjoy) the forthcoming adventure as much as I shall.

(iii) May I know what your activities (be), lately?

(iv) All the questions already (solve) when I entered the room.

(v) I would be so happy if only he (come) here to help us tomorrow.

(vi) While Sita and her friends (eat) their lunch, they saw a little boy fall from the wall.

(vii) Will you speak to him warmly when you (meet) him next week in Madurai?

(viii) Who (be) to the cinema with you yesterday if you had behaved arrogantly?

(ix) He looks as if he quite (please) with what we are going through.

(x) I (surprise) to see a shy friend of mine talking freely with a pretty girl yesterday.

(xi) I gave up studying Sanskrit after I (work) at it for a long time.

(xii) Nobody shall disturb us while confidential matters (discuss) in the meeting. **(12)**

3. Rewrite the following sentences making corrections wherever necessary. *Do not make needless changes.*

(i) You should know that I neither insulted him nor anyone else who came with him.

(ii) Our chairman should be more liberal rather than strict.

(iii) What he is looking for is a wife.

(iv) Your milk is getting cold; drink up it quickly.

(v) Some of the books, which you seem to have read, have become best-sellers already.

(vi) His plans were known by everyone in his office.

(vii) Would you bring me two three bananas now that you are going to the market?

(viii) Strangely enough it was the thief who needed protection from the police!

(ix) I don't care whether or not your car breaks down.

(x) I will devote myself to help the poor and the downtrodden of my country. **(10)**

4. Rewrite the following sentences as directed:

(i) "Why don't you work hard enough for me?", she demanded of him. (Put the sentence into Reported Speech.)

(ii) People should not make the Holi celebrations an excuse for rowdy behaviour. (Put the sentence into the passive voice.)

(iii) Surya has been one of the best leaders of our time. (Rewrite the sentence beginning with 'Few' without changing the meaning.)

(iv) The grammar is so difficult that we can't understand it. (Rewrite the sentence using 'too' without changing the meaning.)

(v) Shyam exclaimed, "What a marvellous dinner!" (Put the sentence into Reported Speech.)

(vi) At the time of his death, he was a rich man. (Rewrite the sentence using just three words but without modifying the meaning.)

(vii) It is fun to be with Rajiv. (Rewrite the sentence beginning with 'Rajiv'.)

(viii) She has been tremendously helpful to all her pupils. (Write a question tag to this sentence.)

(ix) I had hardly understood the problem when he hustled me into giving my views. (Rewrite the sentence beginning with 'Hardly'.)

(x) The committee is unanimous on this issue. (Write a sentence expressing the opposite meaning of this sentence without using any negative forms.) **(10)**

5. The sentences in the following paragraph are jumbled. Rewrite the entire paragraph in the correct sequence. Merely numbering the sentences will not fetch any marks.

Every experience in life makes an impression on some of the cells of the brain, or other nerve centres. Moreover, the methods by which it can be cultivated are all founded on practical common sense. The other part involves filing them away in such an orderly manner that many will remain intact. Of course, the memory can be improved. One part of the problem of memory improvement is to emphasize these impressions. It can be improved by anyone with normal intelligence. **(5)**

## PART B

6. Write an essay in about 350 words on *one* of the following:

(i) The kind of god I believe in.

(ii) Playing, not winning, is what matters.
(iii) Terrorism does not help.
(iv) The way I have changed over the past five years. **(20)**

7. Explain the meaning of *one* of the following statements in about 150 words:
   (i) Only the knowledgeable know how little they know.
   (ii) A man is known by the company he does not keep.
   (iii) Love is healing. **(10)**

8. Write a paragraph on *one* of the following in about 150 words. Write either *for* or *against* the topic:
   (i) I cannot have a worse enemy than myself.
   (ii) Freedom is an end in itself.
   (iii) 'An eye for an eye' is fair play. **(10)**

9. Read through the following passage and answer *in your own words* the questions given. No credit will be given if portions are lifted from the passage.

Boredom as a factor in human behaviour has been, I believe, one of the great motive powers throughout the historical epoch, and is so at the present day more than ever. The desire for excitement is very deep-seated in human beings, especially in males. I suppose that in the hunting stage it was more easily gratified than it has been since. The chase was exciting, war was exciting, courtship was exciting. But with the coming of agriculture, life began to grow dull. In old days, after supper everybody sat round and had what was called "a happy family time". This meant that the head of the family went to sleep, his wife knitted and the daughters wished they were dead or at Timbuktu. All this weight of boredom should be borne in mind in estimating the world of a hundred years ago.

We are less bored than our ancestors were, but we are more afraid of boredom. We have come to believe that boredom is not part of the natural lot of man, but can be avoided by a sufficiently vigorous pursuit of excitement. A wish to escape from boredom is natural; indeed, all races of mankind have displayed it as opportunity occurred. Wars, genocide and persecutions have all been part of the flight from boredom; even quarrels with neighbours have been found better than nothing.

Boredom, however, is not to be regarded as wholly evil. A certain power of enduring boredom is essential to a happy life. All great books contain boring portions, and all great lives have contained uninteresting stretches. Imagine a modern publisher confronted with the Old Testament as a new manuscript submitted to him for the first time. What would his comments be, for instance, on the genealogies? "My dear sir," he would say, "you can't expect your reader to be interested in a mere string of proper names of persons about whom you tell so little. You have begun your story, I will admit, in fine style, and at first I was very favourably impressed, but you have altogether too much wish to tell it all. Pick out the highlights, take out the superfluous matter and bring me back the manuscript when you have reduced it to a reasonable length". All the best novels contain boring passages. A novel which sparkles from the first page to

the last is pretty sure not to be a great book. Nor have the lives of most great men been exciting except at a few great moments. Socrates could enjoy a banquet now and again and must have derived considerable satisfaction from his conversations while the hemlock was taking effect, but most of his life he lived quitely with Xanthippe, taking a constitutional in the afternoon and perhaps meeting a few friends by the way. Immanuel Kant is said never to have been more than ten miles from Konigsberg in all his life. Charles Darwin, after going round the world, spent the whole of the rest of his life in his own house. Karl Marx, after stirring up a few revolutions, decided to spend the remainder of his days in the British Museum. Altogether it will be found that a quiet life is characteristics of great men, and that their pleasures have not been of the sort that would look exciting to the outward eye.

The capacity to endure a more or less monotonous life is one which should be acquired in childhood. Modern parents do not realise the importance to a child of having one day like another, except, of course, for somewhat rare occasions. The pleasures of childhood should in the main be such as the child extracts from his environment by means of some effort and inventiveness. Pleasures which are exciting and at the same time involve no physical exertion, such, for example, as the theatre, should occur very rarely. A child develops best when, like a young plant, he is left undisturbed in the same soil. Too much travel, too great a variety of impressions, are not good for the young and cause them as they grow up to become incapable of enduring fruitful monotony. A boy or young man who has some serious constructive purpose will endure voluntarily a great deal of boredom if he finds that it is necessary by the way. But constructive purposes do not easily from themselves in a boy's mind if he is living a life of distractions and dissipations, for in that case his thoughts will always be directed towards the next pleasure rather than the distant achievement. A generation that cannot endure boredom will be a generation of little men, of men unduly divorced from the slow processes of nature.

(i) How did life become dull with the coming of agriculture?

(ii) What, according to the passage, are the means man has used to get away from boredom?

(iii) Why would a modern publisher refuse to publish the Old Testament if it were submitted to him as a new manuscript?

(iv) What point does the author make by mentioning Socrates, Kant, Darwin and Marx together?

(v) How would a life of mere pleasure and excitement affect a young mind?

**(10)**

# MODEL SOLUTIONS

## PHYSICS

### PART A

1. (i) (D) $\frac{MgL}{18}$
   (ii) (B) remains constant
   (iii) (A), (C); 31.25, 93.75
   (iv) (B) 50
   (v) (A), (B), (D);
   $E=0$, $B=0$, $E=0$, $B \neq 0$, $E \neq 0$, $B \neq 0$
   (vi) (A), (D); $Q > Q_0$, $U > U_0$
   (vii) (D) the grid is positive and plate is positive
   (viii) (C) more towards the wire
   (ix) (D) having all wavelengths lying between a minimum and a maximum wavelength
   (x) (B), (C); the balance $B$ will read more than 5 kg, the balance $A$ will read less than 2 kg and $B$ more than 5 kg
2. (i) 8, 6
   (ii) 15 cm
   (iii) $M^1L^2T^{-1}$
   (iv) angular momentum
   (v) a mixture of solid and liquid
   (vi) 4000 Å, $5 \times 10^{14}$ Hz
3. (i) False

   When the source is moving towards the observer with velocity $V$, its apparent frequency will increase due to Doppler effect. The sound of same frequency, 256 Hz, will be elastically reflected from the wall and will travel with the same velocity $V$ towards the observer. As the apparent frequency of the reflected sound will be exactly equal to the apparent frequency of the direct sound as heard by the observer, no beats will be heard.

   (ii) True

   The resistance of metallic wire, as found from the ratio of $V$ and $I$, in Fig. 4, is larger at temperature $T_2$ as compared to that at temperature $T_1$. The temperature $T_2$ is, therefore, greater than $T_1$.

(iii) False

The value of acceleration due to gravity decreases owing to the motion of the earth so that effective weight of a body of mass $M$ is equal to $M\left(g-\frac{v^2}{R}\right)$, where $v$ is the effective velocity of this body located at a distance $R$ from the centre of the earth. As two identical trains are moving on rails along the equator of the earth in opposite directions with the same speed, the velocity of earth will be added to the velocity of one of the trains moving in the same direction and it will be subtracted from the velocity of the second train. The effective velocity $v$, and therefore, the effective value of pressure on the rails, $M\left(g-\frac{v^2}{R}\right)$, will be different.

(iv) True

An isothermal process is represented by $PV$=constant and an adiabatic process is given by $PV^\gamma$=constant, where $\gamma$ has a value more than unity. As the slope for curve $A$ is less than that for curve $B$, the isothermal process is represented by curve $A$.

(v) False

The radius $R$ of circular path described by a particle of charge $e$ and momentum $mv$, when subjected to a magnetic field '$B$' is given by $R=\frac{mv}{Be}$.

Though both these particles have got equal kinetic energy $E$, the momentum of the heavier particle, i.e. the proton, is different from the momentum of the lighter electron as momentum, $mv=\sqrt{2mE}$. Thus the radius of the circular path of the electron will be different from the radius of the path of the proton.

(vi) False

As an equal force $F$ is applied at the middle point in both the cases, it yields equal torques about A and B such that $I_A\alpha_A = I_B\alpha_B$. The moment of inertia of triangular plate about $A$, $I_A$, is obviously larger than its moment of inertia about $B$, $I_B$. Therefore, the angular acceleration, $\alpha_A$, in case (a) is less than angular acceleration, $\alpha_B$, in case (b).

## PART B

4. (a) The total time-period of the block is equal to the sum of time-periods of the block to cover to and fro, distance $CD$, with constant velocity and periods of oscillation of the spring-block system for each of the springs $k_1$ and $k_2$.

The distance covered by the block from $C$ to $D$ (shown in Fig. 12) and back to complete one period of oscillation $= 2\times CD = 2\times 0.60 = 1.20$ m.

Constant velocity of the block $= 1.20$ m s$^{-1}$

The time of oscillation of the block, on smooth horizontal table,

$$t_1=\frac{1.20}{1.20}=1.0 \text{ sec.}$$

The time-period of oscillation of block-spring system for a spring of spring constant, $k_2=3.2 \text{ N m}^{-1}$, is equal to $2\pi\sqrt{\frac{m}{k_2}}=2\pi\sqrt{\frac{0.2}{3.2}}$.

As the block will be in contact with spring $k_2$ for a time-period equal to one half of this time-period at $D$, the effective time-period of oscillation of block-spring system at $D$,

$$t_2=\pi\sqrt{\frac{0.2}{3.2}}.$$

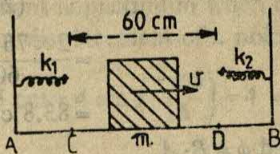



Fig. 12

Also the effective time-period of oscillation of the block-spring system with spring constant $k_1$ at $C$, $t_3=\pi\sqrt{\frac{0.2}{1.8}}$

$\therefore$ Total time-period $=t_1+t_2+t_3$

$$=1.0+\pi\sqrt{\frac{1}{16}}+\pi\sqrt{\frac{1}{9}}$$

$$=1.0+\pi\left(\frac{1}{4}+\frac{1}{3}\right)=1.0+\frac{11}{6}=2.83 \text{ sec.}$$

(b) Let $P_1$, $V_1$ and $T_1$ be respective values of pressure, volume and temperature of gas in each of the glass bulbs having $n$ number of moles.

$$P_1V_1=nRT_1$$

where $R$ is the universal gas constant.

$\therefore$ number of moles in each bulb $=n=\frac{P_1V_1}{RT_1}$

Total number of moles in both the bulbs $=n+n$

$$=\frac{P_1V_1}{RT_1}+\frac{P_1V_1}{RT_1}$$

Let the new value of pressure inside both the bulbs be $P_2$ when temperature of second bulb, $T_2$, is raised to 62°C.

Let $n_1$ be number of moles in the first bulb and $n_2$ be number of moles in the second bulb.

$$n_1=\frac{P_2V_1}{RT_1}$$

and $$n_2 = \frac{P_2V_1}{RT_2}$$

As the total number of moles remains constant,

$$2n = n_1 + n_2$$

or $$\frac{2P_1V_1}{RT_1} = \frac{P_2V_1}{RT_1} + \frac{P_2V_1}{RT_2}$$

or $$\frac{2P_1V_1}{T_1} = P_2V_1\left[\frac{1}{T_1} + \frac{1}{T_2}\right]$$

or $$P_2 = \frac{2P_1}{T_1}\left[\frac{T_1 \times T_2}{T_1 + T_2}\right] = \frac{2P_1 \times T_2}{T_1 + T_2}$$

$$= \frac{2 \times 76 \times (273 + 62)}{(273 + 335)}$$

$$= \frac{2 \times 76 \times 335}{608}$$

$$= 83.8 \text{ cm of Hg column}$$

5. (a) (i) Flux linked $\phi = B \cdot A$

Induced emf $E = -\frac{d\phi}{dt} = -B\frac{dA}{dt}$

The change in area per unit time when a loop of radius $r$ rotates with angular velocity $\omega\left(= \frac{d\theta}{dt}\right) = \frac{dA}{dt} = \frac{1}{2}r\left(r\frac{d\theta}{dt}\right)$ $= \frac{r^2\omega}{2}$.

Induced emf $E = -\frac{Br^2\omega}{2}$

If $R$ is the effective resistance of the loop, magnitude of current in the loop $= \frac{E}{R} = \frac{Br^2\omega}{2R}$.

(ii) The direction of current is opposite to the clockwise direction of rotation of the loop and is anticlockwise as shown in Fig. 13.

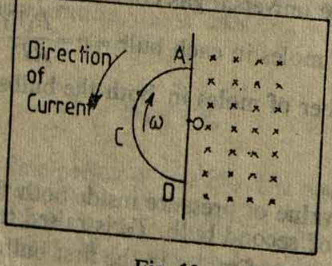



Fig. 13

(iii) The time-period of rotation is equal to $\frac{2\pi}{\omega}$. The graph between the induced emf and time of rotation is shown in Fig. 14.

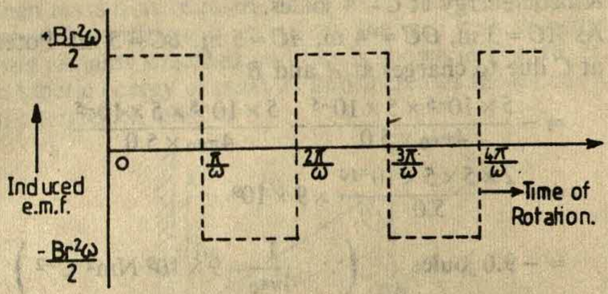



Fig. 14

(b) (i) Displacement is maximum when $\cos(96\pi t) = 1$
Maximum displacement of a point at $x$ equal to

$$5 \text{ cm} = 4 \sin\left(\frac{\pi \times 5}{15}\right) = 4 \sin\left(\frac{\pi}{3}\right)$$

$$= 4 \times \frac{\sqrt{3}}{2} = 3.46 \text{ cm}$$

(ii) Nodes are located at those values of distance $x$ where

$$\sin\left(\frac{\pi x}{15}\right) = 0$$

or $$\frac{\pi x}{15} = n\pi \quad (n \text{ being an integer})$$

or $$x = 15n$$

$\therefore$ Nodes will be present at distances $x = 0$, 15, 30, 45 and 60 cm.

(iii) $$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

$\therefore$ Velocity $\frac{dy}{dt} = -(4 \times 96\pi) \times \sin\left(\frac{\pi x}{15}\right) \times \sin(96\pi t)$

At $x = 7.5$ cm and $t = 0.25$ sec,

$$\sin(96\pi t) = \sin 24\pi$$
$$= 0$$

$\therefore$ $$\frac{dy}{dt} = 0$$

(iv) The component waves are given by the following equations:

$$y_1 = 2 \sin\left(\frac{\pi x}{15} + 96\pi t\right)$$

and $$y_2 = 2 \sin\left(\frac{\pi x}{15} - 96\pi t\right)$$

6. (a) Kinetic energy at $C = 4$ joules.

As $AO = 3$ m, $OC = 4$ m, $AC = 5$ m, $BC = 5$ m; Potential energy at $C$ due to charges at $A$ and $B$

$$= -\frac{5\times10^{-5}\times5\times10^{-5}}{4\pi\epsilon_0\times5.0} - \frac{5\times10^{-5}\times5\times10^{-5}}{4\pi\epsilon_0\times5.0}$$

$$= -\frac{2\times5\times5\times10^{-10}}{5.0}\times9\times10^9$$

$$= -9.0 \text{ joules} \qquad \left(\because \frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ Nm}^2\text{ C}^{-2}\right)$$

$\therefore$ Total energy at $C = 4 - 9$

$$= -5 \text{ joules}$$

The negative charge will reach the farthest point $D$ such that its total energy is equal to $-5$ joules (Fig. 15).

$\therefore$ Total energy at $D$ due to charges at $A$ and $B$

$$= -\frac{q^2}{4\pi\epsilon_0\times AD} - \frac{q^2}{4\pi\epsilon_0\times AD}$$

$$= -\frac{2q^2}{4\pi\epsilon_0\times AD}$$

$$= -\frac{2\times(5\times10^{-5})^2\times9\times10^9}{AD}$$

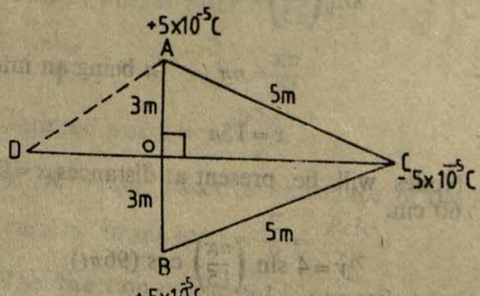



Fig. 15

Equating this energy with the energy at $C$,

$$-\frac{2\times25\times10^{-10}\times9\times10^9}{AD} = -5$$

or

$$AD = \frac{2\times25\times9\times10^{-1}}{5}$$

$$= 9 \text{ m}$$

$\therefore$ Distance of point $D$ from point $O = OD$

$$= \sqrt{AD^2 - AO^2}$$

$$= \sqrt{81-9}$$

$$= \sqrt{72} = 8.5 \text{ m (approx.)}$$

(b) When mass $m$ is released from rest, the heavier mass $M$ loses potential energy and it strikes the wall with velocity $v$ and the string remains stretched and this is shown in Fig. 16. It increases the kinetic energy of mass $M$ and increases the kinetic and potential energies of mass $m$.

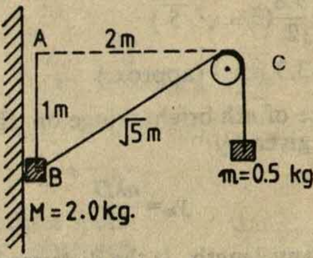



Fig. 16

Decrease in potential energy of mass

$$\begin{aligned} M &= Mg \times 1 \\ &= 2.0 \times g \times 1 \\ &= 2g \qquad \text{where } g = 9.8 \text{ ms}^{-2} \end{aligned}$$

Increase in kinetic energy of mass $M = \frac{1}{2}Mv^2$

$$= \frac{1}{2} \times 2.0 \times v^2$$

$$= v^2$$

Increase in potential energy of mass $m = mg(\sqrt{5} - 1)$

$$= \frac{g}{2}(\sqrt{5} - 1)$$

$\because$ Distance through which mass $m$ is raised

$$\begin{aligned} &= AB + BC - AC \\ &= 1 + \sqrt{5} - 2 \\ &= (\sqrt{5} - 1) \text{ m} \end{aligned}$$

Increase in kinetic energy of mass $m = \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 0.5 \times v^2$$

$$= \frac{v^2}{4}$$

As the total energy must be conserved, the decrease in the potential energy of the mass $M$ equals the increase in the potential and kinetic energies of both the masses.

$$\therefore \quad Mg \times 1 = \frac{1}{2} Mv^2 + mg(\sqrt{5} - 1) + \frac{1}{2}mv^2$$

$$\therefore \quad 2g = v^2 + \frac{g}{2}(\sqrt{5}-1) + \frac{v^2}{4}$$

$$\text{or} \quad 5\frac{v^2}{4} = g\left[2 - \frac{(\sqrt{5}-1)}{2}\right]$$

$$\text{or} \quad 5\frac{v^2}{4} = \frac{9.8}{2}(3-\sqrt{5})$$

$$\text{or} \quad v = 3.3\ \text{ms}^{-1}\ \text{(approx.)}$$

7. (a) (i) The distance of $n$th bright fringe on the screen from central maxima is given by

$$y_n = \frac{n\lambda D}{d}$$

where $\lambda$ is wavelength, $D$ the distance between the plane of slits and screen and $d$ the distance between slits.

$$\therefore \quad y_3 = \frac{3\lambda D}{d}$$

$$= \frac{3 \times 6500 \times 10^{-8} \times 120}{0.2}$$

$$= 0.117\ \text{cm}$$

(ii) Let the $n$th fringe of wavelength $\lambda n$ coincide with the $m$th fringe of wavelength $\lambda m$ for the same distance, $y$, from the central maxima

$$\therefore \quad y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$

$$\text{or} \quad \frac{m}{n} = \frac{\lambda_n}{\lambda_m} = \frac{6500}{5200} = \frac{5}{4}$$

$$\therefore \quad m = 5 \quad \text{and} \quad n = 4$$

Thus

$$y = \frac{m\lambda_m D}{d}$$

$$= \frac{5 \times 5200 \times 10^{-8} \times 120}{0.2}$$

$$= 0.156\ \text{cm}$$

Or

(a) (i) The energy difference of electron in $Li^{++}$ between the first and the third orbit $= E_3 - E_1$.

$$\therefore \quad E_3 - E_1 = 13.6 \times Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 13.6 \times (3)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 13.6 \times 9 \times \frac{8}{9} \times 1.6 \times 10^{-19}\ \text{joule}$$

Therefore, the equivalent wavelength $\lambda$ is given by

$$E_3 - E_1 = \frac{hc}{\lambda}$$

or
$$\lambda = \frac{hc}{E_3 - E_1}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{13.6 \times 8 \times 1.6 \times 10^{-19}}$$

$$= 1.137 \times 10^{-8} \text{ m}$$

$$= 113.7 \text{ A}^\circ$$

(ii) The following three spectral lines are observed due to the following transitions (Fig. 17):

3rd to 1st orbit
3rd to 2nd orbit
2nd to 1st orbit

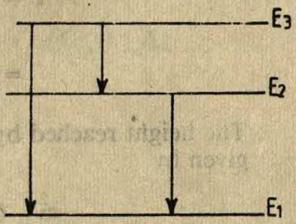



Fig. 17

(b) The total time of flight of the masses will be equal to the sum of the time of flight of the masses till they collide, the time taken to reach maximum height by the coalesced masses and the time taken for the two masses to fall freely to the ground.

Let $y_1$ be the distance travelled by the ball moving vertically upwards and $y_2$ the distance travelled by the falling ball in the same time $t$ till these collide such that

$$y_1 + y_2 = 98 \text{ m}$$

$\therefore$
$$y_1 = 49t - \frac{1}{2} \times 9.8 \times t^2$$

$$= 49t - 4.9t^2$$

and
$$y_2 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$= 4.9t^2$$

$$y_1 + y_2 = 49t = 98$$

or $\quad t = 2 \text{ sec} \quad$ (i)

The velocity of the ball moving upwards at the time of collision

$$v_1 = 49 - 9.8 \times 2$$

$$= 29.4 \text{ m s}^{-1}$$

The velocity of ball moving downwards at the time of collision

$$v_2 = 0 + 9.8 \times 2$$

$$= 19.6 \text{ m s}^{-1}$$

As $v_1$ in the upward direction is larger than $v_2$ in the downward direction, the total mass of 200 gm will move upwards with velocity $v_3$ such that

$$200 \times v_3 = 29.4 \times 100 - 19.6 \times 100$$
$$= 9.8 \times 100$$

Therefore, the velocity of total mass of 200 gm moving upwards

$$v_3 = 4.9 \text{ m s}^{-1}$$

Time taken by the 200 gm mass moving upwards with velocity $v_3$ to come to rest

$$t' = \frac{v_3}{g}$$
$$= \frac{4.9}{9.8} = 0.5 \text{ s} \qquad \text{(ii)}$$

The height reached by mass of 200 gms till it comes to rest is given by

$$\frac{v_3^2}{2g} = \frac{(4.9)^2}{2 \times 9.8}$$
$$= \frac{4.9}{4} = 1.23 \text{ m}$$

Height at which these masses collide is given by

$$y_1 = 49t - 4.9 \times t^2$$
$$= 49 \times 2 - 4.9 \times 4$$
$$= 78.4 \text{ m}$$

$\therefore$ Total height from which the coalesced mass of 200 gm falls

$$= 78.4 + 1.23$$
$$= 79.63 \text{ m}$$

Time taken for the total mass of 200 gm to fall freely from a height of 79.63 m,

$$t'' = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 79.63}{9.8}} = 4.03 \text{ s} \qquad \text{(iii)}$$

$\therefore$ Total time of flight of masses $= t + t' + t''$

$$= 2 + 0.5 + 4.03$$
$$= 6.53 \text{ s}$$

# CHEMISTRY

## PART A

1. (i) (B) spectrum of an atom or ion containing one electron only
   (ii) (A) NO
   (iii) (D) $KNO_{3(aq)} + NaCl_{(aq)} = KCl_{(aq)} + NaNO_{3(aq)}$
   (iv) (D) inversely proportional to the square root of its molecular weight
   (v) (B) the relative lowering of vapour pressure is equal to the mole fraction of solute
   (vi) (C) Pt | $H_2$(g) | HCl (soln) | AgCl(s) | Ag
   (vii) (B) $10^{-13}$ (cm)
   (viii) (D) phenolphthalein (8 to 9.6)
   (ix) (A) $KNO_3$
   (x) (B) $Na_2O_2$
   (xi) (D) $Na_2SO_4 \cdot 10H_2O$
   (xii) (C) $SO_2$
   (xiii) (A) $NH_3$
   (xiv) (A) ethyne
   (xv) (C) diethyl ketone
   (xvi) (C) but—2—ene
   (xvii) (C) $BeCl_2$
   (xviii) (A) toluene
   (xix) (D) heated aluminium oxide
   (xx) (B) radio wave

2. (a) (i) Aqueous HCl, being an ionic compound, is a good conductor of electricity. However, anhydrous HCl, being a covalent compound, does not conduct any electricity.
   (ii) O-Nitrophenol contains intramolecular hydrogen bonding whereas no such bonding is present in p-nitrophenol. Hence only O-nitrophenol is steam volatile.
   (iii) The methyl group present only in acetic acid is electron releasing whereas formic acid does not contain any methyl group.
   (iv) In the case of graphite, which has a layered structure, one layer slides over another owing to friction, overcoming the weak Van der Waal's forces between layers and making it lubricative.
   (v) It is because the element fluorine is most electronegative in character and hence cannot be prepared from its fluorides by chemical oxidation.

   (b) (i) concentration of the reactants
   (ii) $Sp^3$
   (iii) electronegativity
   (iv) eight
   (v) The solution has solvated electrons

3. (a)

| | |
|---|---|
| (i) False | (vi) True |
| (ii) False | (vii) True |
| (iii) False | (viii) True |
| (iv) False | (ix) True |
| (v) False | (x) False |

(b) (i) $O^{2-} > F^- > Na^+ > Mg^{2+}$

(ii) $Na_2O_2 < MgO < ZnO > P_2O_5$

(iii) $Na < Al < Mg < Si$

(iv) $C_2H_5COCH_3 < CH_3COCH_3 < CH_3CHO < HCHO$

(v) $N_2 < O_2 < F_2 < Cl_2$

4.

| | | |
|---|---|---|
| (a) | (A) spinel, | (i) $MgAl_2O_4$ |
| | (B) feldspar, | (iii) $KAlSi_3O_8$ |
| | (C) cerussite, | (ii) $PbCO_3$ |
| | (D) malachite, | (v) $Cu(OH)_2 \cdot CuCO_3$ |
| | (E) kisserite, | (iv) $MgSO_4 \cdot H_2O$ |
| (b) | (F) liquid air, | (viii) Claude process |
| | (G) $Na_2CO_3$, | (x) Solvay process |
| | (H) nitric oxide, | (ix) Ostwald process |
| | (I) silver, | (vii) Perke process |
| | (J) chlorine | (vi) Deacon process |
| (c) | (K) phenol, | (xiv) antiseptic |
| | (L) $Na_2S_2O_3$, | (xii) antichlor |
| | (M) salicylic acid, | (xv) analgesic |
| | (N) quicklime, | (xiii) refractory material |
| | (O) CuO, | (xi) coloured glass |
| (d) | (P) Aston, | (xx) mass spectrum |
| | (Q) Priestley, | (xviii) oxygen |
| | (R) Ramsay, | (xix) inert gas |
| | (S) Marie Curie, | (xvi) radium |
| | (T) Becquerel, | (xvii) radioactivity |

## PART B

5. (a) It is because ground state outermost electronic configuration should not violate Hund's rule.

(b) ten

(c) three pi bonds and twelve sigma bonds

(d) the resonance structures for nitrous oxide are:

$$N^- = N^+ = O \leftrightarrow N \equiv N^+ - O^-$$

(e) the boiling points of butanol and butanal are different because butanol has hydrogen bonding whereas no such bonding occurs in butanal.

6. (a)

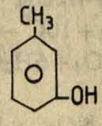
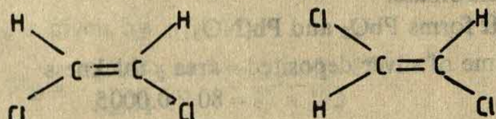



Fig. 18

(b) (i) In order to distinguish between 2-butyne and 1-butyne, they are treated with an ammonial solution of $CuSO_4$. Only 1-butyne reacts whereas no such reaction occurs with 2-butyne.

(ii) In order to distinguish between methanol and ethanol, we perform iodoform test with them which confirms the presence of ethanol.

(c) Two possible isomers of dichloroethene are:




(i) (ii)

Fig. 19

Only (ii) has zero dipole moment.

(d) (i)

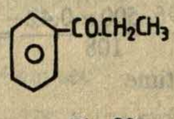



Fig. 20

(ii) $H_3C \cdot CH_2 \cdot CH = \underset{\displaystyle CH_3}{\underset{|}{C}} \cdot CHO$

(iii) $CHI_3$

(e)

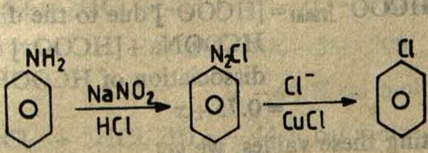
(i) 


Fig. 21

(ii) $HC \equiv CH \xrightarrow[Hg^{2+}]{H_2SO_4} CH_3CHO \xrightarrow[\text{Ca salt}]{\text{Heating with}} CH_3COCH_3$

7. (a) (i) $2KClO_3 + H_2C_2O_4 + H_2SO_4 \rightarrow 2ClO_2 + K_2SO_4 + 2CO_2 + 2H_2O$
(ii) $2KMnO_4 + MnO_2 + 4KOH \rightarrow 3K_2MnO_4 + 2H_2O$
(iii) $2Ca_3(PO_4)_2 + 6SiO_2 + 10C \rightarrow 6CaSiO_3 + 10CO + P_4$
(iv) $K_4Fe(CN)_6 + 6H_2SO_4 + 6H_2O \rightarrow 2K_2SO_4 + FeSO_4 + 6CO + 3(NH_4)_2SO_4$
(v) $(NH_4)_2SO_4 + NO + NO_2 \rightarrow H_2SO_4 + 3H_2O + 2N_2$

(b) (i) Sulphur dioxide is reduced to sulphur and hydrogen sulphide gets oxidised to sulphur.
(ii) When aqueous ammonia is added to a solution of copper sulphate, bluish white precipitates of $Cu(OH)_2$ are formed which then result in the copper ammonia complex having deep blue colouration.
(iii) When tin is treated with concentrated nitric acid, $HNO_3$ gets reduced to $NO_2$ and tin is oxidised to $H_2SnO_3$.
(iv) Precipitates of $Cr(OH)_3$ having green colouration are formed which ultimately dissolve to produce a yellow solution of the chromate.
(v) It forms $PbO_2$ and $Pb(NO_3)_2$.

8. (a) Volume of silver deposited = area × thickness
$= 80 \times 0.0005$
$= 0.04$ cc

Mass of silver deposited = volume × density
$= 0.04 \times 10.5$
$= 0.42$ g

Atomic mass of silver = 108

Amount of charge required to deposit 108 g of silver = 96,500 C

Amount of charge required to deposit 0.42 g of silver

$$= \frac{96,500 \times 0.42}{108} = 375 \text{ C}$$

As current = charge/time

time required to allow a current of 3A $= \frac{375}{3}$

$= 125$ Sec

(b) Suppose the concentration of hydrogen ion ($H^+$) after adding sodium formate be $x$

$[HCOOH]_{final} = 0.2 - x$

$[HCOO^-]_{final} = [HCOO^-]$ due to the dissociation of $HCOONa + [HCOO^-]$ due to the dissociation of HCOOH

$= 0.75 + x$

Substituting these values, we get

$$2.4 \times 10^{-4} = \frac{x \times (0.75 + x)}{0.2 - x}$$

$$= \frac{x \times 0.75 + x^2}{0.2 - x}$$

or $\quad 0.2\times 2.4\times 10^{-4}-2.4\times 10^{-4}x=x\times 0.75+x^2$

or $\quad 0.2\times 2.4\times 10^{-4}=0.75\times x$

or $\quad x=\dfrac{0.2\times 2.4\times 10^{-4}}{0.75}$ (neglecting $x^2$ and $2.4\times 10^{-4}x$, which will be very small)

$=0.64\times 10^{-4}$ mol/litre

As pH $=-\log[H^+]$, pH of the solution will be given by,

$$pH=-\log[0.64\times 10^{-4}]$$

$$\therefore \quad pH=3.2$$

9. (a) Root mean square velocity $\bar{C}$ is given by $\bar{C}=\sqrt{\dfrac{3PV}{M}}$, where $P$ is the pressure, $V$ the volume and $M$ the molecular weight of the gas.

Using the gas equation, $\dfrac{P_1V_1}{T_1}=\dfrac{P_2V_2}{T_2}$, the volume of Ozone at 20°C, $V_{20}$, is given by

$$V_{20}=\frac{P_1V_1T_{20}}{P_{20}T_1}=\frac{76\times 22400\times 293}{82\times 273}$$

$$=2.28\times 10^4 \text{ cm}^3 \qquad \text{(i)}$$

Pressure of Ozone at 20°C, $P_{20}$, will be given by

$$P_{20}=82\times 13.6\times 981 \qquad (\because\ P=hdg)$$

$$=1.094\times 10^6 \text{ dynes/cm}^2$$

Substituting these values of $P_{20}$ and $V_{20}$, we get

$$\bar{C}=\sqrt{\frac{3\times 1.094\times 10^6\times 2.28\times 10^4}{48}}$$

$$=3.9\times 10^4 \text{ cm sec}^{-1}$$

(b) $H_2 \rightarrow 2H,\ \Delta H=104$ K cal mol$^{-1}$ (i)

$Cl_2 \rightarrow 2Cl,\ \Delta H=58$ K cal mol$^{-1}$ (ii)

$HCl \rightarrow H+Cl,\ \Delta H=103$ K cal mol$^{-1}$ (iii)

Adding (i) and (ii) and subtracting $2\times$(iii) from the result, we get :

$$H_2+Cl_2-2HCl \rightarrow 2H+2Cl-2H-2Cl;$$

$$\Delta H=104+58-206$$

$$=-44 \text{ K cal mol}^{-1}$$

or $\quad H_2+Cl_2 \rightarrow 2HCl,\ \Delta H=-44$ K cal mol$^{-1}$

or $\quad \frac{1}{2}H_2(g)+\frac{1}{2}Cl_2(g) \rightarrow HCl(g),\ \Delta H=-\frac{44}{2}$

$$=-22 \text{ K cal mol}^{-1}$$

Thus the enthalpy of formation of HCl gas is $-22$ K cal mol$^{-1}$.

(c) The given chemical reaction is:

$$A_2(g) + B_2(g) \rightleftharpoons 2AB(g)$$

Total volume under consideration = 3 litres

Initial concentration of $A$ before reaction, $[A] = \frac{1}{3}$ mol litre$^{-1}$

Initial concentration of $B$ before reaction, $[B] = \frac{2}{3}$ mol litre$^{-1}$

Suppose $x$ mol litre$^{-1}$ of $A$ and $B$ have reacted. Then,

| | $A_2(g)$ | + | $B_2(g)$ | $\rightleftharpoons$ | $2AB(g)$ |
|---|---|---|---|---|---|
| Initial concentration: | $x$ | | $x$ | | 0 |
| Final concentration: | $\left(\frac{1}{3}-x\right)$ | | $\left(\frac{2}{3}-x\right)$ | | $2x$ |

At equilibrium, $$K = \frac{(2x)^2}{\left(\frac{1}{3}-x\right)\left(\frac{2}{3}-x\right)}$$

But $K = 50$

$\because$ $$50 = \frac{4x^2}{\left(\frac{1}{3}-x\right)\left(\frac{2}{3}-x\right)}$$

or $$50 = \frac{36x^2}{(1-3x)(2-3x)}$$

or $36x^2 = 50(2 - 9x + 9x^2)$

or $450x^2 - 450x - 36x^2 + 100 = 0$

or $414x^2 - 450x + 100 = 0$

or $207x^2 - 225x + 50 = 0$

Solving this quadratic equation, we get

$$x = \frac{225 \pm \sqrt{(225)^2 - 4 \times 207 \times 50}}{2 \times 207}$$

$$= \frac{225 \pm 96}{414}$$

$$= 0.78,\ 0.3$$

If $x = 0.78$,

$$[A_2] = \frac{1}{3} - 0.78$$

$$= \frac{1 - 2.34}{3}$$

$$= -0.45$$

which does not make any sense

$\therefore$ $x = 0.3$ mol litre$^{-1}$

10. (a) The total volume of the solution formed = 2 litres.

Let $x$ ml be the volume of $H_2SO_4$ used for making the solution.

$$\text{Strength of sodium carbonate solution} = \frac{\text{g litre}^{-1}}{\text{Eq. wt.}}$$

$$= \frac{\frac{1 \times 1000}{100}}{\frac{286}{2}} \text{N}$$

$$= \frac{10}{143} \text{N}$$

$$\text{Strength of the acid mixture} = \frac{42.9 \times 10}{143} \times \frac{1}{30} \text{N}$$

$$= \frac{1}{10} \text{N}$$

According to the data given in the question, 2 litres of the acid mixture comprises 5 ml of 8 N nitric acid, 4.8 ml of 5 N hydrochloric acid and $x$ ml of 17 M sulphuric acid.

$$\therefore \quad \frac{5 \times 8}{2000} + \frac{4.8 \times 5}{2000} + \frac{17 \times 2 \times x}{2000} = \frac{1}{10}$$

or $$\frac{1}{50} + \frac{3}{250} + \frac{34x}{2000} = \frac{1}{10}$$

or $$\frac{40 + 24 + 34x}{2000} = 0.1$$

or $$\frac{64 + 34x}{2000} = 0.1$$

or $$34x = 200 - 64$$
$$= 136$$

or $$x = \frac{136}{34} = 4 \text{ ml}$$

Thus only 4 ml of sulphuric acid has taken part in the acid mixture solution.

$\therefore$ Amount of sulphate ions ($SO_4^{-?}$) in the acid mixture

$$\text{solution} = 17 \times 4 \times \frac{96}{1000}$$

$$= \frac{6528}{1000}$$

$$= 6.53 \text{ gm}$$

(b) As the graph of logarithm of partial pressure versus time is linear, the parameters that can be obtained from the graph are: (i) specific reaction rate, and (ii) order of reaction.

# MATHEMATICS

## PART A

1. (i) (D) none of these
   (ii) (A) AP
   (iii) (C) 0
   (iv) (B) are similar
   (v) (A) $(2, \infty)$
2. (i) (A) $p+q+r=0$, (B) $p^2+q^2+r^2=pq+qr+rp$, (C) $p^3+q^3+r^3 = 3pqr$
   (ii) (C) $f(x)$ is not differentiable at $x=0$
   (iii) (A) $|w_1|=1$, (B) $|w_2|=1$, (C) $\text{Re}(w_1\bar{w}_2)=0$
3. (i) False
   (ii) True
   (iii) False
   (iv) False
   (v) True
   (vi) True
4. (i) zero
   (ii) $-1$
   (iii) $P(A)=P(B)=P(A\cap B)=P(A\cup B)$
   (iv) zero
   (v) $n^n$, $\underline{|n}$
   (vi) $(5, \infty)$
   (vii) arithmetic
   (viii) 8 square units
   (ix) $\frac{5}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$
   (x) first
   (xi) $(-2, 1)$, $[-1, 1]$
   (xii) $\frac{1}{e}$
   (xiii) $\frac{1}{9}$
   (xiv) $x^2+y^2-x=0$

## PART B

5. Let $P(n)=2\cdot 7^n+3\cdot 5^n-5$.
   Substitute

$$n=1.$$

$$P(1)=2\cdot 7^1+3\cdot 5^1-5$$

$$=24$$

which is divisible by 24.

Suppose the identity be true for $n = m$.

$$\therefore \qquad P(m) = 2 \cdot 7^m + 3 \cdot 5^m - 5$$

which is divisible by 24.

Consider the expression for $n = m+1$.

$$\therefore \qquad P(m+1) = 2 \cdot 7^{m+1} + 3 \cdot 5^{m+1} - 5$$
$$= 2 \cdot 7^m \cdot 7^1 + 3 \cdot 5^m \cdot 5^1 - 5$$

Thus
$$P(m+1) - P(m) = 2 \cdot 7^m \cdot 7^1 + 3 \cdot 5^m \cdot 5^1 - 5 - 2 \cdot 7^m - 3 \cdot 5^m + 5$$
$$= 2 \cdot 7^m \cdot 6 + 3 \cdot 5^m \cdot 4$$
$$= 12(7^m + 5^m)$$

But $7^m$ and $5^m$ are both odd and the sum of two odd numbers is even. Thus $(7^m + 5^m)$ is divisible by 2. Further, the difference between $P(m+1)$ and $P(m)$ is divisible by 24 and $P(m)$ itself is divisible by 24. Thus $P(m+1)$ is divisible by 24. From the principle of mathematical induction, the identity is, therefore, true for all $n \in I$ and $n > 0$.

6. (i)

Let
$$I = \int_0^{\pi/2} \frac{x \sin x \cos x \, dx}{\cos^4 x + \sin^4 x}$$

Using
$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right) dx}{\cos^4\left(\frac{\pi}{2} - x\right) + \sin^4\left(\frac{\pi}{2} - x\right)}$$

$$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \cos x \sin x \, dx}{\sin^4 x + \cos^4 x}$$

$$= \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x \, dx}{\sin^4 x + \cos^4 x} - \int_0^{\pi/2} \frac{x \sin x \cos x \, dx}{\sin^4 x + \cos^4 x}$$

$$= \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x \, dx}{\sin^4 x + \cos^4 x} - I$$

or
$$I = \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin x \cos x \, dx}{\sin^4 x + \cos^4 x}$$

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin x \cos x \, dx}{1 - 2 \sin^2 x \cos^2 x}$$

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin x \cos x \, dx}{1 - \frac{\sin^2 2x}{2}}$$

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin 2x \, dx}{1 + (1 - \sin^2 2x)}$$

$$= \frac{\pi}{4} \int_0^{\pi/2} \frac{\sin 2x \, dx}{1 + \cos^2 2x}$$

Put $\cos 2x = t$

or $-2\sin 2x\,dx = dt$

when $x = 0, \quad t = 1.$

when $x = \frac{\pi}{2}, \ t = -1.$

Thus,

$$I = \frac{\pi}{4}\int_{1}^{-1} \frac{-dt}{2(1+t^2)}$$

$$= -\frac{\pi}{8}[\tan^{-1}t]_1^{-1}$$

$$= -\frac{\pi}{8}[\tan^{-1}(-1) - \tan^{-1}(1)]$$

$$= -\frac{\pi}{8}[-2\tan^{-1}(1)] = -\frac{\pi}{8}[-2 \times \pi/4]$$

$$= \frac{\pi^2}{16}$$

Hence,

$$\int_0^{\pi/2} \frac{x \sin x \cos dx}{\sin^4 x + \cos^4 x} = \frac{\pi^2}{16}$$

6. (ii) Let $I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

Substitute

$$\sqrt{x} = \sin\theta.$$

$\therefore$

$$\frac{1}{2\sqrt{x}}\,dx = \cos\theta\,d\theta$$

Thus,

$$I = \int \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times 2\sin\theta\cos\theta\,d\theta$$

$$= \int \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times \sqrt{\frac{1-\sin\theta}{1-\sin\theta}} \times 2\sin\theta\cos\theta\,d\theta$$

$$= \int \frac{(1-\sin\theta)}{\cos\theta} \times 2\sin\theta\cos\theta\,d\theta$$

$$= 2\int (\sin\theta - \sin^2\theta)\,d\theta$$

$$= 2\int \sin\theta\,d\theta - 2\int \frac{(1-\cos 2\theta)}{2}\,d\theta$$

$$= -2\cos\theta - \int (1-\cos 2\theta)\,d\theta$$

$$= -2\cos\theta - \theta + \frac{\sin 2\theta}{2} + C$$

$$= -\theta - 2\cos\theta + \sin\theta\cos\theta + C$$

where $C$ is constant of integration,

Thus,

$$I = -\sin^{-1}\sqrt{x} - 2\sqrt{1-x} + \sqrt{x}\sqrt{1-x} + C, \text{ as}$$

$$\sin\theta = \sqrt{x}$$

7. Suppose $l$ is the length of the ladder.

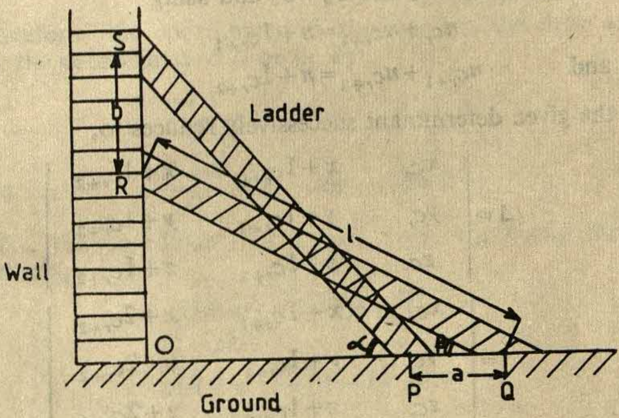



Fig. 22

From Fig. 22,

$$OQ = l\cos\beta$$
$$OP = l\cos\alpha$$

$\therefore$ $$OQ - OP = l(\cos\beta - \cos\alpha)$$

or $$a = l(\cos\beta - \cos\alpha) \quad \text{(i)}$$

Similarly,

$$OS = l\sin\alpha$$
$$OR = l\sin\beta$$

$\therefore$ $$OS - OR = l(\sin\alpha - \sin\beta)$$

or $$b = l(\sin\alpha - \sin\beta) \quad \text{(ii)}$$

Dividing (ii) by (i),

$$\frac{a}{b} = \frac{(\cos\beta - \cos\alpha)}{(\sin\alpha - \sin\beta)}$$

$$= \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

or $$\frac{a}{b} = \tan\left(\frac{\alpha+\beta}{2}\right)$$

Thus,

$$a = b\tan\left(\frac{\alpha+\beta}{2}\right)$$

8. (a) Let the given determinant be written as

$$\Delta = \begin{vmatrix} {}^{x}C_{r} & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_{r} & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_{r} & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix}$$

Operating $C_2+C_1$ and $C_3+C_2$ and using

$${}^{n}C_{r}+{}^{n}C_{r+1}={}^{n+1}C_{r+1}$$

and $${}^{n}C_{r+1}+{}^{n}C_{r+2}={}^{n+1}C_{r+2}$$

the given determinant successively reduces to,

$$\Delta = \begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^{z}C_{r} & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$$

$$= \begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_{r} & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

Thus,

$$\begin{vmatrix} {}^{x}C_{r} & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_{r} & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_{r} & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix} = \begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_{r} & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$

(b) Let $O$ be the centre of the circle. $M$ is the mid-point of $AB$ whose coordinates are (1, 4). The equation of the diameter of the circle is $4y=x+7$. The centre of the circle lies on the perpendicular bisector of $AB$. Therefore, the equation of the perpendicular bisector having $-1/0$ as slope and passing through (1, 4) is given by Fig. 23,

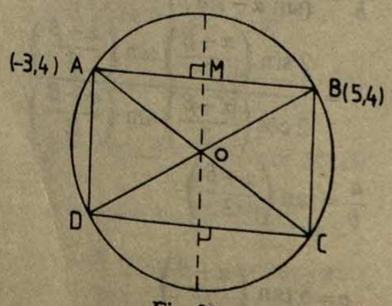



Fig. 23

$$y-4=\frac{-1}{0}(x-1)$$

$$\Rightarrow x=1$$

as $\quad 4y=x+7$

$$y=\frac{1+7}{4}=2$$

Therefore, the coordinates of the centre of the circle are (1, 2). Let the coordinates of $D$ be $(a, b)$

$\therefore \quad \frac{a+5}{2}=1 \quad$ or $\quad a=-3$

and $\quad \frac{b+4}{2}=2 \quad$ or $\quad b=0$

Thus, the coordinates of $D$ are $(-3, 0)$. Using the distance formula,

$$AD=\sqrt{(-3+3)^2+(4-0)^2}$$
$$=4$$
$$AB=\sqrt{(5+3)^2+(4-4)^2}$$
$$=8$$

Area of rectangle $ABCD = AD \times AB$
$$=4\times 8$$
$$=32 \text{ square units}$$

9. Let us find the point of intersection of the two curves (shown in Fig. 24).

$$y=|x-1|$$

and $\quad y=\sqrt{5-x^2}$

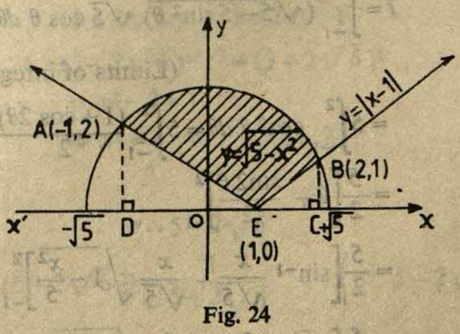



Fig. 24

For $x-1 \geqslant 0$,

$|x-1|=x-1 \quad$ (It will also include solutions for $x-1|=1-x$)

$\Rightarrow \quad x-1=\sqrt{5-x^2}$

or $\quad x^2-2x+1=5-x^2$

or $\quad x^2-x-2=0 \quad$ which gives $x=2 \;\; -1$

$\therefore \qquad y=|2-1|=1$

and $\qquad y=|-1-1|=2$

Thus the points of intersection are (2, 1) and (−1, 2).

If $\qquad y=0 \quad$ where $\quad y=\sqrt{5-x^2}$

then $\qquad 5-x^2=0 \Rightarrow x=\pm\sqrt{5}.$

If $\qquad y=0 \quad$ where $\quad y=|x-1|, \quad$ then $x=1.$

If $\qquad y=\sqrt{5-x^2} \quad$ or $\quad y^2=5-x^2 \Rightarrow x^2+y^2=5.$

This is a circle with radius $\sqrt{5}$.

$$\text{Area } ABCD=\int_{-1}^{2}\sqrt{5-x^2}\,dx$$

$$\text{Area of } \Delta ADE=\frac{1}{2}\times 2\times 2$$

$$=2 \text{ square units}$$

$$\text{Area of } \Delta BCE=\frac{1}{2}\times 1\times 1$$

$$=\frac{1}{2}\text{square unit}$$

Let us now find area *ABCD* by solving

$$\int_{-1}^{2}\sqrt{5-x^2}\,dx.$$

Consider $\qquad I=\int_{-1}^{2}\sqrt{5-x^2}\,dx.$

Put $\qquad x=\sqrt{5}\sin\theta,\ dx=\sqrt{5}\cos\theta\,d\theta.$

$\therefore \qquad I=\int_{-1}^{2}(\sqrt{5-5\sin^2\theta})\sqrt{5}\cos\theta\,d\theta$

(Limits of integral are still in $x$)

$$=5\int_{-1}^{2}\cos^2\theta\,d\theta=5\int_{-1}^{2}\frac{(1+\cos 2\theta)}{2}\,d\theta$$

$$=\frac{5}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]_{-1}^{2}$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{x}{\sqrt{5}}+\frac{x}{\sqrt{5}}\sqrt{1-\frac{x^2}{5}}\right]_{-1}^{2}$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\frac{2}{\sqrt{5}}\sqrt{1-\frac{4}{5}}-\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right)+\frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{5}}\right]$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}+\frac{2}{5}+\frac{2}{5}\right]$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}+\frac{4}{5}\right]$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right]+2$$

$\therefore$ Required area $AEB$ (shown shaded in Fig. 24)

$$= \text{Area } ABCD - \text{Area } ADE - \text{Area } BCE$$

$$=\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right]+2-2-\frac{1}{2}$$

$$=\left\{\frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right]-\frac{1}{2}\right\} \text{ square units}$$

10. $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ (i)

Put $5+2\sqrt{6}=t$ and $x^2-3=y$

then (i) can be written as

$$t^y+\left(\frac{1}{t}\right)^y=10$$

because

$$\frac{1}{t}=\frac{5-2\sqrt{6}}{5+2\sqrt{6}}\times\frac{1}{5-2\sqrt{6}}=5-2\sqrt{6}$$

$\therefore$ $t^{2y}-10t^y+1=0$

or $z^2-10z+1=0$ where $z=t^y$

or $z=\dfrac{10\pm\sqrt{100-4}}{2}=\dfrac{10\pm4\sqrt{6}}{2}=5\pm2\sqrt{6}$

Take $t^y=5+2\sqrt{6}$

or $(5+2\sqrt{6})^y=(5+2\sqrt{6})^1$

or $(5+2\sqrt{6})^{x^2-3}=(5+2\sqrt{6})^1$ $(\because\ y=x^2-3)$

$\Rightarrow x^2-3=1$

$\Rightarrow x^2=4$ or $x=\pm2$

Again,

$t^y=5-2\sqrt{6}$

or $(5+2\sqrt{6})^y=\dfrac{1}{5+2\sqrt{6}}$ $\left(\because\ 5-2\sqrt{6}=\dfrac{1}{5+2\sqrt{6}}\right)$

or $(5+2\sqrt{6})^{y+1}=1=(5+2\sqrt{6})^0$

or $(5+2\sqrt{6})^{x^2-3+1}=(5+2\sqrt{6})^0$ $(\because\ y=x^2-3)$

$\Rightarrow$ $x^2-2=0$

or $x=\pm\sqrt{2}$

Thus the roots of the given equation are $\pm\sqrt{2}$ and $\pm2$.

11. The equations of the given lines are

$$y = x + 2$$

and $\quad y = 7x + 3$

Thus the equations of bisectors of given lines are given by:

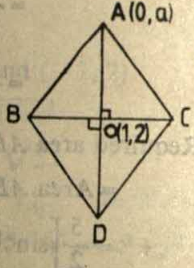



Fig. 25

$$\frac{y-x-2}{\sqrt{2}} = \pm\frac{y-7x-3}{\sqrt{50}}$$

or $$\frac{y-x-2}{1} = \pm\frac{y-7x-3}{5}$$

which gives

$$5(y-x-2) = +(y-7x-3)$$

or $$4y = -2x + 7$$

or $$y = -\frac{1}{2}x + \frac{7}{4} \qquad \text{(i)}$$

Also

$$5(y-x-2) = -(y-7x-3)$$

or $$6y = 12x + 13$$

or $$y = 2x + \frac{13}{6} \qquad \text{(ii)}$$

Slopes of these lines represented by (i) and (ii) are $\frac{-1}{2}$ and 2 respectively.

$$\text{Slope of } AO = \frac{2-a}{1-0} = 2-a$$

As the diagonals of a rhombus bisect each other at right angles

$$2-a = -\frac{1}{2} \text{ or } 2$$

For $$(2-a) = -\frac{1}{2}$$

or $$4-2a = -1$$

or $$a = 5/2$$

For $$(2-a) = 2$$

$\Rightarrow$ $$a = 0$$

Thus the possible coordinates of $A$ are (0, 0) and (0, 5/2).

12. 7 relatives of a man comprise 4 ladies and 3 gentlemen. Also 7 relatives of the man's wife comprise 3 ladies and 4 gentlemen. We are required to find the total number of ways for inviting 3 ladies and 3 gentlemen such that the invited members are 3 from the man's relatives and 3 from his wife's relatives. There are four mutually exclusive ways in which the members can be invited

| | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| Man's men relatives | 0 | 1 | 2 | 3 |
| Man's women relatives | 3 | 2 | 1 | 0 |
| Wife's men relatives | 3 | 2 | 1 | 0 |
| Wife's women relatives | 0 | 1 | 2 | 3 |

Number of ways in case (i) $= {}^4C_3 \times {}^4C_3$

$$= \frac{\underline{|4}}{\underline{|3} \times \underline{|1}} \times \frac{\underline{|4}}{\underline{|3} \times \underline{|1}}$$

$$= 16$$

Number of ways in case (ii) $= {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1$

$$= 3 \times \left(\frac{\underline{|4}}{\underline{|2} \times \underline{|2}}\right)^2 \times 3$$

$$= 3 \times (6)^2 \times 3$$

$$= 324$$

Number of ways in case (iii) $= {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2$

$$= 3 \times 4 \times 4 \times 3$$

$$= 144$$

Number of ways in case (iv) $= {}^3C_3 \times {}^3C_3$

$$= 1 \times 1$$

$$= 1$$

$\therefore$ Total number of ways required $= 16 + 324 = 144 + 1$

$$= 485 \text{ ways}$$

13. As $AD$ is the median,

$$BD = DC = x \text{ (say)} \qquad \text{(i)}$$

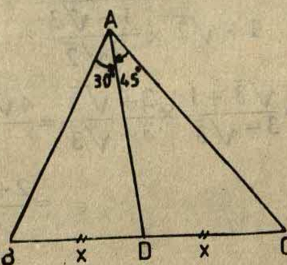



Fig. 26

Using the sine formula in triangles $ABD$ and $ADC$, we have

$$\frac{x}{\sin 45°} = \frac{AD}{\sin \theta} \qquad \text{(ii)}$$

$$\frac{x}{\sin 30°} = \frac{AD}{\sin (105° - \theta)} \qquad \text{(iii)}$$

From (ii) and (iii),

$$x=\frac{AD\sin 45^\circ}{\sin\theta}=\frac{AD\sin 30^\circ}{\sin(105^\circ-\theta)}$$

or
$$\frac{1}{\sqrt{2}\sin\theta}=\frac{1}{2\sin(105^\circ-\theta)}$$

or
$$\sin(105^\circ-\theta)=\frac{1}{\sqrt{2}}\sin\theta$$

or
$$\sin 105^\circ\cos\theta-\cos 105^\circ\sin\theta=\frac{1}{\sqrt{2}}\sin\theta$$

or
$$\sin\theta(1+\sqrt{2}\cos 105^\circ)=\sqrt{2}\sin 105^\circ\cos\theta$$

or
$$\tan\theta=\frac{\sqrt{2}\sin 105^\circ}{1+\sqrt{2}\cos 105^\circ} \qquad \text{(iv)}$$

$$\begin{aligned}\sin(105^\circ)&=\sin(60^\circ+45^\circ)\\&=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ\\&=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}\\&=\frac{\sqrt{3}+1}{2\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\cos(105^\circ)&=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ\\&=\frac{1}{2}\times\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}=\frac{1-\sqrt{3}}{2\sqrt{2}}\end{aligned}$$

Substituting these values in (iv) we get,

$$\begin{aligned}\tan\theta&=\frac{\sqrt{2}\times\frac{\sqrt{3}+1}{2\sqrt{2}}}{1+\sqrt{2}\times\frac{1-\sqrt{3}}{2\sqrt{2}}}\\&=\frac{\sqrt{3}+1}{3-\sqrt{3}}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}=\frac{4\sqrt{3}+6}{6}\\&=\frac{2+\sqrt{3}}{\sqrt{3}}\end{aligned}$$

Refer to Fig. 27.

$$\begin{aligned}PR^2&=3+(2+\sqrt{3})^2\\&=3+4+3+4\sqrt{3}\\&=10+4\sqrt{3}\end{aligned}$$

or
$$PR=\sqrt{10+4\sqrt{3}}$$

$$\sin\theta=\frac{2+\sqrt{3}}{\sqrt{10+4\sqrt{3}}}$$

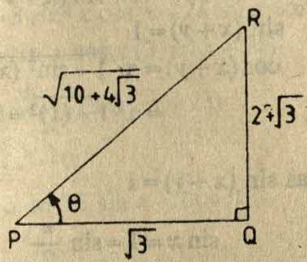



Fig. 27

From (i) and (ii),

$$BC = 2x$$

$$= 2\,\frac{AD}{\sin\theta} \times \sin 45^\circ \qquad \left(\because\ AD = \frac{1}{\sqrt{11-6\sqrt{3}}}\right)$$

$$= 2 \times \left(\frac{1}{\sqrt{11-6\sqrt{3}}} \times \frac{\sqrt{10+4\sqrt{3}}}{2+\sqrt{3}} \times \frac{1}{\sqrt{2}}\right)$$

$$= 2 \times \left(\frac{1}{\sqrt{11-6\sqrt{3}}} \times \frac{\sqrt{5+2\sqrt{3}}}{\sqrt{(2+\sqrt{3})^2}}\right)$$

$$= 2 \times \left(\frac{\sqrt{5+2\sqrt{3}}}{\sqrt{(11-6\sqrt{3})(7+4\sqrt{3})}}\right)$$

$$= 2 \times \left(\frac{\sqrt{5+2\sqrt{3}}}{\sqrt{77+44\sqrt{3}-42\sqrt{3}-72}}\right)$$

$$= 2 \times \left(\frac{\sqrt{5+2\sqrt{3}}}{\sqrt{5+2\sqrt{3}}}\right) = 2 \times 1 = 2 \text{ units}$$

14. Equation of the curve is

$$y = \cos(x+y) \qquad \text{(i)}$$

Differentiating wrt '$x$'

$$\frac{dy}{dx} = -\sin(x+y)\left\{1 + \frac{dy}{dx}\right\}$$

or $$\frac{dy}{dx}\{1+\sin(x+y)\} = -\sin(x+y)$$

or $$\frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)} \qquad \text{(ii)}$$

Since the tangents are parallel to $x+2y=0$ or $y = -\dfrac{x}{2}$ (Slope $= -1/2$) their slopes must be equal.

$$\therefore \qquad \frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$

or $$2\sin(x+y)=1+\sin(x+y)$$

or $$\sin(x+y)=1$$

$\therefore$ $$\cos(x+y)=\sqrt{1-\sin^2(x+y)}$$
$$=\sqrt{1-(1)^2}=0$$

From (i),

$$y=0 \text{ as } \sin(x+y)=1$$

or $$\sin x=1=\sin\frac{\pi}{2}$$

$\therefore$ $$x=n\pi+(-1)^n\frac{\pi}{2},\ n\in I$$

For $$n=0\Rightarrow x=0$$

For $$n=1\Rightarrow x=\frac{\pi}{2}$$

For $$n=2\Rightarrow x=\frac{5\pi}{2}\notin[-2\pi,2\pi]$$

For $$n=-1\Rightarrow x=-\frac{3\pi}{2}$$

For $$n=-2\Rightarrow x=-\frac{3\pi}{2}$$

For $$n=3\Rightarrow x=-\frac{7\pi}{2}$$

Thus $x=\frac{\pi}{2}$ and $-\frac{3\pi}{2}$ are only permissible as $-2\pi\leqslant x\leqslant 2\pi$.

Thus the points on the curve are $\left(\frac{\pi}{2},0\right)$ and $\left(-\frac{3\pi}{2},0\right)$. The equations of the tangents are given by

$$y-0=-\frac{1}{2}\left(x-\frac{\pi}{2}\right)$$

or $$y=-\frac{1}{2}\left(x-\frac{\pi}{2}\right)$$

or $$4y+2x-\pi=0$$

Also,

$$y-0=-\frac{1}{2}\left(x+\frac{3\pi}{2}\right)$$

or $$2y=-x-\frac{3\pi}{2}$$

or $$4y+2x+3\pi=0$$

Thus the required equations of tangents are given by:

$$4y+2x-\pi=0$$
$$4y+2x+3\pi=0$$

15. $$f(x)=\sin^3 x+\lambda \sin^2 x \quad \text{(i)}$$

$$f'(x)=3\sin^2 x\cos x+2\lambda \sin x\cos x$$
$$=\sin x\cos x(3\sin x+2\lambda)$$
$$=\frac{\sin 2x}{2}(3\sin x+2\lambda) \quad \text{(ii)}$$

For extreme values, $f'(x)=0$.

$\therefore$ $$\sin 2x=0 \text{ and } 3\sin x+2\lambda=0 \quad \text{(iii)}$$

Thus,

$$\sin 2x=0$$
$$\Rightarrow \quad 2x=n\pi$$
$$\Rightarrow \quad x=0 \text{ for } n=0 \text{ as } -\frac{\pi}{2}<x<\frac{\pi}{2}$$

Again,

$$3\sin x+2\lambda=0$$
$$\Rightarrow \quad \sin x=-\frac{2\lambda}{3}\Rightarrow \lambda=0 \qquad (\because \ |\sin x|<1 \Rightarrow \left|\frac{2\lambda}{3}\right|<1)$$

Thus the two equations in (iii) would be identical for two distinct critical values in $-\frac{\pi}{2}<x<\frac{\pi}{2}$ for $\lambda\in(-3/2, 0)\cup(0, 3/2)$.

Differentiating (ii) wrt $x$ we get,

$$f''(x)=\cos 2x(3\sin x+2\lambda)+\frac{\sin 2x}{2}(3\cos x)$$

or $$f''(x)=3\sin x\cos 2x+2\lambda\cos 2x+3\sin x\cos^2 x$$
$$=3\sin x(1-2\sin^2 x)+2\lambda(1-2\sin^2 x)+3\sin x(1-\sin^2 x)$$
$$=6\sin x-9\sin^3 x-4\lambda\sin^2 x+2\lambda$$

Therefore for $\sin x=0, f''(x)=2\lambda$; and for

$$\sin x=-\frac{2\lambda}{3}, f''(x)=-2\lambda\left(1-\frac{4\lambda^2}{9}\right).$$

As $f''(x)$ has opposite signs for $\sin x=0$ and $\sin x=-\frac{2\lambda}{3}$ for all $\lambda\in(-3/2, 0)\cup(0, 3/2)$, therefore, there lies exactly one maximum and exactly one minimum in the interval above.

16. Let $S$ be the sum of the series.

$$\therefore S=\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\ldots \text{ up to } m \text{ terms}\right]$$

$$=\sum_{r=0}^{n}(-1)^r\, {}^nC_r\frac{1}{2^r}+\sum_{r=0}^{n}(-1)^r\, {}^nC_r\frac{3^r}{2^{2r}}+\sum_{r=0}^{n}(-1)^r\, {}^nC_r\frac{7^r}{2^{3r}}$$

$$+\ldots+\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left(\frac{2^m-1}{2^m}\right)^n \quad \text{(i)}$$

Consider $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\,\frac{1}{2^r}$

$$=(-1)^0\,{}^nC_0\,\frac{1}{2^0}+(-1)^1\,{}^nC_1\,\frac{1}{2^1}+(-1)^2\,{}^nC_2\,\frac{1}{2^2}+\dots$$

$$=1-n\times\frac{1}{2}+n(n-1)\frac{1}{2^2}+\dots$$

$$=\left(1-\frac{1}{2}\right)^n$$

Similarly, other terms in the expression can be simplified, and therefore (i) can be written as

$$S=\left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{2^2}\right)^n+\left(1-\frac{7}{2^3}\right)^n+\dots+\left(1-\frac{2^m-1}{2^m}\right)^n$$

$$=\left(\frac{1}{2}\right)^n+\left(\frac{1}{2^2}\right)^n+\left(\frac{1}{2^3}\right)^n+\dots+\left(\frac{1}{2^m}\right)^n$$

$$=\frac{1}{2^n}+\frac{1}{2^{2n}}+\frac{1}{2^{3n}}+\dots+\frac{1}{2^{mn}}$$

Using $a+ar+ar^2+ar^3+\dots+ar^n=\frac{a(1-r^n)}{1-r},\ r<1$

$$S=\frac{\frac{1}{2^n}\left[1-\left(\frac{1}{2^n}\right)^m\right]}{1-\frac{1}{2^n}}$$

$$=\frac{\frac{1}{2^n}\left(1-\frac{1}{2^{mn}}\right)}{\frac{2^n-1}{2^n}}$$

$$=\frac{1-\frac{1}{2^{mn}}}{2^n-1}=\frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

Thus the sum of the given series,

$$S=\frac{2^{mn}-1}{2^{(nm+1)}-2^{mn}}$$

17. $f(t)=t^3-t^2+t+1$

$f'(t)=3t^2-2t+1$

for $f'(t)=0\Rightarrow t=\frac{2\pm\sqrt{4-12}}{6}$ or $t=\frac{2\pm\sqrt{-8}}{6}$.

Therefore, only the end points need be checked.

At $x=0$, $0\leqslant t\leqslant 0\Rightarrow t=0\Rightarrow f(t)=1$ and $g(x)=1$ at $x=0$.

At $x=1$, $0\leqslant t\leqslant 1$ and $t=1\Rightarrow f(x)=1-1+1+1=2$;

$t=0\Rightarrow f(x)=1$ and $g(x)=2$ at $x=1$.

Also, the left-hand limit, therefore, is 2.

Right-hand limit $=\lim_{x\to 1}(3-x)$

$=3-1=2$

As right-hand limit $= g(x)\Big|_{x=1}$ = left-hand limit = 2; $g(x)$ is a continuous function at $x=1\in(0, 2)$.

Let us now check the differentiality.

$$g'(x) \quad (\text{for } 1 < x \leqslant 2) = -1$$

$$= \text{right-hand derivative at } x=1$$

$$f'(x) = 3x^2 - 2x + 1$$

At $x = \dfrac{1}{2}$,

$$f'(x) = 3 \times \frac{1}{4} - 2 \times \frac{1}{2} + 1$$

$$= \frac{3}{4} > 0$$

The function $f(x)$ is, therefore, increasing in (0, 1).

Thus,

$$\forall \quad 0 \leqslant x \leqslant 1, g(x) = x^3 - x^2 + x + 1$$

$$\Rightarrow \qquad g'(x) = 3x^2 - 2x + 1$$

At $x = 1$,

$$g'(x) = 3 \times 1 - 2 \times 1 + 1$$

$$= 2$$

Thus, right-hand derivative $\neq$ left-hand derivative. Therefore, the function $g(x)$ is not differentiable at $x = 1\in(0, 2)$.

18. The candidate may tick one or more of the alternatives. As each alternative may or may not be chosen, the total number of possibilities at his disposal are given by:

$$2^4 - 1 = 16 - 1$$

$$= 15$$

Therefore, the probability that the questions are correctly answered by the candidate is 1/15.

However, the candidate may be correct on the first, second, third or fourth round. As these events are mutually exclusive, total probability will be given by:

$$\frac{1}{15} + \frac{14}{15} \times \frac{1}{14} + \frac{14}{15} \times \frac{13}{14} \times \frac{1}{13}$$

$$= \frac{1}{15} + \frac{1}{15} + \frac{1}{15}$$

$$= \frac{3}{15}$$

$$= \frac{1}{5}$$

**Thus the probability that the candidate will get marks in the question is 1/5.**

# ENGLISH

## PART A

1. (i) imminent
   (ii) artiste
   (iii) complement
   (iv) tortuous
   (v) intuition
   (vi) responsive
   (vii) Barring
   (viii) facets
   (ix) recurrence
   (x) brutishly
   (xi) deference
   (xii) cowardly
   (xiii) incarnate
2. (i) has been
   (ii) would enjoy
   (iii) have been
   (iv) had been solved
   (v) came
   (vi) were eating
   (vii) meet
   (viii) would have been
   (ix) is/was pleased
   (x) was surprised
   (xi) had worked
   (xii) are being discussed
3. (i) You should know that I *insulted neither him nor anyone else* who came with him.
   (ii) Our chairman *should be liberal rather than strict*.

   *Or*

   Our chairman *should be more liberal than strict*.
   (iii) What he is looking for is a wife. (It does not need any corrections).
   (iv) Your milk is getting cold, *drink it up quickly*.
   (v) Some of the *books which you seem to have read have become* best-sellers already.
   (vi) His plans were *known to* everyone in his office.
   (vii) Would you bring me *two or three* bananas now that you are going to the market?
   (viii) Strangely enough, it was the thief who needed protection from the police!

(ix) I don't care whether or not your car breaks down. (It does not need any corrections).

(x) I will devote myself *to helping the* poor and the downtrodden of my country.

4. (i) She demanded (of him) why he didn't work hard enough for her.

(ii) The Holi celebrations should not be made an excuse for rowdy behaviour.

(iii) Few of the leaders of our time have been as good as Surya.

(iv) This grammar is too difficult (for us) to understand.

(v) Shyam exclaimed with happiness that it was a marvellous dinner.

(vi) He died rich.

(vii) Rajiv is fun to be with.

(viii) Hasn't she?

(ix) Hardly had I understood the problem when he hustled me into giving my views.

(x) The committee are divided on this issue.

5. Of course, the memory can be improved.
It can be improved by anyone with normal intelligence.
Moreover, the methods by which it can be cultivated are all founded on practical common sense.
Every experience in life makes an impression on some of the cells of the brain, or other nerve centres.
One part of the problem of memory improvement is to emphasize these impressions.
The other part involves filling them away in such an orderly manner that many will remain intact.

## PART B

*Important note*: The candidate is expected to answer questions of 'PART-B' in his own words as far as possible. Marks for compositions are awarded on the criteria of (i) relevance of subject matter, (ii) control over language and (iii) organization, which involves the development of the subject in a logical and coherent manner. Unduly long answers (in case the word-limit is mentioned) are penalised. Serious errors of grammar and usage will also be penalised. A mere reproduction of sentences from the original passage (in case of Question 9) does not fetch any marks. For getting good marks in 'Part B' answers should be brief and relevant. Above all, the candidate must have a good control over the English language.

# 1986

# QUESTION PAPERS

## PHYSICS

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS: 100

*N.B.*—(1) Answers must be written in English.
(2) Figures in brackets on the right hand margin indicate marks for the questions.
(3) Answer to Part 'A' *must* be given only on the *first six pages* of the answer-book *in the order in which the questions appear in the question paper.*
(4) Answer to each question of Part 'B' should begin on a fresh page.
(5) Answer all parts of a question at one place.
(6) You are advised not to spend more than sixty minutes in answering Part 'A'.

Useful physical data—

Velocity of sound in air = 340 m/s
Velocity of light in vacuum = $3 \times 10^8$ m/s
Acceleration due to gravity (g) = 9.8 m/s$^2$
Planck's constant = $6.6 \times 10^{-34}$ J s
1 eV = $1.6 \times 10^{-19}$ J
$\mu_0 = 1.26 \times 10^{-6}$ H/m
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m$^2$/C$^2$
Mass of proton = $1.67 \times 10^{-27}$ kg
Mass of neutron = $1.675 \times 10^{-27}$ kg
Specific heat of water = 1.0 kcal/kg/°C
Latent heat of steam at 100°C = 540 kcal/kg

### PART A

**Marks**

1. In each of the statements given below several alternatives are given. In some cases more than one alternative is correct. Select the correct

alternative(s) in each case and write down the corresponding letter(s) (A), (B), (C) or D in your answer book. For each part marks will be awarded only if ALL the correct alternatives are selected and written:

$(2 \times 10 = 20)$

(i) The dimensions of the quantities in one (or more) of the following pairs are the same, identify the pair(s).

(A) Torque and work
(B) Angular momentum and work
(C) Energy and Young's modulus
(D) Lightyear and wavelength.

(ii) A ball hits the floor and rebounds after an inelastic collision. In this case:

(A) the momentum of the ball just after the collision is the same as that just before the collision.
(B) the mechanical energy of the ball remains the same in the collision.
(C) the total momentum of the ball and the earth is conserved.
(D) the total energy of the ball and the earth is conserved.

(iii) A shell is fired from a cannon with a velocity $V$ (m/s) at an angle $\theta$ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/s) of the other piece immediately after the explosion is:

(A) $3\,V\cos\theta$ (B) $2\,V\cos\theta$ (C) $\frac{3}{2}\,V\cos\theta$
(D) $\frac{\sqrt{3}}{2}\,V\cos\theta$

(iv) A reference frame attached to the earth

(A) is an inertial frame by definition.
(B) can not be an inertial frame because the earth is revolving round the sun.
(C) is an inertial frame because Newton's laws are applicable in this frame.
(D) can not be an inertial frame because the earth is rotating about its own axis.

(v) A simple pendulum of length $L$ and mass (bob) $M$ is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $\theta(\,|\theta| < \phi)$, the tension in the string and the velocity of the bob are $T$ and $V$ respectively. The following relations hold good under the above conditions:

(A) $T\cos\theta = Mg$
(B) $T - Mg\cos\theta = \frac{MV^2}{L}$
(C) the magnitude of the tangential acceleration of the bob $|a_T| = g\sin\theta$
(D) $T = Mg\cos\theta$

(vi) Steam at 100°C is passed into 1.1 kg of water contained in a

calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilograms is

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

(vii) Two thin long parallel wires separated by a distance $b$ are carrying a current $i$ A each. The magnitude of the force per unit length exerted by one wire on the other is

(A) $\frac{\mu_0 i^2}{b^2}$ (B) $\frac{\mu_0 i^2}{2\pi b}$ (C) $\frac{\mu_0 i}{2\pi b}$ (D) $\frac{\mu_0 i}{2\pi b^2}$

(viii) A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen

(A) half the image will disappear
(E) complete image will be formed
(C) intensity of the image will increase
(D) intensity of the image will decrease

(ix) A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz).

(A) 1024 (B) 512 (C) 256 (D) 128

(x) The mass number of a nucleus is

(A) always less than its atomic number.
(B) always more than its atomic number.
(C) sometimes equal to its atomic number.
(D) sometimes more than and sometimes equal to its atomic number.

2. Read the statements carefully. Indicate the sub-division number and write down the matter corresponding to each blank strictly in the order in which it appears in the statement: $(2 \times 6 = 12)$

(i) When the number of electrons striking the anode of an X-ray tube is increased the. . . . . . of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased the cut-off wavelength of the emitted X rays. . . . . . .

(ii) In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude $A$ and of wavelength $\lambda$. In another experiment with the same set-up the two slits are sources of equal amplitude $A$ and wavelength $\lambda$, but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is. . . . .

(iii) Two small balls having equal positive charges $Q$ (C) on each are suspended by two insulating strings of equal length $L$ (m) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is . . . . . . . . . . and the tension in each string is. . . . . . . N.

(iv) Two simple harmonic motions are represented by the equations

$$y_1 = 10 \sin (4\pi t + \pi/4) \text{ and}$$
$$y_2 = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

Their amplitudes are in the ratio of . . . . . . .

(v) When Boron nucleus ($^{10}_{5}B$) is bombarded by neutrons, $\alpha$-particles are emitted. The resulting nucleus is of the element. . . . . . and has the mass number. . . . .

(vi) Atoms having the same. . . . . . . . but different . . . . . are called isotopes.

3. State whether the following statements are TRUE or FALSE, giving reasons in brief to support your answer. No marks will be awarded if correct reasons are not given. $(3 \times 6 = 18)$

(i) Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference $V$ as shown in the figure. The forces on the two protons are identical.

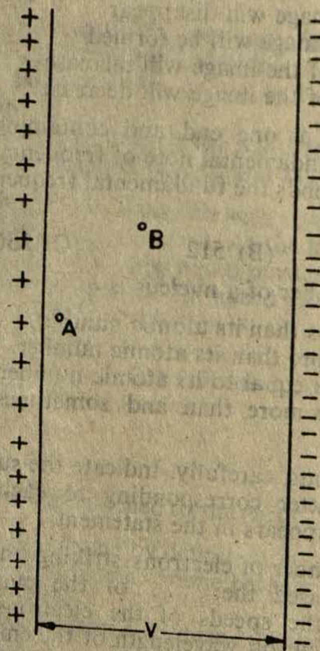



Fig. 1

(ii) In a photoelectric emission process the maximum energy of the photo-electrons increases with increasing intensity of the incident light.

(iii) A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises.

(iv) A coil of metal wire is kept stationary in a non-uniform magnetic field. An emf is induced in the coil.

(v) A thin uniform circular disc of mass $M$ and radius $R$ is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity $\omega$. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$.

(vi) For a diode the variation of its anode current $I_a$ with the anode voltage $V_a$ at two different cathode temperatures $T_1$ and $T_2$ is shown in the figure. The temperature $T_2$ is greater than $T_1$.

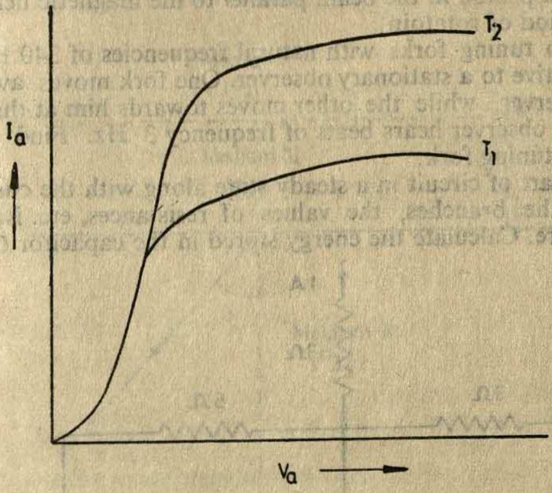



Fig. 2

## PART B

4. Answer any *two*.

(a) Two satellites $S_1$ and $S_2$ revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of the orbit of $S_1$ is $10^4$ km. When $S_2$ is closest to $S_1$, find, (6)

(i) the speed of $S_2$ relative to $S_1$.

(ii) the angular speed of $S_2$ as actually observed by an astronaut in $S_1$.

(b) A body falling freely from a given height $H$ hits an inclined plane in its path at a height $h$. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of $(h/H)$ the body will take maximum time to reach the ground? (6)

(c) A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal ends containing air at the same pressure $P$. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46 cm and 44.5 cm respectively. Calculate the pressure $P$ in centimetres of mercury. (The temperature of the system is kept at 30°C). (6)

5. (a) There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? (6)

(b) A beam of protons with a velocity $4 \times 10^5$ m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotatoin). (6)

6. (a) Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. (8)

(b) A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances, etc. is shown in the figure. Calculate the energy stored in the capacitor $C$ ($4\mu$ F). (4)

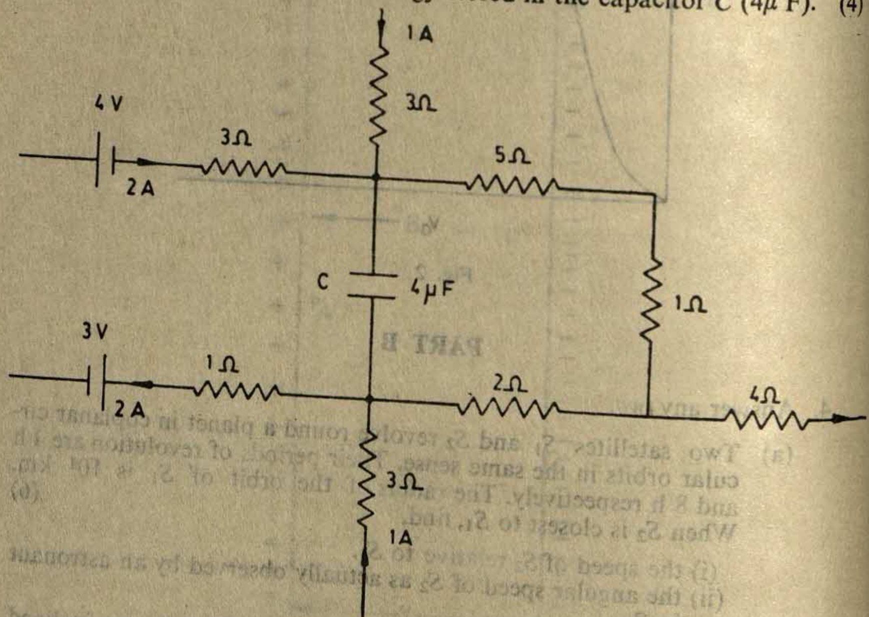



Fig. 3

7. (a) An electric heater is used in a room of total wall area 137 m² to maintain a temperature of +20°C inside it, when the outside temperature is −10°C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m/°C respectively. (8)

(b) Monochromatic light is incident on a plane interface $AB$ between two media of refractive indices $n_1$ and $n_2$ ($n_2 > n_1$) at an angle of incidence $\theta$ as shown in the figure. The angle $\theta$ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab DEFG of uniform thickness and of refractive index $n_3$ is introduced on

the interface (as shown in the figure), show that for any value of $n_3$ all light will ultimately be reflected back again into medium II. Consider separately the cases (6)

(i) $n_3 < n_1$ and (ii) $n_3 > n_1$

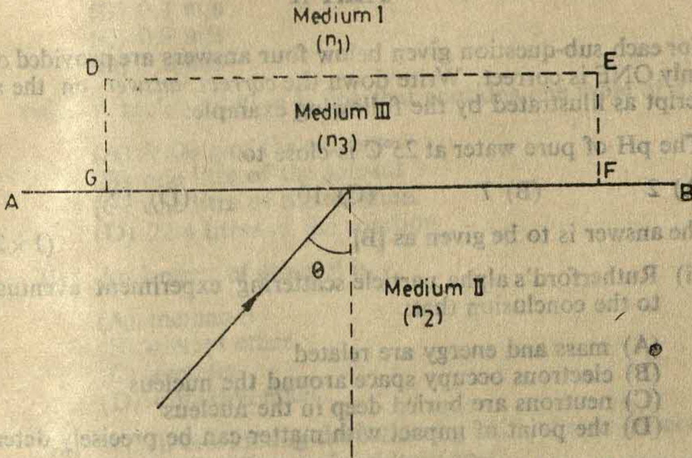



Fig. 4

# CHEMISTRY

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

*Note:* (1) Answer must be written in English.
(2) Figures in brackets on the right hand margin indicate marks for the questions.
(3) Answers to Part 'A' *must* be given only on the *first six pages* of the answer book *in the order in which the questions appear in the question paper*.
(4) Answer to each question of Part 'B' should begin on a fresh page.
(5) Answer all parts of a question at one place.
(6) You are advised not to spend more than sixty minutes in answering Part A.
(7) Use of CALCULATORS is NOT permitted. Log tables may be used.
(8) Values of some physical constants:

Avogadro's number = $6.022 \times 10^{23}$ per mole
Gas constant = 2 cal/mole. deg = 8.314 J/mole. deg
= 0.082 litre-atm/mole. deg
Planck's constant = $6.626 \times 10^{-27}$ erg sec
Velocity of light = $3.0 \times 10^{10}$ cm/sec
1 Å = $10^{-8}$ cm; Faraday constant = 96,500 coul/equivalent

*Atomic masses*

H = 1; C = 12; N = 14; O = 16; Mg = 24; S = 32

*Atomic numbers*

H = 1; C = 6; N = 7; O = 8; F = 9; Ar = 18; Ca = 20;
Mg = 12; S = 16; Cl = 17; CO = 27; Sn = 50; Hg = 80

## PART A

1. For each sub-question given below four answers are provided of which only ONE is correct. Write down the *correct answer* on the answer-script as illustrated by the following example:

[The pH of pure water at 25°C is close to

(A) 2 (B) 7 (C) 10 (D) 12

The answer is to be given as [B]

(1 × 20 = 20)

(i) Rutherford's alpha particle scattering experiment eventually led to the conclusion that:

(A) mass and energy are related
(B) electrons occupy space around the nucleus
(C) neutrons are buried deep in the nucleus
(D) the point of impact with matter can be precisely determined

(ii) The bond between two identical non-metal atoms has a pair of electrons:

(A) unequally shared between the two
(B) transferred fully from one atom to another
(C) with identical spins
(D) equally shared between them

(iii) Which one of the following sets of quantum numbers represents as impossible arrangement?

| | $n$ | $l$ | $m_l$ | $m_s$ |
|---|---|---|---|---|
| (A) | 3 | 2 | −2 | 1/2 |
| (B) | 4 | 0 | 0 | 1/2 |
| (C) | 3 | 2 | −3 | 1/2 |
| (D) | 5 | 3 | 0 | −1/2 |

(iv) The compound insoluble in acetic acid is:

(A) calcium oxide
(B) calcium carbonate
(C) calcium oxalate
(D) calcium hydroxide

(v) The pair of compounds which cannot exist together in solution is:

(A) $NaHCO_3$ and NaOH
(B) $Na_2CO_3$ and $NaHCO_3$
(C) $Na_2CO_3$ and NaOH
(D) $NaHCO_3$ and NaCl

(vi) The compound whose 0.1 M solution is basic is:

(A) ammonium acetate
(B) ammonium chloride

(C) ammonium sulphate
(D) sodium acetate

(vii) The average velocity of an ideal gas molecule at 27°C is 0.3 m/s. The average velocity at 927°C will be:

(A) 0.6 m/s
(B) 0.3 m/s
(C) 0.9 m/s
(D) 3.0 m/s

(viii) A molal solution is one that contains one mole of a solute in:

(A) 1000 g of the solvent
(B) one litre of the solvent
(C) one litre of the solution
(D) 22.4 litres of the solution

(ix) An isomer of ethanol is:

(A) methanol
(B) diethyl ether
(C) acetone
(D) dimethyl ether

(x) Which one among the following pairs of ions cannot be separated by $H_2S$ in dilute hydrochloric acid?

(A) $Bi^{3+}$, $Sn^{4+}$
(B) $Al^{3+}$, $Hg^{2+}$
(C) $Zn^{2+}$, $Cu^{2+}$
(D) $Ni^{2+}$, $Cu^{2+}$

(xi) The compound which gives off oxygen on moderate heating is:

(A) cupric oxide
(B) mercuric oxide
(C) zinc oxide
(D) aluminium oxide

(xii) The ratio of the energy of a photon of 2000 Å wavelength radiation to that of 4000 Å radiation is:

(A) 1/4
(B) 4
(C) 1/2
(D) 2

(xiii) The half-life period of a radioactive element is 140 days. After 560 days, one gram of the element will reduce to:

(A) 1/2 g
(B) 1/4 g
(C) 1/8 g
(D) 1/16 g

(xiv) The hydrogen bond is strongest in:

(A) O−H . . . .S
(B) S−H . . . .O
(C) F−H . . . .F
(D) F−H. . . .O

(xv) The hybridisation of sulphur dioxide is:

(A) $sp$
(B) $sp^3$
(C) $sp^2$
(D) $dsp^2$

(xvi) HBr reacts fastest with—

(A) 2-methylpropan-2-ol
(B) propan-1-ol
(C) propan-2-ol
(D) 2-methylpropan-1-ol

(xvii) The reaction of toluene with chlorine in presence of ferric chloride gives predominantly—

(A) benzoyl chloride
(B) m-chlorotoluene
(C) benzyl chloride
(D) *o*- and *p*-chlorotoluene

(xviii) The reaction conditions leading to the best yields of $C_2H_5Cl$ are:

(A) $C_2H_6\ (\text{excess}) + Cl_2 \xrightarrow{\text{uv light}}$

(B) $C_2H_6 + Cl_2 \xrightarrow[\text{room temperature}]{\text{dark}}$

(C) $C_2H_6 + Cl_2\ (\text{excess}) \xrightarrow{\text{uv light}}$

(D) $C_2H_6 + Cl_2 \xrightarrow{\text{uv light}}$

(xix) The highest boiling point is expected for—

(A) *iso*-octane
(B) *n*-octane
(C) 2,2,3,3-tetramethylbutane
(D) *n*-butane

(xx) The bonds present in $N_2O_5$ are—

(A) only ionic
(B) covalent and coordinate
(C) only covalent
(D) covalent and ionic

2. (a) Amongst the four alternate answers given for the sub-questions below, *one* or *more* are correct. Write down *all* the correct answers.

[Example: Solid sodium chloride is (A) ionic (B) brittle (C) non-conductor (D) covalent. Correct answers: (A, B, C)]. $(1 \times 8 = 8)$

(i) If a gas is expanded at constant temperature—

(A) the pressure decreases
(B) the kinetic energy of the molecules remains the same
(C) the kinetic energy of the molecules decreases
(D) the number of molecules of the gas increases

(ii) When $NaNO_3$ is heated in a closed vessel, oxygen is liberated and $NaNO_2$ is left behind. At equilibrium:

(A) addition of $NaNO_2$ favours reverse reaction
(B) addition of $NaNO_3$ favours forward reaction
(C) increasing temperature favours forward reaction
(D) increasing pressure favours reverse reaction

(iii) Phenol is less acidic than:

(A) acetic acid
(B) *p*-methoxyphenol
(C) *p*-nitrophenol
(D) ethanol

(iv) Dipole moment is shown by:

(A) 1.4-dichlorobenzene
(B) cis 1,2-dichloroethene
(C) trans 1,2-dichloroethene
(D) trans 1,2-dichloro-2-pentene

(v) Only two isometric monochloro derivatives are possible for:

(A) *n*-butane
(B) 2,4-dimethylpentane
(C) benzene
(D) 2-methylpropane

(vi) $CO_2$ is isostructural with:

(A) $HgCl_2$
(B) $SnCl_2$
(C) $C_2H_2$
(D) $NO_2$

(vii) The sum of the number of neutrons and proton in the isotope of hydrogen is:

(A) 6
(B) 5
(C) 4
(D) 3

(viii) In the electrolysis of alumina, cryolyte is added to:

(A) lower the melting point of alumina
(B) increase the electrical conductivity
(C) minimise the anode effect
(D) remove impurities from alumina

2. (b) Fill in the blanks with appropriate words: $(1 \times 4 = 4)$

(i) The rate of diffusion of a gas is . . . . . proportional to both . . . . and square root of molecular mass.
(ii) The more . . . . the standard reduction potential, the . . . . is its ability to displace hydrogen from acids.
(iii) The hydrolysis of ethyl acetate in . . . . medium is a . . . . . order reaction.
(iv) A . . . . diol has two hydroxyl groups on . . . . carbon atoms.

8. (a) The EMF of a cell corresponding to the reaction (4)

$Zn(s) + 2H^{+}(aq) \rightarrow Zn^{2+} + (0.1\ M) + H_2\ (g,\ 1\ atm)$ is 0.28 V at 25°C.

Write the half-cell reactions and calculate the pH of the solution at the hydrogen electrode.

$E^{\circ}_{Zn^{2+}/Zn} = -0.76\ V;\quad E^{\circ}_{H^{+}/H_2} = 0$

8. (b) The standard molar heats of formation of ethane, carbon dioxide and liquid water are −21.1, −94.1, and −68.3 kcal respectiviey. Calculate the standard molar heat of combustion of ethane. (2)

9. (a) $^{234}Th_{90}$ disintegrates to give $^{206}Pb_{82}$ as the final product. How many alpha and beta particles are emitted during this process? (2)

9. (b) The vapour pressures of ethanol and methanol are 44.5 mm and 88.7 mm Hg respectively. An ideal solution is formed at the same temperature by mixing 60 g of ethanol with 40 g of methanol. Calculate the total vapour pressure of the solution and the mole fraction of methanol in the vapour. (4)

10. (a) The solubility of $Mg(OH)_2$ in pure water is $9.57 \times 10^{-3}$ g/litre. Calculate its solubility (in g/litre) in 0.02 M $Mg(NO_3)_2$ solution. (5)

10. (b) During the discharge of a lead storage battery, the density of sulphuric acid fell from 1.294 to 1.139 g/ml. Sulphuric acid of density 1.294 g/ml is 39% $H_2SO_4$ by weight and that of density 1.139 g/ml is 20% $H_2SO_4$ by weight. The battery holds 3.5 litres of the acid and the volume remained practically constant during the discharge.

Calculate the number of ampere-hours for which the battery must have been used. The charging and discharging reactions are:

$Pb + SO_4^{2-} = PbSO_4 + 2e^-$ (charging)

$PbO_2 + 4H^+ + SO_4^{2-} + 2e^- = PbSO_4 + 2H_2O$ (discharging)

# MATHEMATICS

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS: 100
(Part A—50)
(Part B—50)

*N.B.*—(1) Answers must be written in English.
(2) Figures in brackets on the right hand margin indicate marks for the questions.
(3) Answers to Part A *must* be given only on the *first six pages* of the answer-book *in the order in which the questions appear in the question paper*.
(4) Answer to each question of Part B should begin on a fresh page.
(5) You are advised not to spend more than sixty minutes in answering Part A.
(6) Answer all parts of a question at one place.
(7) Use of calculator, logarithmic and trigonometric tables and graph paper is not allowed.

## PART A

*N.B.*—1. Attempt all questions in this part.
2. The student is advised to show the rough work used to arrive at the conclusion.

Marks

1. There are *seventeen* parts in this question. Each part has *one* or *more than one* correct answer. For each part, write letters from A, B, C, D, E corresponding to correct answer. $(17 \times 2 = 34)$

(i) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ be a polynominal in a real variable $x$ with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. The function $P(x)$ has

(A) neither a maximum nor a minimum (B) only one maximum (C) only one minimum (D) only one maximum and only one minimum (E) none of these.

(ii) The function $f(x) = 1 + |\sin x|$ is:

(A) continuous nowhere (B) continuous everywhere (C) differentiable (D) not differentiable at $x = 0$ (E) not differentiable at infinite number of points.

(iii) The principal value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is

(A) $-\frac{2\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$ (E) none of these

(iv) If $S$ is the set of all real $x$ such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive, then $S$ contains:

(A) $\left(-\infty, -\frac{3}{2}\right)$ (B) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$ (C) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

(D) $\left(\frac{1}{2}, 3\right)$ (E) none of these

(v) If $C_r$ stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)C_n^2\right],$$

where $n$ is an even positive integer, is equal to

(A) 0 (B) $(-1)^{n/2}(n+1)$ (C) $(-1)^n(n+2)$ (D) $(-1)^n n$ (E) none of these

(vi) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$ (E) none of these

(vii) The points $\left(0, \frac{8}{3}\right)$, (1,3) and (82, 30) are vertices of:

(A) an obtuse angled triangle (B) an acute angled triangle (C) a right angled triangle (D) an isosceles triangle (E) none of these

(viii) The determinant

$$\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$$ is equal to zero, if

(A) $a, b, c$ are in A.P. (B) $a, b, c$ are in G.P.
(C) $a, b, c$ are in H.P.
(D) $\alpha$ is a root of the equation $ax^2+bx+c=0$
(E) $(x-\alpha)$ is a factor of $ax^2+2bx+c$

(ix) All points lying inside the triangle formed by the points (1, 3), (5, 0) and (−1, 2) satisfy:

(A) $3x+2y \geqslant 0$ (B) $2x+y-13 \geqslant 0$ (C) $2x-3y-12 \leqslant 0$
(D) $-2x+y \geqslant 0$ (E) none of these

(x) The expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+a)\right]-2\left[\sin^6\left(\frac{\pi}{2}+a\right)+\sin^6(5\pi-\alpha)\right]$$

is equal to:

(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha+\cos 6\alpha$
(E) none of these

(xi) Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three nonzero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$ is equal to

(A) 0 (B) 1 (C) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)$ $(b_1^2+b_2^2+b_3^2)$
(D) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)$ $(b_1^2+b_2^2+b_3^2)$ $(c_1^2+c_2^2+c_3^2)$
(E) none of these

(xii) Let $[x]$ denote the greatest integer less than or equal to $x$. If $f(x)=[x \sin \pi x]$, then $f(x)$ is:

(A) continuous at $x=0$ (B) continuous in $(-1, 0)$
(C) differential at $x=1$ (D) differential in $(-1, 1)$
(E) none of these

(xiii) There exists a triangle $ABC$ satisfying the conditions

(A) $b \sin A=a,\ A<\frac{\pi}{2}$ (B) $b \sin A>a,\ A>\frac{\pi}{2}$

(C) $b \sin A>a,\ A<\frac{\pi}{2}$ (D) $b \sin A<a,\ A<\frac{\pi}{2}, b>a$

(E) $b \sin A<a,\ A>\frac{\pi}{2},\ b=a$

(xiv) Let $z_1$ and $z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $z_1$ has positive real part and $z_2$ has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$, may be

(A) zero (B) real and positive (C) real and negative
(D) purely imaginary (E) none of these

(xv) A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If, with respect to the new system, $\vec{a}$ has components $p+1$ and 1, then

(A) $p=0$ (B) $p=1$ or $p=\frac{1}{3}$ (C) $p=-1$ or $p=\frac{1}{3}$
(D) $p=1$ or $p=-1$ (E) none of these:

(xvi) If $a$, $b$ and $c$ are distinct positive numbers, then the expression $(b+c-a)(c+a-b)(a+b-c)-abc$ is

(A) positive (B) negative (C) nonpositive
(D) nonnegative (E) none of these

(xvii) A student appears for tests I, II, and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are $p$, $q$ and $\frac{1}{2}$, respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(A) $p=q=1$ (B) $p=q=\frac{1}{2}$ (C) $p=1,\ \ q=0$
(D) $p=1,\ \ q=\frac{1}{2}$ (E) none of these

2. This question contains *eight* incomplete statements. Fill in the blanks so that the statements are correct. Write only the answers.

(i) The solution of equation $\log_7 \log_5 (\sqrt{x+5}+\sqrt{x})=0$ is . . .

(ii) The solution set of the system of equations $x+y=\frac{2\pi}{3}$, $\cos x+\cos y=\frac{3}{2}$, where $x$ and $y$ are real, is . . .

(iii) The equation of the line passing through the points of intersection of the circles $3x^2+3y^2-2x+12y-9=0$ and $x^2+y^2+6x+2y-15=0$ is . . .

(iv) If the quadratic equations $x^2+ax+b=0$ and $x^2+bx+a=0$ $(a \neq b)$ have a common root, then the numerical value of $a+b$ is . . .

(v) The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x=\frac{1}{2}$ is . . .

(vi) If
$$f(x)=\sin x,\quad x\neq n\pi,\qquad n=0, \pm 1, \pm 2, \pm 3, \ldots$$
$$=2, \text{ otherwise}$$
and
$$g(x)=x^2+1,\quad x\neq 0, 2$$
$$=4,\quad x=0$$
$$=5,\quad x=2$$
then $\lim_{x\to 0} g[f(x)]$ is . . .

(vii) If $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of the three mutually exclusive events, then the set of all values of $p$ is . .

(viii) From the point $A$ (0, 3) on the circle $x^2+4x+(y-3)^2=0$, a chord $AB$ is drawn and extended to a point $M$ such that $AM=2AB$. The equation of the locus of $M$ is . . .

## PART B

*N.B.*—1. There are *fifteen* questions in this part. Attempt any *TEN* questions.

2. Justify your answers with mathematical arguments:

3. (a) The position vectors of the points $A$, $B$, $C$ and $D$ are

$$3\hat{i}-2\hat{j}-\hat{k}, \quad 2\hat{i}+3\hat{j}-4\hat{k} \quad -\hat{i}+\hat{j}+2\hat{k} \text{ and } 4\hat{i}+5\hat{j}+\lambda\hat{k}$$

respectively. If the points $A$, $B$, $C$ and $D$ lie on a plane, find the value of $\lambda$. (3)

(b) A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw? (2)

4. (a) Show that the area of the triangle on the Argand diagram formed by the complex numbers $z$, $iz$ and $z+iz$ is $\frac{1}{2}\,|z|^2$. (2)

(b) Complex numbers $z_1$, $z_2$, $z_3$ are the vertices $A$, $B$, $C$ respectively of an isosceles right angled triangle with right angle at $C$. Show that

$$(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$$

(3)

5. Consider the system of linear equations in $x$, $y$, $z$:

$$\begin{aligned}(\sin 3\theta)\ & x-y+z=0\\ (\cos 2\theta)\ & x+4y+3z=0\\ & 2x+7y+7z=0.\end{aligned}$$

Find the values of $\theta$ for which this system has nontrivial solutions. (5)

6. If in a triangle $ABC$, $\cos A \cos B+\sin A \sin B \sin C=1$, show that $a:b:c=1:1:\sqrt{2}$. (5)

7. The equations of the perpendicular bisectors of the sides $AB$ and $AC$ of a triangle $ABC$ are $x-y+5=0$ and $x+2y=0$, respectively. If the point $A$ is $(1, -2)$, and the equation of the line $BC$. (5)

8. Find the area bounded by the curves

$$x^2+y^2=4,\ x^2=-\sqrt{2}\,y \quad \text{and} \quad x=y.$$ (5)

9. The sum of the squares of three distinct real numbers, which are in G.P., is $S^2$. If their sum is $\alpha S$, show that

$$\alpha^2 \in (\tfrac{1}{3}, 1) \cup (1, 3)$$

(5)

10. Using mathematical induction prove that (5)

$$\sum_{k=0}^{n} k^2\, {}^nC_k=n(n+1)\,2^{n-2} \qquad \text{for } n \geqslant 1$$

11. For $a \leqslant 0$, determine all real roots of the equation (5)

$$x^2-2a\,|x-a|-3a^2=0$$

12. A man observes a tower $AB$ of height $h$ from a point $P$ on the ground.

He moves a distance $d$ towards the foot of the tower and finds that the angle of elevation has doubled. He further moves a distance $\frac{3}{4}d$ in the same direction and finds that the angle of elevation is three times that at $P$. Prove that $36h^2 = 35d^2$. (5)

13. Evaluate:

(i) $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

(ii) $\int_0^{\pi} \frac{x\,dx}{1+\cos\alpha \sin x}, \quad 0 < \alpha < \pi$ (2+3)

14. Let $A(p^2, -p)$, $B(q^2, q)$, $C(r^2, -r)$ be the vertices of the triangle $ABC$. A parallelogram $AFDE$ is drawn with vertices $D$, $E$ and $F$ on the line segments $BC$, $CA$ and $AB$ respectively. Using calculus, show that maximum area of such a parallelogram is: (5)

$\frac{1}{4}(p+q)(q+r)(p-r)$

15. Let $f(x)$ be defined in the interval $[-2,2]$ such that

$$f(x) = -1, \quad -2 \leqslant x \leqslant 0$$
$$= x-1, \quad 0 < x \leqslant 2$$

and $$g(x) = f(|x|) + |f(x)|$$

Test the differentiable of $g(x)$ in $(-2, 2)$ (5)

16. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle $C_1$ of diameter 6. If the centre of $C_1$ lies in the first quadrant, find the equation of the circle $C_2$ which is concentric with $C_1$ and cuts intercepts of length 8 on these lines. (5)

17. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelfth testing? (5)

## ENGLISH

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

*Note:* (1) Answers must be written in English.
(2) Figures in brackets on the right hand margin indicate marks for the questions.
(3) Answers to Part A *must* be given only on the *first six pages* of the answer book *in the order in which the questions appear in the question paper*.
(4) Answer to each question of Part B should begin on a fresh page.
(5) Answer all parts of a question at one place.
(6) You are advised not to spend more than sixty minutes in answering Part A.

### PART A

1. Fill in the blanks in each of the following sentences with the correct preposition/adverbial particle. Write the number of the sentence and the word. *Do not reproduce the sentence.* (10)

(i) Professor Kumar will take . . . . . as the new principal tomorrow.
(ii) We all have to adjust ourselves . . . . . . new circumstances.
(iii) The loud music is getting . . . . . . my nerves.
(iv) Because of heavy losses in business he ran . . . . . . . huge debts.
(v) Your experience counts . . . . . . nothing in this job.
(vi) His behaviour borders . . . . . . lunacy.
(vii) Young fans clustered . . . . . the film star.
(viii) My son is apprenticed . . . . . Mr. Lal, a chartered accountant.
(ix) You will always be short of money if you live . . . . . yours means.
(x) They took . . . . . . . an insurance policy soon after they were married.

2. Fill in the blank with the right word in the second sentence in each of the following pairs of sentences. The word should be connected with the part *italicized* in the first sentence of each pair. Write the number of the pair and the word. *Do not reproduce the sentence.* (10)
(Example:

He *knew* history very well.
He had a good . . . . . of history.

Answer: knowledge.)

(i) The corrupt politician is about to be *exposed.*
The corrupt politician faces . . . . . .
(ii) Most of the cups *broke* in transit.
Most of the cups suffered . . . . in transit.
(iii) The passengers were *annoyed* at the delay.
The passengers expressed their . . . . at the delay.
(iv) The court ordered the building to be *demolished.*
The court ordered the . . . . . . . of the building.
(v) This strange word is *repeated* in all his poems.
There is a . . . . . of this strange word in all his poems.
(vi) He told me not to be in such *haste.*
He told me not to . . . . . so much.
(vii) What was your *deduction* from the evidence?
What did you . . . . . from the evidence?
(viii) He had the *distinction* of winning the first prize.
He . . . . . himself by winning the first prize.
(ix) My grandfather *fell* down.
My grandfather had a . . . . .
(x) The visitor *frightened* the child.
The visitor gave the child a . . . .

3. Rewrite the following sentences using the correct forms of the verbs given in brackets: (10)

(i) He would have passed the examination last year if only he (work) a little harder.
(ii) He has given up (look) for the lost dog.
(iii) By the time the police arrived, the thief (bolt) over the wall.
(iv) The chemist claims (discover) a new element.
(v) The house still (burn) when the firemen arrived.
(vi) Don't leave the room until (ask) to do so.
(vii) I know that he (practise) karate for the last five years, but he has not won any belt so far.
(viii) By the time the football season ends this year, he (play) ten important matches.

(ix) No one likes (tell) that he is the second best.
(x) As soon as he reached home yesterday, he (hang) the trophy on the wall.

4. In each of the following groups of sentences (i) to (x) only one sentence is correct. Write the number of the group and the letter which stands for the correct sentence. *Do not reproduce the sentence.* (10)

(i) (A) The class comprised of students with different levels of language ability. (B) The class comprised students with different levels of language ability. (C) The class was comprising students with different levels of language ability.
(ii) (A) Many Indian parents do not let their children choose their own profession. (B) Many Indian parents do not let their children to choose their own profession. (C) Many Indian parents do not let their children in choosing their own profession.
(iii) (A) The speaker stressed the need for raising the educational standards. (B) The speaker stressed on the need for raising the educational standards. (C) The speaker stressed over the need for raising the educational standards.
(iv) (A) The driver paused the bus for a short while. (B) The driver held on the bus for a short while. (C) The driver halted the bus for a short while.
(v) (A) She was named Smita by her grandparents. (B) She was named as Smita by her grandparents. (C) She was named to be Smita by her grandparents.
(vi) (A) Having eaten our lunch, the train came in. (B) After we had eaten our lunch, the train came in. (C) Our lunch having been eaten, the train came in.
(vii) (A) He may not be an eminent scholar, but he is an excellent teacher. (B) He may not be an eminant scholer, but he is an excellent teacher. (C) He may not be an eminent scholer, but he is an excellent teacher.
(viii) (A) You hardly need remind to me to behave in a responsible manner. (B) You hardly need reminding me to behave in a responsible manner. (C) You hardly need remind me to behave in a responsible manner.
(ix) (A) We were asked did we want to eat or not. (B) We were asked whether we wanted to eat or not. (C) We were asked about our wanting to eat or not.
(x) (A) I said 'and'; I did not say 'or'. If I had said or I should have been wrong. (B) I said 'and'; I did not say 'or'. If I had said 'or' I should have been wrong. (C) I said 'and'; I did not say or if I had said 'or'; I should have been wrong.

5. Each of the following five groups of words can be written in only one particular order to form a sentence. Arrange all the words in each group and form a sentence. (10)

(i) happy made news good the him I piece brought of
(ii) population shaky a to leads increasing economy
(iii) their verified bicycle and numbers of parts the separated he the various
(iv) with the with fundamental are begin let concerned problem we me
(v) difficult conducted many of he the him nowhere led experiments

## PART B

6. Write an essay in about 300 words on any *one* of the following: (20)
   - (i) My ideal teacher
   - (ii) Computers and unemployment
   - (iii) A case of injustice
   - (iv) Many disasters are man-made

7. Expand the idea in any *one* of the following statements in about 150 words: (10)
   - (i) Laughter is the shortest distance between two persons.
   - (ii) If only the young knew, if only the old could.
   - (iii) The strongest man is he who stands most alone.

8. Write a paragraph on any *one* of the following in about 150 words. Argue either *for* or *against* the topic. (10)
   - (i) The past is useless.
   - (ii) There is more good than evil in the world.
   - (iii) India's large population is her greatest strength.

9. Read the following passage carefully and answer the questions below it *in your own words*. Do not use more than two sentences for each answer. (10)

It seems to me that there is a good deal of ballyhoo about scientific method. I venture to think that the people who talk most about it are the people who do least about it. Scientific method is what working scientists do, not what other people or even they themselves may say about it. No working scientist when he plans an experiment in the laboratory asks himself whether he is being properly scientific, nor is he interested in whatever method he may be using *as method*. When the scientist ventures to criticize the work of his fellow scientist, as is not uncommon, he does not base his criticism on such glittering generalities as failure to follow the 'scientific method', but his criticism is specific, based on some feature characteristic of the particular situation. The working scientist is always too much concerned with getting down to brass tacks to be willing to spend his time on generalities.

Scientific method is something talked about by people standing on the outside and wondering how the scientist manages to do it. These people have been able to uncover various generalities applicable to at least most of what the scientist does, but it seems to me that these generalities are not very profound, and could have been anticipated by anyone who knew enough about scientists to know what is their primary objective. I think that the objectives of all scientists have this in common—that they are trying to get the correct answer to the particular problem in hand. This may be expressed in more pretentious language as the pursuit of truth. Now if the answer to the problem is correct there must be some way of knowing and proving that it is correct—the very meaning of truth implies the possibility of checking or verification. Hence the necessity for checking his results always inheres in what the scientist does. Furthermore, this checking must be exhaustive, for the truth of a general proposition may be disproved by a single exceptional case. A long experience has shown the scientist that various things are inimical to getting the correct answer. He has found that it is not sufficient to trust the word of his neighbour, but that if he wants to be sure, he must be able to check a result for himself. Hence the scientist is the

enemy of all authoritarianism. Furthermore, he finds that he often makes mistakes himself and he must learn how to guard against them. He cannot permit himself any preconception as to what sort of results he will get, nor must he allow himself to be influenced by wishful thinking or any personal bias. All these things together give that 'objectivity' to science which is often thought to be the essence of the scientific method.

But to the working scientist himself all this appears obvious and trite. What appears to him as the essence of the situation is that he is not consciously following any prescribed course of action, but feels complete freedom to utilize any method or device whatever which in the particular situation before him seems likely to yield the correct answer. In his attack on his specific problem he suffers no inhibitions of precedent or authority, but is completely free to adopt any course that his ingenuity is capable of suggesting to him. No one standing on the outside can predict what the individual scientist will do or what method he will follow. In short, science is what scientists do, and there are as many scientific methods as there are individual scientists.

(i) This passage is based on a contrast between two views on scientific method. What are these two views?
(ii) What is the common goal of all scientists?
(iii) What are the things the scientist should be watchful against?
(iv) Why does the scientist not rely on authority?
(iv) What do you understand by the following phrases?

(a) 'getting down to brass tacks'.
(b) 'suffers no inhibitions'.

# MODEL SOLUTIONS

## PHYSICS

### PART A

**Ans. 1**

(i) (A) Torque and work
(D) Light year and wavelength
(ii) (C) The total momentum of the ball and the earth is conserved
(D) The total energy of the ball and the earth is conserved
(iii) (A) $3V \cos \theta$
(iv) (B) cannot be an inertial frame because the earth is revolving round the sun
(D) cannot be an inertial frame because the eath is rotating about its own axis
(v) (B) $T - Mg \cos \theta = \frac{MV^2}{L}$
(C) the magnitude, of the tangential acceleration of the bob $| a_T | = g \sin \theta$
(vi) (A) 0.130
(vii) (B) $\frac{\mu_0 i^2}{2\pi b}$
(viii) (B) complete image will be formed
(D) intensity of the image will decrease
(ix) (A) 1024
(x) (C) sometimes equal to its atomic number
(D) sometimes more than and sometimes equal to its atomic number

**Ans. 2**

(i) intensity, decreases
(ii) two
(iii) 180°, $\frac{Q^2}{16\pi\epsilon_0 L^2}$
(iv) 1 : 1
(v) Lithium, 7
(vi) atomic number, mass number

**Ans. 3**

(i) *True.* The electric field inbetween the two plates of a parallel plate capacitor charged to a potential difference $V$ is uniform throughout. Therefore, the forces in the two protons $A$ and $B$ are identical irrespective of their distance from the plates.

(ii) *False:* The maximum kinetic energy, $E_{max}$ of photoelectrons is given by

$$E_{max} = h\nu - W_0$$

where $h\nu$ is the energy of incident photons and $W_0$ is work function. The maximum energy of photoelectrons does not depend upon the intensity of incident light.

(iii) *False:* When the ice melts completely, the level of water in the vessel will fall instead of rising. As the density of lead is large, the ice block with the lead shot imbedded in it displaces a large amount of water equivalent to the weight of the lead shot and weight of the ice block. When ice melts, the lead shot sinks displacing water equivalent to its own volume and melting ice also displaces water equivalent to its volume with the result that the level of water falls when the lead shot sinks.

(iv) *False:* An emf is induced in the coil only when the coil moves in a non-uniform magnetic field. A stationary coil does not change magnetic flux, and therefore, emf is not induced.

(v) *False:* Initial angular momentum of mass $M$ and radius $R$ moving with angular velocity $\omega = I\omega = \dfrac{MR^2\omega}{2}$. Final angular momentum of masses $M$ and $\dfrac{M}{4}$ moving with angular velocity $\omega' = \left(\dfrac{MR^2}{2} + \dfrac{MR^2}{4\times 2}\right)\omega'$

As angular momentum is conserved,

$$\left(\frac{MR^2}{2} + \frac{MR^2}{4\times 2}\right)\omega' = \left(\frac{MR^2}{2}\right)\omega$$

or

$$\omega' = \frac{4}{5}\omega$$

Therefore, the angular velocity of the system is $\dfrac{4\omega}{5}$ and not $\dfrac{2\omega}{\sqrt{5}}$.

(vi) *True:* As the temperature of the cathode of the diode increases, a large number of electrons will be emitted. The anode current will, therefore, be higher when $T_2$ is greater than $T_1$.

## PART B

**Ans. 4**

(a) Let $R_1$ and $R_2$ be radii, $v_1$ and $v_2$ be speeds and $T_1$, $T_2$ be periods of revolution of satellites $S_1$ and $S_2$ respectively.

(i) $T_1 = 1$ h

$T_2 = 8$ h

$R_1 = 10^4$ km

Speed of $S_1 = \dfrac{2\pi R_1}{T_1} = \dfrac{2\pi}{1} \times 10^4$

$= 2\pi \times 10^4$ km/h

According to Kepler's law:

$$\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{T_1}{T_2}\right)^3$$

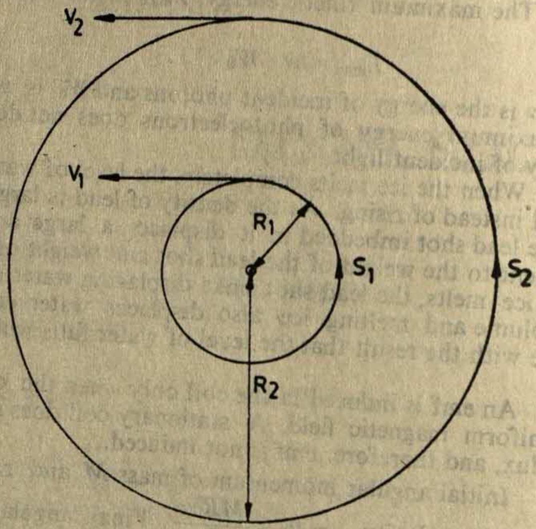



Fig. 5

or
$$R_2 = R_1 \left(\frac{T_2}{T_1}\right)^{2/3}$$
$$= 10^4 \left(\frac{8}{1}\right)^{2/3} = 4 \times 10^4 \text{ km}$$

$$\text{Speed of } S_2 = v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8}$$
$$= \pi \times 10^4 \text{ km/h}$$

The relative speed of $S_2$ with respect to
$$S_1 = v_2 - v_1 = 2\pi \times 10^4 - \pi \times 10^4$$
$$= \pi \times 10^4$$
$$= 3.14 \times 10^4 \text{ km/h}$$

(ii) The angular speed of $S_2$ as actually observed by an astronaut in
$$S_1 = \frac{v_2 - v_1}{R_2 - R_1} = \frac{\pi \times 10^4}{4 \times 10^4 - 10^4}$$
$$= \frac{\pi}{3} \text{ rad/h}$$
$$= \frac{\pi}{3} \times \frac{180°}{\pi} = 60°/\text{h}$$

**Ans. 4**

(b) Let $t_1$ be the time taken by the body to fall through a height $(H - h)$.

$\therefore$
$$H - h = \tfrac{1}{2} g t_1^2$$

or
$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

As the direction of velocity of the body after impact is horizontal, its velocity along the vertical direction is zero.

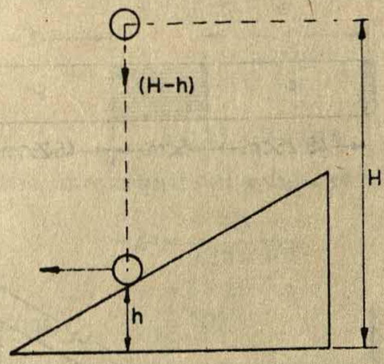



Fig. 6

Let $t_2$ be the time taken by the body to reach the ground after falling through a height $h$.

$\therefore$ $$t_2 = \sqrt{\frac{2h}{g}}$$

Total time, $$t = t_1 + t_2 = \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}$$

In order that $t$ is maximum, $\frac{dt}{dh} = 0$

or $$\frac{1}{2}\left[\frac{2(H-h)}{g}\right]^{-1/2}\left(-\frac{2}{g}\right) + \frac{1}{2}\left(\frac{2h}{g}\right)^{-1/2}\left(\frac{2}{g}\right) = 0$$

or $$\sqrt{\frac{1}{(H-h)}} = \sqrt{\frac{1}{h}}$$

or $$H - h = h$$

or $$H = 2h$$

$\therefore$ $$\frac{h}{H} = \frac{1}{2}$$

**Ans. 4**

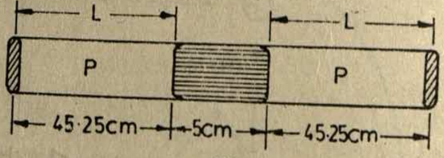
(c) Let $P$ be the pressure in both the columns of air in the horizonta position. When the tube is horizontal, length $L$ of both the ai columns is the same and is given by:

$$L = \frac{46 + 44.5}{2} = 45.25 \text{ cm}$$

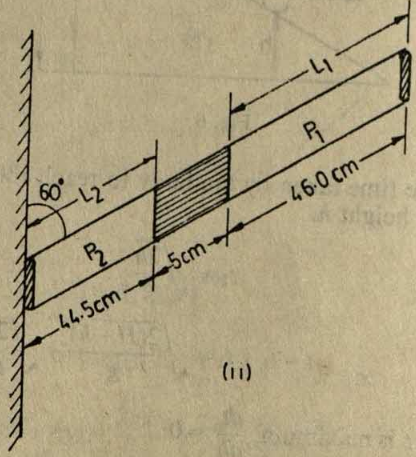
When the tube is inclined at an angle of 60° to the vertical, let $P_1$, $P_2$ be pressures and $L_1$, $L_2$ be lengths of upper and lower air columns respectively. If $A$ is the area of cross-section of the tube., Boyle's law requires that

$$PAL = P_1AL_1 = P_2AL_2$$

or
$$PL = P_1L_1 = P_2L_2$$




(i)




(ii)

Fig. 7

As the column of mercury exerts extra pressure on the lower air column,

$$P_2 = P_1 + 5 \cos 60°$$

$\therefore$
$$P_1L_1 = P_2L_2 = (P_1 + 5 \cos 60°)L_2$$

or
$$P_1(L_1 - L_2) = 5L_2 \cos 60°$$

or
$$P_1 = \frac{5\ L_2 \cos 60°}{(L_1 - L_2)}$$

$\therefore$
$$P = \frac{P_1L_1}{L} = \frac{5L_2L_1 \cos 60°}{L(L_1 - L_2)}$$

$$= \frac{5 \times 46.0 \times 44.5 \times 0.5}{45.25 \times (46.0 - 44.5)}$$

$$= 75.39 \text{ cm of Hg}$$

**Ans. 5**

(a) Kinetic energy of neutrons, $E = 0.0327$ eV

$= 0.0327 \times 1.6 \times 10^{-19}$ J

Mass of a neutron, $m = 1.675 \times 10^{-27}$ kg

If $v$ is the velocity of neutrons, $E=\frac{1}{2}mv^2$

or
$$v=\sqrt{\frac{2E}{m}}$$
$$=\sqrt{\frac{2\times 0.327\times 1.6\times 10^{-19}}{1.675\times 10^{-27}}}$$
$$=0.25\times 10^4 \text{ m/s}$$

Time taken by the neutrons to travel a distance of 10 m with speed

$$v=\Delta t=\frac{10}{0.25\times 10^4}$$
$$=4\times 10^{-3}\text{ s}$$

Half-life of neutrons, $T_{1/2}=700$ s

$\therefore$ Fraction of neutrons decaying per second $=\dfrac{\Delta N}{N_0}$

$$=\frac{\ln 2}{T_{1/2}}$$

Fraction of neutrons decaying in time $\Delta t=\dfrac{\ln 2}{T_{1/2}}\times \Delta t$

$$=\frac{0.693\times 4\times 10^{-3}}{700}$$
$$=3.96\times 10^{-6}$$

**Ans. 5**

(b) Component of velocity of proton perpendicular to the direction of field

$$=v\sin\theta$$
$$=4\times 10^5\times \sin 60^\circ$$
$$=2\times\sqrt{3}\times 10^5\text{ m/s}$$

If $R$ is the radius of helical path of proton beam, force on the proton

$$=e(v\sin\theta)B$$
$$=\frac{m(v\sin\theta)^2}{R}$$

or
$$R=\frac{mv\sin\theta}{eB}$$
$$=\frac{1.67\times 10^{-27}\times 2\times\sqrt{3}\times 10^5}{1.6\times 10^{-19}\times 0.3}$$
$$=0.012\text{ m}$$

If $T$ is time-period to complete one rotation with angular velocity $\omega$,

$$T=\frac{2\pi}{\omega}=\frac{2\pi R}{v\sin\theta}$$
$$=\frac{2\pi m}{eB}\quad\left[\because\ \omega=\frac{2\pi}{T}=\frac{v\sin\theta}{R}=\frac{eB}{m}\right]$$

$$\therefore \qquad T = \frac{2\pi \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3}$$
$$= 2.19 \times 10^{-7} \text{ s}$$

Thus the distance travelled by the proton in the beam parallel to the direction of the magnetic field during one period of rotation

$$= T \times v \cos \theta$$
$$= 2.19 \times 10^{-7} \times 4 \times 10^5 \times \cos 60°$$
$$= 8 \cdot 76 \times 10^{-2} \times \tfrac{1}{2}$$
$$= 4.38 \times 10^{-2} \text{ m}$$

**Ans. 6**

(a) Let $c$ be the velocity of sound in air and $u_s$ be the speed of the tuning fork. When the source moves towards a stationary observer with velocity $u_s$, its frequency $\nu'$ becomes

$$\nu' = \frac{c\nu}{(c - u_s)} \tag{i}$$

When the tuning fork moves away from the stationary observer, its frequency $\nu''$ becomes

$$\nu'' = \frac{c\nu}{(c + u_s)} \tag{ii}$$

When both the tuning forks move relative to a stationary observer, the number of beats/second produced $= \nu' - \nu''$

From (i) and (ii) we have,

$$\nu' - \nu'' = c\nu\left[\frac{1}{c - u_s} - \frac{1}{c + u_s}\right]$$

But $\qquad \nu' - \nu'' = 3$

$$\therefore \qquad 3 = c\nu\left[\frac{c + u_s - c + u_s}{c^2 - u_s^2}\right]$$
$$= \frac{2cu_s\nu}{c^2 - u_s^2}$$

or $\qquad 3 = \dfrac{2 \times 340 \times u_s \times 340}{(340)^2 - u_s^2}$

As $(340)^2 \gg u_s^2$, $\qquad (340)^2 - u_s^2 \approx (340)^2$

$$\therefore \qquad \frac{(340)^2 \times 2u_s}{(340)^2} = 3$$

or $\qquad u_s \simeq \dfrac{3}{2} \simeq 1.5 \text{ m/s}$

Thus the speed of the tuning fork is about 1.5 m/s.

**Ans. 6**

(b) Let $I_1$ be the current flowing through resistances 5 $\Omega$ and 1 $\Omega$. Let $I_2$ be the current flowing through resistance of 2 $\Omega$.

According to Kirchhoff's law,

At $A$, $\quad \sum I = 0$

$$I_1 = 1 + 2 = 3 \text{ A}$$

At $B$, $\quad \sum I = 0$

$$I_2 = 2 - 1 = 1 \text{ A}$$

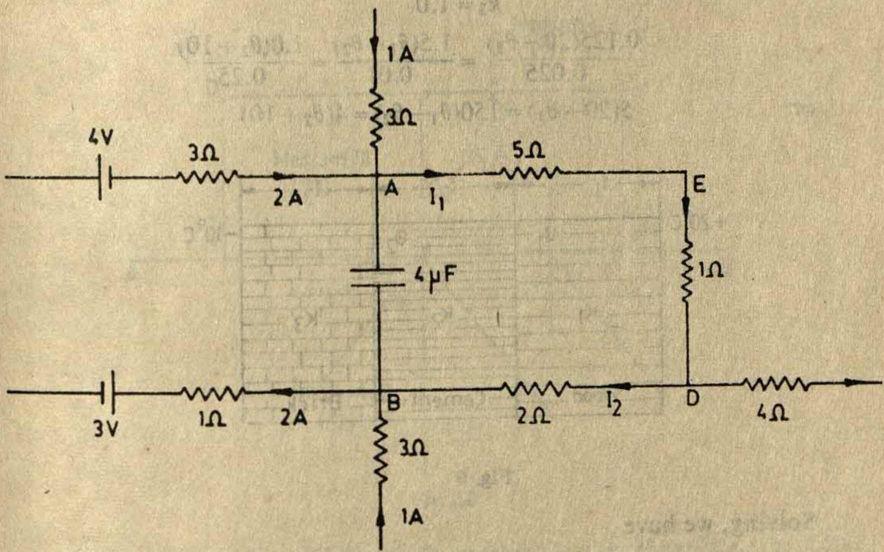



Fig. 8

$\therefore$ Potential difference across capacitor $C$ = Potential difference across the mesh $ABDE$

$$= 5 \times 3 + 1 \times 3 + 2 \times 1$$
$$= 15 + 3 + 2$$
$$= 20 \text{ V}$$

$\therefore$ Energy stored in the capacitor $= \frac{1}{2} CV^2$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (20)^2$$
$$= 2 \times 400 \times 10^{-6}$$
$$= 8 \times 10^{-4} \text{ J}$$

Ans. 7

(a) The amount of heat flowing per unit area per unit time at any surface

$$= \frac{k(\theta_1 - \theta_2)}{l}$$

where $k$ is coefficient of thermal conductivity, $\theta_1$ and $\theta_2$ are temperature at both ends separated by a distance $l$. Let the temperature at the interface between wood and cement be $\theta_1$ and the temperature between cement and brick layer be equal to $\theta_2$. As the amount of heat

passing through all the layers per unit area per second in steady-state conditions is the same,

$$\frac{Q}{At}=\frac{k_1(20-\theta_1)}{0.025}=\frac{k_2(\theta_1-\theta_2)}{0.01}=\frac{k_2(\theta_3+10)}{0.25}$$

But
$$k_1=0.125$$
$$k_2=1.5$$
$$k_3=1.0$$

$$\therefore \quad \frac{0.125(20-\theta_1)}{0.025}=\frac{1.5(\theta_1-\theta_2)}{0.01}=\frac{1.0(\theta_2+10)}{0.25}$$

or
$$5(20-\theta_1)=150(\theta_1-\theta_2)=4(\theta_2+10)$$

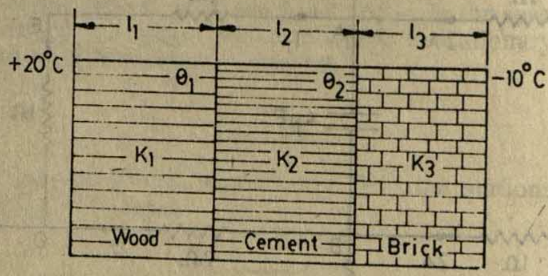



Fig. 9

Solving, we have

$$100-5\theta_1=150\theta_1-150\theta_2$$

or
$$100=155\theta_1-150\theta_2 \qquad \text{(i)}$$

and
$$60=5\theta_1+4\theta_2 \qquad \text{(ii)}$$

Multiplying (i) by 4 and (ii) by 150, we get

$$400=620\theta_1-600\theta_2$$
$$9000=750\theta_1+600\theta_2$$

Adding, we have

$$9400=1370\theta_1$$

or
$$\theta_1=\frac{9400}{1370}=\frac{940}{137}$$

$\therefore$ Amount of heat flowing per unit area per unit time

$$=5\left(20-\frac{940}{137}\right)$$

$$=5\times\frac{1800}{137}\ \text{W/m}^2$$

Total amount of heat flowing through a wall of area

$$137\ \text{m}^2=5\times\frac{1800}{137}\times 137\ \text{W}$$
$$=9000$$
$$=9\ \text{kW}$$

**Ans. 7**

(b) The critical angle, $\theta_c$, for monochromatic light incident from a denser medium of refractive index $n_2$ to a rarer medium of refractive index $n_1$ is given by

$$\sin \theta_c = \frac{n_1}{n_2}$$

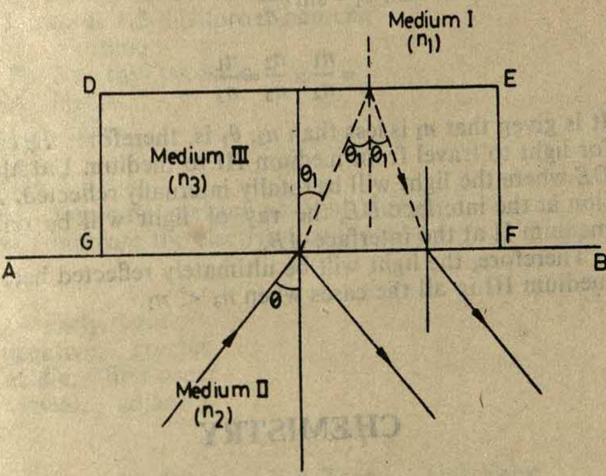



Fig. 10

It is given that $\theta$ is infinitesimally greater than $\theta_c$; therefore, the total internal reflection takes place at the interface *AB*. When a transparent slab *DEFG* of refractive index $n_3$ is introduced, the following possibilities exist.

(i) $n_3 < n_1$

Critical angle, $\theta'_c$, for monochromatic light incident from a denser medium of refractive index $n_2$ to a still rarer medium of refractive index $n_3$ is given by:

$$\sin \theta'_c = \frac{n_3}{n_2}$$

As $\quad n_3 < n_1, \dfrac{n_3}{n_2} < \dfrac{n_1}{n_2}$

or $\quad \theta'_c < \theta_c < \theta$

$\therefore$ The light is totally internally reflected into medium II.

(ii) $n_3 > n_1$

There are two possibilities.

(a) $n_3 < n_2$: When $n_3 < n_2$, critical angle, $\theta''_c$, for light to travel from a denser medium of refractive index $n_2$ to a rarer medium of refractive index $n_3$ is given by:

$$\sin \theta''_c = \frac{n_3}{n_2}$$

As angle $\theta_c''$ is less than the angle of incidence $\theta$, light will be totally internally reflected at the interface $AB$ itself.

(b) $n_3 > n_2$: Let $\theta_1$ be angle of refraction in medium III.

$$\therefore \qquad \frac{\sin \theta_1}{\sin \theta} = \frac{n_2}{n_3} \qquad \text{(By Snell's law)}$$

or

$$\sin \theta_1 = \sin \theta \frac{n_2}{n_3}$$

$$= \frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{n_1}{n_3}$$

It is given that $n_1$ is less than $n_3$, $\theta_1$ is, therefore, the critical angle for light to travel from medium III to medium I at the interface $DE$ where the light will be totally internally reflected. After reflection at the interface $DE$, the ray of light will be refracted into medium II at the interface $AB$.

Therefore, the light will be ultimately reflected back again into medium III in all the cases when $n_3 < n_1$.

# CHEMISTRY

## PART A

**Ans. 1**

(i) (B) electrons occupy space around the nucleus
(ii) (D) equally shared between them
(iii) (C) $n=3, \quad l=2, \quad m_l=-3, \quad m_s=\frac{1}{2}$
(iv) (C) calcium oxalate
(v) (A) $NaHCO_3$ and NaOH
(vi) (D) sodium acetate
(vii) (A) 0.6 m/s
(viii) (A) 1000 g of the solvent
(ix) (D) dimethyl ether
(x) (A) $Bi^{3+}$, $Sn^{4+}$
(xi) (B) mercuric oxide
(xii) (D) 2
(xiii) (D) $\frac{1}{16}$ g
(xiv) (D) F – H . . . . F
(xv) (C) $sp^2$
(xvi) (A) 2-methyl propan-2-ol
(xvii) (D) *o*- and *p*-chlorotoluene
(xviii) (A) $C_2H_6$ (excess) + $Cl_2 \xrightarrow{uv \text{ light}}$
(xix) (B) *n*-octane
(xx) (B) covalent and coordinate

**Ans. 2**

(a) (i) (A) the pressure decreases
(B) the kinetic energy of the molecules remains the same
(ii) (C) increasing temperature favours forward reaction
(D) increasing pressure favours reverse reaction
(iii) (A) acetic acid
(C) *p*-nitrophenol
(iv) (B) cis 1,2-dichloroethene
(D) trans 1,2-dichloro-2-pentene
(v) (A) *n*-butane
(D) 2-methyl propane
(vi) (A) $HgCl_2$
(C) $C_2H_2$
(vii) (C) 4
(D) 3
(viii) (A) lower the melting point of alumina
(B) increase the electrical conductivity

**Ans. 2**

(b) (i) inversely, time
(ii) negative, greater
(iii) acidic, first order
(iv) vicinal, adjacent

**Ans. 3**

(a) (i) True (ii) False (iii) True (iv) False

**Ans. 3**

(b) (i) Oxidation number of oxygen in $H_2O_2 = -1$
Oxidation number of oxygen in $H_2O = -2$
As the oxidation number of oxygen in hydrogen peroxide is more as compared to that in water, hydrogen peroxide is a better oxidising agent than water.
(ii) The transition metal compounds have unpaired electrons which absorb light in the visible range. Thus these compounds are coloured.
(iii) In strongly acidic medium, hydrazine gets easily protonated which lowers its nucleophilicity. That is why hydrazones of aldehydes and ketones are not prepared in highly acidic medium.
(iv) Ionization of acetic acid is suppressed by the addition of acetate ion due to common ion effect. Hence acetic acid is less acidic in sodium acetate solution than in sodium chloride solution.

**Ans. 4**

(a) (i) $Ca^{2+}$, Ar, $Cl^-$, $S^{2-}$
(ii) HClO, $HClO_2$, $HClO_3$, $HClO_4$
(iii) *p*-nitroaniline, aniline, *p*-toluidine, N,N-dimethyl-*p*-toluidine
(iv) HI, HBr, HCl, HF
(v) HI, $I_2$, ICl, $HIO_4$
(vi) $CH_3CONH_2$, $CH_3COOC_2H_5$, $(CH_3CO)_2O$, $CH_3COCl$

**Ans. 4**

(b) (i) Lewis acid—$BF_3$
(ii) Philosopher's wool—ZnO

(iii) Electrophile—$SO_3$
(iv) Preservative—HCHO
(v) Electron Emission—small proton to neutron ratio
(vi) Brönsted Acid—$NH_4^+$
(vii) Black Jack—Zinc ore
(viii) X-ray emission—K electron capture

## PART B

**Ans. 5**

(a) (i) $2Mn^{2+} + 5PbO_2 + 4H^+ \longrightarrow 2MnO_4^- + 2H_2O + 5Pb^{2+}$

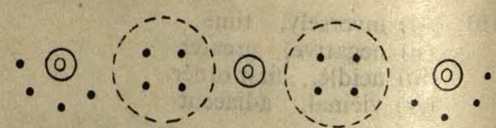
(ii) $4S + 6OH^- \longrightarrow 2S^{2-} + S_2O_3^{2-} + 3H_2O$

(iii) $ClO_3^- + 6I^- + 6H_2SO_4 \longrightarrow Cl^- + 3I_2 + 6HSO_4^- + 3H_2O$

(iv) $6Ag^+ + AsH_3 + 3H_2O \longrightarrow H_3AsO_3 + 6Ag + 6H^+$

**Ans. 5**

(b) Lewis dot structure for $O_3$ is



Lewis dot structure for $COCl_2$ is

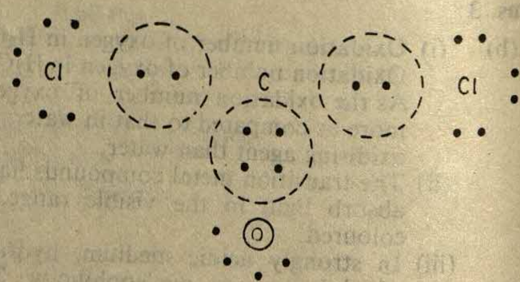


Fig. 11

**Ans. 5**

(c) (i) $FeCl_3$, $Br_2$
(ii) $Na_2ZnO_2$, $H_2O$
(iii) $SnCl_2$ $I_2$
(iv) $Na_2SO_4$, HCl

**Ans. 6**

(a) Molecular formula of (A) is $C_6H_{13}Cl$.
As it slowly yields white precipitates with alcoholic $AgNO_3$, (A) must be a *secondary alkyl halide*.
The structural formula for (A) is

$$\begin{array}{ccccccc} & & CH_3 & & Cl & & \\ & & | & & | & & \\ H_3C & — & CH & — & CH & — & CH_2—CH_3 \end{array}$$

The structural formula for (B) is

$$H_3C-\underset{}{\overset{\displaystyle CH_3}{\overset{|}{C}H}}-CH=CH-CH_3$$

The structural formula for (C) is

$$H_3C-\overset{\displaystyle CH_3}{\overset{|}{C}}=CH-CH_2-CH_3$$

**Ans. 6**

(b) The structural formula for (X) is

$$C_2H_5-\overset{\displaystyle O}{\overset{\|}{C}}-O-CH_2-CH_2-CH_3$$

The structural formula for (Y) is

$$CH_3-CH_2-COOH \quad \text{or} \quad C_2H_5COOH$$

The structural formula for (Z) is

$$CH_3-CH_2-CH_2OH \quad \text{or} \quad C_3H_7OH$$

**Ans. 7**

(a) (i)

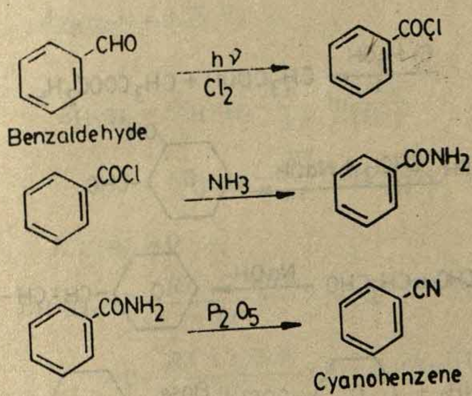



Fig. 12

(ii) $C_2H_5OH \xrightarrow{\text{Conc. } H_2SO_4} H_2C=CH_2$

$$H_2C=CH_2 \xrightarrow{Br_2} H_2\overset{\displaystyle Br}{\overset{|}{C}}-\overset{\displaystyle Br}{\overset{|}{C}}H_2$$

$$H_2\overset{\displaystyle Br}{\overset{|}{C}}-\overset{\displaystyle Br}{\overset{|}{C}}H_2 \xrightarrow{\text{alcoholic KOH}} HC\equiv CH$$

$$HC\equiv CH \xrightarrow[(CH_3COO)_2\,Hg]{CH_3COOH} \underset{\text{Vinyl acetate}}{H_2C=CH-(CH_3COO)}$$

**Ans. 7**

(b) 2-pentenoic acid

**Ans. 7**

(c)

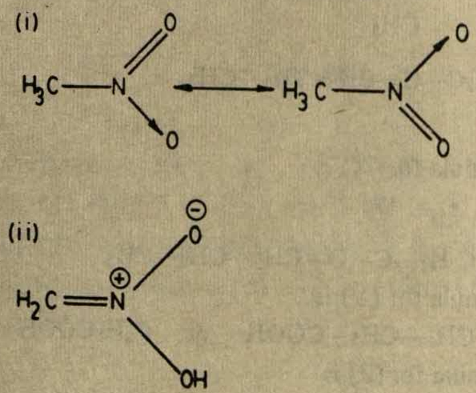


Fig. 13

**Ans. 7**

(d)

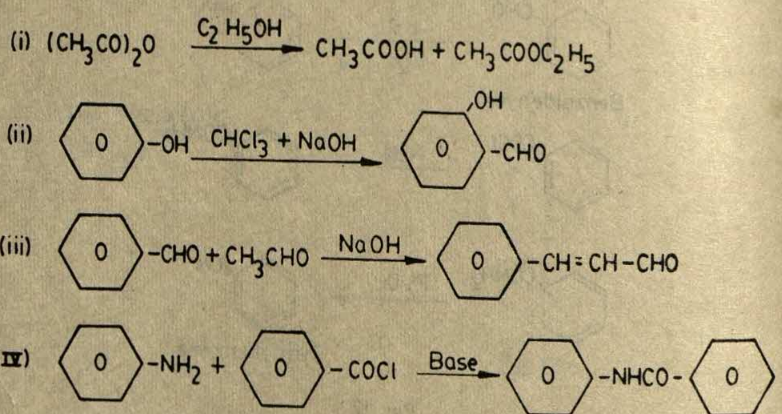


Fig. 14

**Ans. 8**

(a) The chemical reaction for the given cell is

$$Zn(S) + 2H^+_{(aq)} \longrightarrow Zn^{2+} + (0.1\ M) + H_2(g,\ 1\ atm) \qquad \text{(i)}$$

E.M.F. of the cell at 25°C = 0.28 V

Half-cell reactions will be

$$Zn^{2+} + 2e \rightleftharpoons Zn \qquad \text{(ii)}$$

$$H^+ + e \rightleftharpoons \tfrac{1}{2}H_2$$

or

$$2H^+ + 2e \rightleftharpoons H_2 \qquad \text{(iii)}$$

The pH of the solution at the hydrogen electrode can be calculated using the Half-cell reactions as done below:

$$E_{Zn/Zn^{2+}} = E^\circ_{Zn/Zn^{2+}} - \frac{RT}{nF} \ln \frac{[Zn^{2+}]}{[Zn]} \quad \text{(iv)}$$

Here, $R = 8.314$ J/mole deg
$T = 273 + 25 = 298$ K
$F = 96500$ Coul/equivalent
$n = 2$
$E^\circ_{Zn/Zn^{2+}} = 0.76$ V

Substituting these values in (iv), we get,

$$E_{Zn/Zn^{2+}} = 0.76 - \frac{8.314 \times 298}{2 \times 96500} \ln \frac{[0.1]}{1}$$

$$= 0.76 - \frac{8.314 \times 298}{193000} \log_e (0.1)$$

$$= 0.76 - \frac{8.314 \times 298 \times 2.3026}{193000} [\log_{10}^{1} - \log_{10}^{10}]$$

$$= 0.76 - 0.0295588\ (0-1)$$

$$= 0.76 + 0.03$$

$$= 0.79 \quad \text{(v)}$$

$\therefore \quad E_{Zn/Zn^{2+}} = 0.79$ V

Similarly,

$$E_{H^+/H_2} = E^\circ_{H^+/H_2} - \frac{RT}{nF} \ln \frac{[H_2]}{[H^+]^2}$$

$$= E^\circ_{H^+/H_2} - \frac{RT}{nF} \log_e \frac{1}{[H^+]^2}$$

Given: $E^\circ_{H^+/H_2} = 0$

$$\therefore \quad E_{H^+/H_2} = 0 - \frac{RT \times 2.3026}{nF} \times [\log_{10}^{1} - 2 \log_{10}[H^+]]$$

$$= 0 - \frac{RT \times 2.3026}{nF} \times [0 - 2 \log_{10}[H^+]]$$

$$= \frac{2\ RT \times 2.3026}{nF} \log_{10} [H^+]$$

$$= \frac{8.314 \times 298 \times 2.3026}{1 \times 96500} \log_{10}[H^+]$$

$$= 0.05915 \log_{10} [H^+]$$

But $\quad pH = -\log_{10}[H^+] \quad$ [By def.] $\quad$ (vi)

$\therefore \quad E_{H^+/H_2} = -0.05915$ pH

Substituting the values calculated at steps (v) and (vi) in (i), we get

$$0.28 = 0.79 - 0.05915 \text{ pH}$$

or $$0.05915\,\text{pH} = 0.79 - 0.28$$
$$= 0.51$$

or $$\text{pH} = \frac{0.51}{0.05915} = 8.62$$

Hence the pH of the given solution is 8.62.

**Ans. 8**

(b) The chemical equations for the formation of ethane, carbon dioxide and liquid water will be

$$2C_{(s)} + 3H_2(g) \longrightarrow C_2H_6(g), \quad \Delta H_f = 21.1 \text{ kcal} \quad \text{(i)}$$
$$C_{(s)} + O_2(g) \longrightarrow CO_2(g), \quad \Delta H_f = -94.1 \text{ kcal} \quad \text{(ii)}$$
$$H_2(g) + \tfrac{1}{2}O_2(g) \longrightarrow H_2O(l), \quad \Delta H_f = -68.3 \text{ kcal} \quad \text{(iii)}$$

Multyplying (ii) by 2, (iii) by 3 and adding, we get

$$2C_{(s)} + 2O_2(g) + \frac{3}{2}O_2(g) + 3H_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$$

or $$2C_{(s)} + \frac{7}{2}O_2(g) + 3H_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$$

where $$\Delta H_f = 2 \times (-94.1) + 3 \times (-68.3)$$
$$= -188.2 - 204.9$$
$$= -393.1 \text{ kcal}$$

$\therefore$ $$2\,C_{(s)} + \frac{7}{2}O_2(g) + 3H_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)_2$$
$$\Delta H_f = -393.1 \text{ kcal} \quad \text{(iv)}$$

Subtracting (i) from (iv), we have

$$\frac{7}{2}O_2(g) \rightarrow -C_2H_2(g) + 2CO_2(g) + 3H_2O(l)$$

or $$C_2H_2(g) + \frac{7}{2}O_2(g) \rightarrow 2\,CO_2(g) + 3H_2O(l),$$

$$\Delta H_f = -393.1 - (-21.1)$$
$$= -393.1 + 21.1$$
$$= -372.0 \text{ kcal}$$

Thus the molar heat of combustion of ethane is $-372.0$ kcal.

**Ans. 9**

(a) When an alpha particle is given out from some parent radioactive element, the mass number and the atomic number of the daughter element so formed decrease by 4 units and 2 units respectively.

$${}^{234}_{90}Th \rightarrow {}^{206}_{82}Pb$$

Decrease in the mass number $= 234 - 206$
$$= 28 \text{ units}$$

$\therefore$ Number of alpha particles given out in the above nuclear disintegration $= \frac{28}{4} = 7$

The atomic number of the daughter element so formed should be, $7 \times 2 = 14$ units less.

$\therefore$ Atomic number of the daughter element $= 90 - 14 = 76$

But the given daughter element has an atomic number 82. This must have been due to the addition of $\beta$-particles. When a $\beta$-particle is given out from some parent element, the atomic number of the daughter element so formed increases by 1 unit but the mass number remains the same.

$\therefore$ Number of $\beta$-particles given out $= 6$

The given radio-active decay can be written as,

$$^{234}_{90}\text{Th} \xrightarrow{-7(^{4}_{2}\text{He})} {}^{206}_{76}\text{X} \xrightarrow{-6(^{\;0}_{-1}\beta)} {}^{206}_{82}\text{Pb}$$

**Ans. 9**

(b) Vapour pressure of ethanol $= 44.5$ mm Hg
Vapour pressure of methanol $= 88.7$ mm Hg

Number of moles of ethanol $(n_1) = \frac{60}{46} = 1.304$

Number of moles of methanol $(n_2) = \frac{40}{32} = 1.250$

$$\text{Mole fraction of methanol} = \frac{n_2}{n_1 + n_2} = \frac{1.250}{1.304 + 1.250} = \frac{1.250}{2.554} = 0.4894$$

$$\text{Mole fraction of ethanol} = \frac{n_1}{n_1 + n_2} = 1 - \left(\frac{n_2}{n_1 + n_2}\right) = 1 - 0.4894 = 0.5106$$

Vapour pressure due to methanol $(p_1) = 88.7 \times 0.4894$ mm Hg $= 43.40$ mm Hg

and vapour pressure due to ethanol $(p_2) = 44.5 \times 0.5106 = 22.73$ mm Hg

Total vapour pressure $(p) = p_1 + p_2 = (43.40 + 22.73)$ mm Hg $= 66.13$ mm Hg

Mole fraction of methanol in the vapours

$$\left(\frac{p_1}{p}\right)=\frac{\text{Vapour pressure due to methanol}}{\text{Total vapour pressure}}$$

$$=\frac{43.40 \text{ mm Hg}}{66.13 \text{ mm Hg}}$$

$$=0.656$$

**Ans. 10**

(a) Let $S$ be the solubility of $Mg(OH)_2$

$\therefore$

$$S=9.57\times10^{-3} \text{ g/litre}$$

$$=\frac{9.57\times10^{-3}}{24+2(16+1)} \text{ moles/litre}$$

$$=\frac{9.57\times10^{-3}}{58} \text{ moles/litre}$$

$$=1.65\times10^{-4} \text{ moles/litre}$$

Dissociation of $Mg(OH)_2$ can be written as

$$Mg(OH)_2 \rightleftharpoons Mg^{2+}+2(OH)^- \qquad \text{(i)}$$

$\therefore$

$$k_{sp}=4s^3$$

$$=4\times(1.65\times10^{-4})^3$$

$$=4\times4.492\times10^{-12}$$

$$=17.96\times10^{-12}$$

$$\simeq 1.8\times10^{-11}$$

Dissociation of $Mg(NO_3)_2$ can be written as

$$Mg(NO_3)_2 \rightleftharpoons Mg^{2+}+2(NO_3)^- \qquad \text{(ii)}$$

Let $S_1$ be the solubility of $Mg(OH)_2$ in $Mg(NO_3)_2$

$\therefore$

$$[Mg^{2+}]=0.02+S_1$$

$$[OH^-]=2S_1$$

From (ii), we have

$$k_{sp}=[Mg^{2+}]\times[OH^-]^2$$

$$=(0.02+S_1)\times(4S_1^2)$$

$$=1.8\times10^{-11}$$

or $$0.02\times4S_1^2+4S_1^3=1.8\times10^{-11}$$

or $$0.08S_1^2=1.8\times10^{-11}$$

[Neglecting terms containing $S_1^3$ as it is very very small]

or $$S_1=\left(\frac{1.8\times10^{-11}}{0.08}\right)^{1/2}$$

$$=(2.25\times10^{-10})^{1/2}$$

$$=1.5\times10^{-5} \text{ mole/litre}$$

Thus the solubility of $Mg(OH)_2$ in $Mg(NO_3)_2$ solution is given by

$$1.5\times10^{-5}\times58 \text{ g/litre}$$

$$=8.7\times10^{-4} \text{ g/litre}$$

**Ans. 10**

(b) Density of 39% sulphuric acid before discharging = 1.294 g/ml
Density of 20% sulphuric acid after discharging = 1.139 g/ml

$\therefore$ Amount of sulphuric acid in the solution before discharging

$$= 3500 \times 1.294 \times \frac{39}{100}$$

$$= 1767.0 \text{ g}$$

Amount of sulphuric acid in the solution after discharging

$$= 3500 \times 1.139 \times \frac{20}{100}$$

$$= 797.1 \text{ g}$$

Amount of sulphuric acid that has been consumed by the lead storage battery $= 1767.0 - 797.1$

$$= 969.9 \text{ g}$$

$$= \frac{969.9}{98}$$

$$= 9.987 \text{ moles}$$

The charging and discharging reactions are:

$$Pb + SO_4^{2-} \rightarrow PbSO_4 + 2e^- \text{ (charging)} \qquad \text{(i)}$$

$$PbO_2 + 4H^+ + SO_4^{2-} + 2e^- \rightarrow PbSO_4 + 2H_2O \text{ (discharging)} \qquad \text{(ii)}$$

Adding (i) and (ii), we have

$$Pb + PbO_2 + 2H_2SO_4 \underset{\text{charging}}{\overset{\text{discharging}}{\rightleftharpoons}} 2PbSG_4 + 2H_2O$$

We find that two moles of sulphuric acid have been fully consumed to yield two electrons, therefore,

1 mole of sulphuric acid = 98 g of sulphuric acid
= 1 g equivalent of sulphuric acid

and 9.987 moles sulphuric acid = 9.987 equivalents of sulphuric acid

As 1 faraday = 96500 coul/equivalent

9.987 faradays = $9.987 \times 96500$

$\therefore$ Required number of ampere-hours for which battery is used

$$= \frac{9.987 \times 96500}{60 \times 60} = 267.71$$

# MATHEMATICS

## PART A

**Ans. 1**

(i) (C) only one minimum
(ii) (B) continuous everywhere
(D) not differentiable at $x=0$
(E) not differentiable at infinite number of points
(iii) (E) none of these
(iv) (A) $(-\infty, -\frac{3}{2})$
(D) $(\frac{1}{2}, 3)$
(v) (E) none of these
(vi) (B) $a > 0, \quad b < 0$
(C) $a < 0, \quad b > 0$
(vii) (E) none of these
(viii) (B) $a, b, c$ are in G.P.
(E) $(x-\alpha)$ is a factor of $ax^2+2bx+c$
(ix) (A) $3x+2y \geqslant 0$
(C) $2x-3y-12 \leqslant 0$
(x) (B) 1
(xi) (C) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)$
(xii) (A) continuous at $x=0$
(D) differentiable in $(-1, 1)$
(xiii) (A) $b \sin A = a, \quad A < \dfrac{\pi}{2}$
(D) $b \sin A < a, \quad A < \dfrac{\pi}{2}, b > a$
(xiv) (A) zero
(D) purely imaginary
(xv) (B) $p=1$ or $p=-\frac{1}{3}$
(xvi) (B) negative
(xvii) (C) $p=1, q=0$

**Ans. 2**

(i) $x=4$
(ii) $\phi$
(iii) $10x-3y-18=0$
(iv) $-1$
(v) $-4$
(vi) 1
(vii) $\frac{1}{3} \leqslant p \leqslant \frac{1}{2}$
(viii) $(x+4)^2+(y-3)^2=16$

## PART B

**Ans. 3**

(a) The position vectors of various points are given as

$$A(3\hat{i}-2\hat{j}-\hat{k}), \qquad B(2\hat{i}+3\hat{j}-4\hat{k}), \qquad C(-\hat{i}+\hat{j}+2\hat{k})$$

and $D(4\hat{i}+5\hat{j}+\lambda\hat{k})$

$\therefore \quad \overrightarrow{AB} = (2\hat{i} - 3\hat{i}) + \{3\hat{j} - (-2\hat{j})\} - 4\hat{k} - (-\hat{k})$

$\qquad = -\hat{i} - 5\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = (-\hat{i} - 2\hat{i}) + (\hat{j} - 3\hat{j}) + 2\hat{k} - (-4\hat{k})$

$\qquad = -3\hat{i} - 2\hat{j} + 6\hat{k}$

$\overrightarrow{CD} = 4\hat{i} - (-\hat{i}) + 5\hat{j} - \hat{j} + \lambda\hat{k} - 2\hat{k}$

$\qquad = 5\hat{i} + 4\hat{j} + (\lambda - 2)\hat{k}$

Points $A$, $B$, $C$ and $D$ or $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$ will lie in one plane if

$$\begin{vmatrix} -1 & 5 & -3 \\ -3 & -2 & 6 \\ 5 & 4 & \lambda - 2 \end{vmatrix} = 0$$

$\Rightarrow -1\{-2(\lambda - 2) - 24\} - 5\{-3(\lambda - 2) - 30\} - 3(-12 + 10) = 0$

$\Rightarrow -1(-2\lambda - 20) - 5(-3\lambda - 24) + 6 = 0$

$\Rightarrow 2\lambda + 20 + 15\lambda + 120 + 6 = 0$

$\Rightarrow 17\lambda + 146 = 0$

$\Rightarrow \lambda = -\dfrac{146}{17}$

**Ans. 3**

(b) Number of balls in the box $= 9$
Total number of possible ways of drawing 3 balls of any colour

$$^9C_3 = \frac{\underline{|9}}{\underline{|3}\ \underline{|6}} = \frac{9 \times 8 \times 7 \times \underline{|6}}{3 \times 2 \times 1 \times \underline{|6}} = 84$$

Total number of ways of 3 balls from the box such that no one of which is black

$$^6C_3 = \frac{\underline{|6}}{\underline{|3} \times \underline{|3}} = \frac{6 \times 5 \times 4 \times \underline{|3}}{\underline{|3} \times 3 \times 2 \times 1} = 20$$

$\therefore$ Number of ways of drawing 3 balls containing at least one black ball $= 84 - 20$

$\qquad = 64$

**Ans. 4**

(a) Given complex numbers are $Z$, $iZ$ and $Z+iZ$.
These complex numbers form the sides of the triangle on the Argand diagram.
When we rotate the complex number $Z$ in the anticlockwise direction through an angle $\pi/2$, we get the complex number $iZ$ given. Thus these complex numbers form the sides of a right angled triangle with right angle at $Z+iZ$.

$\therefore$ Area of the triangle $= \frac{1}{2} \mid Z \mid \cdot \mid Z \mid$ [$\because$ Area of a triangle $= \frac{1}{2} \times$ base $\times$ height]

$$= \frac{1}{2} \mid Z \mid^2$$

**Ans. 4**

(b) Let $A(Z_1)$, $B(Z_2)$ and $C(Z_3)$ be the vertices of an isosceles right angled triangle $ABC$.

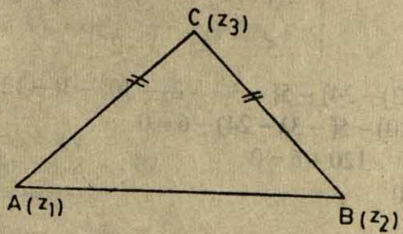



Fig. 15

As the given triangle is right-angled at $C$ and $\mid BC \mid = \mid AC \mid$, therefore,

$$(Z_2 - Z_3) = \pm i(Z_1 - Z_3) \qquad \text{(i)}$$

Consider,

$$\begin{aligned}(Z_1 - Z_2)^2 &= [(Z_1 - Z_3) + (Z_3 - Z_2)]^2 \\ &= (Z_1 - Z_3)^2 + (Z_3 - Z_2)^2 + 2(Z_1 - Z_3)(Z_3 - Z_2) \\ &= (Z_1 - Z_3)^2 + \{\pm i(Z_1 - Z_3)\}^2 + 2(Z_1 - Z_3)(Z_3 - Z_2) \\ &= (Z_1 - Z_3)^2 + \{-(Z_1 - Z_3)^2\} + 2(Z_1 - Z_3)(Z_3 - Z_2) \\ &= 2(Z_1 - Z_3)(Z_3 - Z_2)\end{aligned}$$

Hence $(Z_1 - Z_2)^2 = 2(Z_1 - Z_2)(Z_3 - Z_2)$

**Ans. 5**

The given system of linear equations can be written as

$$(\sin 3\theta)\, x - y + z = 0 \qquad \text{(i)}$$
$$(\cos 2\theta)\, x + 4y + 3z = 0 \qquad \text{(ii)}$$
$$2x + 7y + 7z = 0 \qquad \text{(iii)}$$

This system of linear equations will have non-trivial solutions if

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta\,(4\times 7-7\times 3)-(-)\{7\cos 2\theta-6\}+1\,(7\cos 2\theta-8)=0$
$\Rightarrow 7\sin 3\theta+7\cos 2\theta-6+7\cos 2\theta-8=0$
$\Rightarrow 7\sin 3\theta=14\cos 2\theta-14=0$
$\Rightarrow \sin 3\theta+2\cos 2\theta-2=0$
$\Rightarrow 3\sin\theta-4\sin^3\theta+2\,(1-2\sin^2\theta)-2=0$
$\Rightarrow 3\sin\theta-4\sin^3\theta-4\sin^2\theta=0$
$\Rightarrow \sin\theta\,(2\sin\theta+3)\,(2\sin\theta-1)=0$
$\Rightarrow$ (i) $\sin\theta=0 \Rightarrow \theta=n\pi, \quad n\in I$

(ii) $(2\sin\theta+3)=0$ or $\sin\theta=-\dfrac{3}{2}$, which is not possible

(iii) $(2\sin\theta-1)=0$

$$\Rightarrow \sin\theta=\frac{1}{2}=\sin\frac{\pi}{6}$$

$$\Rightarrow \theta=m\pi+(-1)^m\frac{\pi}{6}, \qquad m\in I$$

Thus $\qquad \theta=n\pi$

and $\qquad \theta=m\pi+(-1)^m\dfrac{\pi}{6}$ are the required values of $\theta$.

**Ans. 6**

The equation is given as:

$$\cos A\cos B+(\sin A\sin B)\sin C=1$$

Multiplying both sides by 2, we have

$$2\cos A\cos B+(2\sin A\sin B)\sin C=2 \qquad \text{(i)}$$

Using: $\quad 2\cos A\cos B=\cos(A+B)+\cos(A-B)$
$\quad\quad 2\sin A\sin B=\cos(A-B)-\cos(A+B)$

Expression (i) becomes,

$$\{\cos(A+B)+\cos(A-B)\}+\{\cos(A-B)-\cos(A+B)\}\sin C=2 \qquad \text{(ii)}$$

Adding and subtracting $\sin C$ on the R.H.S. of (ii) we have

$$\{\cos(A+B)+\cos(A-B)\}+\{\cos(A-B)-\cos(A+B)\}\sin C$$
$$=(1+\sin C)+(1-\sin C)$$

or $\quad (1+\sin C)\cos(A-B)+(1-\sin C)\cos(A+B)$
$$=(1+\sin C)+(1-\sin C)$$

Dividing by $(1+\sin C)$ on both sides, we get

$$\cos(A-B)+\frac{(1-\sin C)}{(1+\sin C)}\cos(A+B)=1+\frac{(1-\sin C)}{(1+\sin C)}$$

But $\qquad A+B+C=180^\circ$

$\therefore \qquad \cos(A+B)=\cos(180^\circ-C)$
$$=-\cos C$$

$$\therefore \quad \cos(A-B)-\cos C\,\frac{(1-\sin C)}{(1+\sin C)}=1+\frac{(1-\sin C)}{(1+\sin C)}$$

or $\qquad \cos(A-B)=\cos C\,\dfrac{(1-\sin C)}{(1+\sin C)}+1+\dfrac{(1-\sin C)}{(1+\sin C)}$

$$=1+\frac{(1-\sin C)(1+\cos C)}{(1+\sin C)}$$

Since $C<180°$, $\cos(A-B)\geqslant 1$

$\therefore \qquad \dfrac{(1-\sin C)(1+\cos C)}{(1+\sin C)}=0$

$\Rightarrow \qquad (1-\sin C)=0 \qquad [\because\ C\neq\pi,\ (1+\cos C)\neq 0]$

or $\qquad \sin C=1$

$\Rightarrow \qquad C=\dfrac{\pi}{2}$

Again, $\cos(A-B)=1=\cos 0°$

or $\qquad A=B$

But $\qquad A+B+C=\pi$

or $\qquad A+B=\pi-\dfrac{\pi}{2}=\dfrac{\pi}{2}$

$\therefore \qquad A=B=\dfrac{\pi}{4}$

Thus $\qquad A=B=\dfrac{\pi}{4}$ and $C=\dfrac{\pi}{2}$

Now, $\qquad \dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

$\therefore \qquad a:b:c::1\,.\,1:\sqrt{2}$

**Ans. 7**

The equation of the perpendicular bisector $XY$ is

$$x-y+5=0 \qquad \text{(i)}$$

Slope of $XY=1$

$\therefore$ Slope of $AB=-1 \qquad (\because\ AB\perp XY)$

$\therefore$ The equation of the line $AB$ is

$$y-(-2)=-1(x-1)$$

or $\qquad y+x+1=0 \qquad$ (ii)

Solving (i) and (ii) simultaneously, we have

$$x=-3$$

and $\qquad y=2$

$\therefore$ The coordinates of $E$ will be $(-3, 2)$

As $E$ is the mid-point of $AB$,

$$\frac{\alpha+1}{2}=-3$$

or $\qquad \alpha=-7$

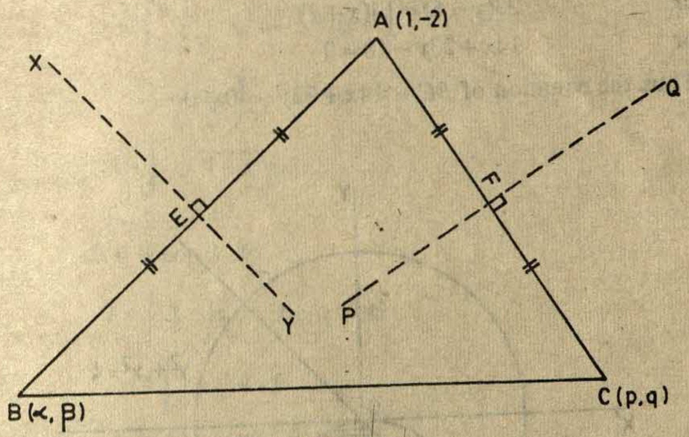



Fig. 16

and $$\frac{\beta-2}{2}=2$$

or $$\beta=6$$

$\therefore$ Coordinates of $B$ will be $(-7, 6)$

The equation of $PQ$ is

$$x+2y=0 \qquad \text{(iii)}$$

Slope of $PQ = -\frac{1}{2}$

$\therefore$ Slope of $AC=2$ $\quad (\because \ AC \perp PQ)$

$\therefore$ The equation of $AC$ will be,

$$y+2=2(x-1)$$

or $$y-2x+4=0 \qquad \text{(iv)}$$

Solving (iii) and (iv), we have the coordinates of $F$ as

$$\left(\frac{8}{5}, \frac{-4}{5}\right)$$

Similarly, $$\frac{8}{5}=\frac{p+1}{2} \quad \text{or} \quad p=\frac{11}{5}$$

and $$\frac{-4}{5}=\frac{q-2}{2}. \quad \text{or} \quad q=\frac{2}{5}$$

$\therefore$ Coordinates of $C$ will be $\left(\frac{11}{5}, \frac{2}{5}\right)$

By using the two-point form, the equation of $BC$ is

$$y-6=\frac{\left(6-\frac{2}{5}\right)}{\left(-7-\frac{11}{5}\right)}(x+7)$$

$$=\frac{28}{-46}(x+7)$$

or $$23(y-6)=14(x+7)$$
or $$14x+23y-40=0$$

Thus the equation of $BC$ is $14x+23y-40=0$

**Ans. 8**

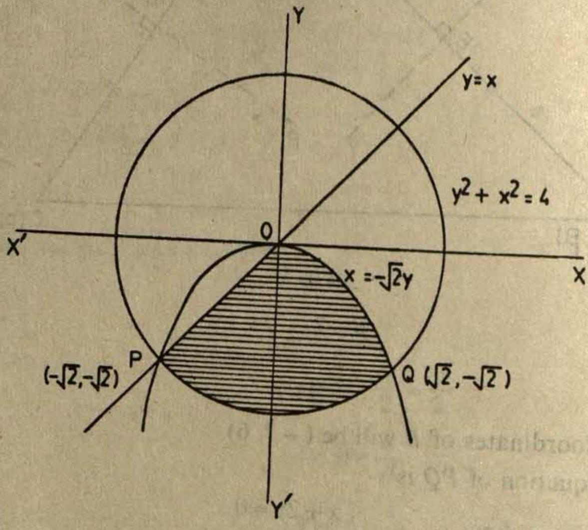



Fig. 17

Let us first find out the coordinates of $P$ and $Q$.

Solving: $$x^2+y^2=4 \quad \text{and} \quad x^2=-\sqrt{2}y$$

$$y^2-\sqrt{2}y-4=0$$

or $$y=\frac{\sqrt{2}\pm\sqrt{2+16}}{2}=2\sqrt{2},\ -\sqrt{2}$$

Taking $x^2=-\sqrt{2}\times 2\sqrt{2}=-4$, which gives imaginary values of $x$ and we reject $y=2\sqrt{2}$

Taking $y=-\sqrt{2}$, we get

$$x^2=-\sqrt{2}\times-\sqrt{2}=2$$

or $$x=\pm\sqrt{2}$$

Thus $$P(-\sqrt{2},-\sqrt{2}) \quad \text{and} \quad Q(\sqrt{2},-\sqrt{2})$$

Required area $=\int_{-\sqrt{2}}^{0}$ ($y$ for circle $-\,y$ for straight line) $dx$

$+\int_{0}^{\sqrt{2}}$ ($y$ for circle $-\,y$ for parabola) $dx$

$$=\int_{-\sqrt{2}}^{0}(\sqrt{4-x^2}-x)\,dx+\int_{0}^{\sqrt{2}}\left(\sqrt{4-x^2}+\frac{x^2}{\sqrt{2}}\right)dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{4-x^2}\, dx - \int_{-\sqrt{2}}^{0} x\, dx$$

$$+ \frac{1}{\sqrt{2}} \int_0^{\sqrt{2}} x^2\, dx \qquad \text{(i)}$$

Let $\qquad I = \int (\sqrt{4-x^2})\, dx$

Put $\qquad x = 2 \sin \theta$

$$dx = 2 \cos \theta \, d\theta$$

$$\therefore \qquad I = \int 2 \cos \theta \times 2 \cos \theta \, d\theta$$

$$= 4 \int \cos^2 \theta \, d\theta$$

$$= 4 \int \frac{(1 + \cos 2\theta)}{2} d\theta$$

$$= 2\theta + \sin 2\theta$$

$$= \left[ 2 \sin^{-1} \frac{x}{2} + \frac{x}{2} \sqrt{4-x^2} \right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= 2 \sin^{-1} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \sqrt{2}$$

$$- \left\{ 2 \left( -\sin^{-1} \frac{1}{\sqrt{2}} \right) - \frac{1}{\sqrt{2}} \times \sqrt{2} \right\}$$

$$= 2 \times \frac{\pi}{4} + 1 + 2 \times \frac{\pi}{4} + 1$$

$$= \pi + 2$$

Putting this value of $I$ in (i).

$$\text{Required area} = \pi + 2 - \left[ \frac{x^2}{2} \right]_{-\sqrt{2}}^{0} + \left[ \frac{x^3}{3\sqrt{2}} \right]_0^{\sqrt{2}}$$

$$= \pi + 2 - [0 - 1] + \left[ \frac{2\sqrt{2}}{3\sqrt{2}} - 0 \right]$$

$$= \pi + 2 + 1 + \frac{2}{3}$$

$$= \pi + \frac{11}{13}$$

**Ans. 9**

Let the three distinct real numbers in G.P. be

$$a,\ ar \quad \text{and} \quad ar^2$$

It is given that

$$a + ar + ar^2 = \alpha S \qquad \text{(i)}$$

and $\qquad a^2 + a^2r^2 + a^2r^4 = S^2 \qquad \text{(ii)}$

Squaring (i) and dividing (i) by (ii), we have

$$\frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)}=\frac{\alpha^2S^2}{S^2}$$

or
$$\frac{(1+r+r^2)^2}{(1+r^2+r^4)}=\alpha^2$$

or
$$\frac{(1+r+r^2)^2}{(1+r+r^2)(1-r+r^2)}=\alpha^2$$

$$[\because\ 1+r+r^4=(1+r+r^2)(1-r+r^2)]$$

or
$$\frac{(1+r+r^2)}{(1-r+r^2)}=\alpha^2$$

or
$$(1-r+r^2)\alpha^2=1+r+r^2$$

or
$$(\alpha^2-1)r^2+(\alpha^2+1)r+(\alpha^2-1)=0$$

It is a quadratic equation in $r$. Let $D$ be the discriminant of this equation. Considering for real roots, $D \geqslant 0 \Rightarrow$

$$(\alpha^2+1)^2-4(\alpha^2-1)^2 \geqslant 0$$

or
$$\alpha^4+1+2\alpha^2-4(\alpha^4-2\alpha^2+1) \geqslant 0$$

or
$$-3\alpha^4+10\alpha^2-3 \geqslant 0$$

or
$$(3\alpha^2-1)(\alpha^2-3) \leqslant 0$$

$\Rightarrow$
$$\alpha^2 \in \left[\frac{1}{3}, 3\right]$$

As the given numbers in G.P. are distinct,

$$r \neq -1, 0, 1$$

$\Rightarrow$
$$\alpha^2 \neq \frac{1}{3}, 1, 3$$

Thus
$$\alpha^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3)$$

**Ans. 10**

$$\sum_{k=0}^{n} k^2\,{}^nC_k = n(n+1)\,2^{n-2}, \qquad n \geqslant 1$$

Let the given sum be denoted by $S_n$

$\therefore$
$$S_n=\sum_{k=0}^{n} k^2\,{}^nC_k=n(n+1)\,2^{n-2}$$

Put
$$n=1$$

$$S_1=\sum_{0}^{1} k^2\,{}^1C_1=1(1+1)\,2^{1-2}$$

$$=1^2\,{}^1C_1=1(2)\,2^{-1}$$

$$=1\times 1=1\times 2^{\circ}$$

or
$S_1=1=1$, which means it is true for $n=1$

If the given relation is true for $n=m$, then $S_{m+1}$ is given by:

$$S_{m+1}=\sum_{k=0}^{m+1} k^2\,{}^{m+1}C_k=\sum_{k=0}^{m+1} k^2\,({}^mC_k+{}^mC_{k-1})$$

$$[\because\ {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r]$$

$$=\sum_{k=0}^{m+1} k^2\,{}^mC_k+\sum_{k=0}^{m+1} k^2\,{}^mC_{k-1}$$

$$=\sum_{k=0}^{m} k^2\,{}^mC_k+\sum_{k=0}^{m} (k+1)^2\,{}^mC_k$$

$$=\sum_{k=0}^{m} k^2\,{}^mC_k+\sum_{k=0}^{m} (k^2+2k+1)\,{}^mC_k$$

$$=\sum_{k=1}^{m} k^2\,{}^mC_k+\sum_{k=0}^{m} k^2\,{}^mC_k+2\sum_{k=0}^{m} k\,{}^mC_k+\sum_{k=0}^{m} {}^mC_k$$

$$=2\sum_{k=0}^{m} k^2\,{}^mC_k+2\sum_{k=0}^{m} k\,{}^mC_k+\sum_{k=0}^{m} {}^mC_k$$

$$=2S_m+2\sum_{k=0}^{m} k\,{}^mC_k+\sum_{k=0}^{m} {}^mC_k$$

But $\quad S_m=m(m+1)\,d^{m-2}$

and $\quad S_{m+1}=(m+1)(m+2)\,2^{(m+1)-2}$
$\quad\quad =(m+1)(m+2)\,2^{m-1}$

$$\therefore\quad \sum_{k=0}^{m+1} k^2\,{}^{m+1}C_k=2m(m+1)2^{m-2}+2\sum_{k=0}^{m} k-{}^mC_k+\sum_{k=0}^{m} {}^mC_k$$

$$=2m(m+1)2^{m-2}+2m\times 2^{m-1}+2^m$$
$$=2^{m-1}m(m+1)+2^m\cdot 2+2^m$$
$$=2^{m-1}\{m(m+1)+2m+2\}$$
$$=2^{m-1}(m(m+1)+2(m+1)\}$$
$$=2^{m-1}(m+1)(m+2)$$
$$=S_{m+1}$$

Thus $S_n$ is true, for $n=(m+1)$ also. But it is true for all $n$. Hence

$$\sum_{k=0}^{n} k^{2n}\,C_k=n(n+1)2^{n-2} \qquad \text{for } n\geqslant 1$$

**Ans. 11**

*Case (i):* Consider $x>a$

The given quadratic equation becomes $x^2-2a(x-a)-3a^2=0$

or $\quad x^2-2xa-a^2=0$

or
$$x=\frac{2a\pm\sqrt{4a^2+4a^2}}{2}$$
$$=\frac{2a\pm2\sqrt{2}\,a}{2}$$
$$=a(1\pm\sqrt{2})$$

As $a\leqslant 0$, the only possible root is $x=a(1-\sqrt{2})$.

*Case* (*ii*): For $x < a$, the given quadratic equation becomes

$$x^2+2a(x-a)-3a^2=0$$

or

$$x^2+2xa-5a^2=0$$

$$\Rightarrow x=\frac{-2a\pm\sqrt{4a^2+20a^2}}{2}$$

$$=\frac{-2a\pm2a\sqrt{6}}{2}$$

$$=a(-1\pm\sqrt{6})$$

As $a \leqslant 0$ and $x < a$, the only possible root is

$$x=a(-1+\sqrt{6})$$

Thus the only possible roots are $a(1-\sqrt{2})$ and $a(-1+\sqrt{6})$.

**Ans. 12**

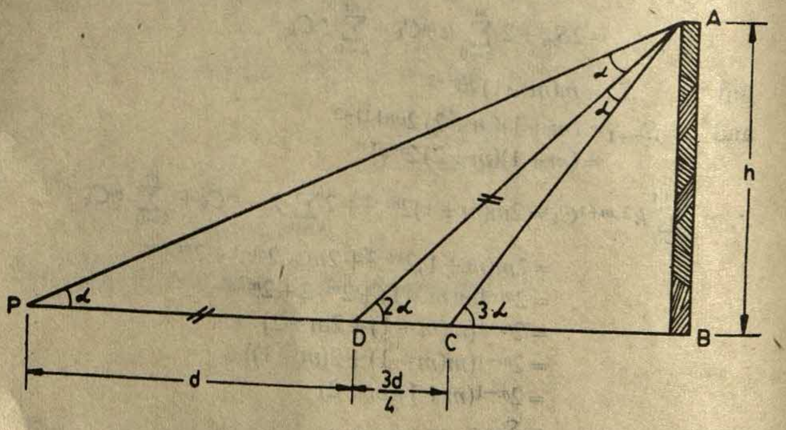



Fig. 18

Let $AB$ be the tower and $P$ be the initial position of the observer such that $\angle APB=\alpha$

$D$ and $C$ are the positions of the observer when he moves towards the foot of the tower such that

$$\angle ADC=2\alpha$$

and

$$\angle ACB=3\alpha$$

It is also given that

$$PD=d$$

and

$$DC=\frac{3d}{4}$$

In $\triangle\ APD$,

$$\angle PAD=2\alpha-\alpha$$

$$=\alpha$$

[$\because$ exterior angle of a triangle is equal to the sum of two interior opposite angles]

Similarly, $\qquad \underline{|DAC} = 3\alpha - 2\alpha = \alpha$

Thus $PAD$ is an isosceles triangle, such that $PD = DA$

From $\triangle ADC$, $\dfrac{DC}{\sin\alpha} = \dfrac{AD}{\sin(180° - 3\alpha)}$

$$= \frac{PD}{\sin 3x}$$

or $$\frac{3d/4}{\sin\alpha} = \frac{d}{\sin 3\alpha}$$

or $$\frac{3}{4\sin\alpha} = \frac{1}{\sin 3\alpha}$$

or $$3\sin 3\alpha = 4\sin\alpha$$

or $$3(3\sin\alpha - 4\sin^3\alpha) = 4\sin\alpha$$

or $$9\sin\alpha - 12\sin^3\alpha = 4\sin\alpha$$

or $$\sin\alpha(5 - 12\sin^2\alpha) = 0$$

As $\sin\alpha \neq 0$, $5 - 12\sin^2\alpha = 0$

$\Rightarrow$ $$\sin^2\alpha = \frac{5}{12}$$

Consider $\triangle ADB$, $\qquad \sin 2\alpha = \dfrac{h}{d}$

or $$2\sin\alpha\cos\alpha = \frac{h}{d}$$

or $$h = 2d\sin\alpha\cos\alpha$$

or $$h^2 = 4d^2\sin^2\alpha\cos^2\alpha$$

But $\qquad \sin^2\alpha = \dfrac{5}{12} \qquad \therefore \ \cos^2\alpha = 1 - \dfrac{5}{12} = \dfrac{7}{12}$

$\therefore$ $$h^2 = 4d^2 \times \frac{5}{12} \times \frac{7}{12}$$

or $$36h^2 = 35d^2$$

**Ans. 13**

(i) Let $\qquad I = \displaystyle\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$

Since $\qquad \sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \dfrac{\pi}{2}$

$$I = \frac{2}{\pi}\int (\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x})\,dx$$

$$= \frac{2}{\pi}\int\left(\frac{\pi}{2} - \cos^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)dx$$

$$= \frac{2}{\pi}\int\left(\frac{\pi}{2} - 2\cos^{-1}\sqrt{x}\right)dx$$

$$= x - \frac{4}{\pi}\int \cos^{-1}\sqrt{x}\,dx \qquad \text{(i)}$$

In order to solve $I'=\int \cos^{-1}\sqrt{x}\, dx$, put $\sqrt{x}=\cos t$

or $$\frac{1}{2\sqrt{x}}dx=-\sin t\, dt$$

$\therefore$ $$I'=\int \cos^{-1}(\cos t)\{-2\sqrt{x}\sin t\, dt\}$$
$$=-\int 2t\cos t\sin t\, dt$$
$$=-\int t\sin 2t\, dt$$
$$=-\left[t\left(-\frac{\cos 2t}{2}\right)+\int\frac{\cos 2t}{2}dt\right]$$
$$=\frac{t}{2}\cos 2t-\frac{1}{4}\sin 2t \qquad \text{(ii)}$$

Putting the value of $I'$ from (ii) into (i), we have

$I=x-\frac{4}{\pi}\left\{\frac{t}{2}\cos 2t-\frac{\sin 2t}{4}\right\}+c$, where $t=\cos^{-1}\sqrt{x}$ and $c$ is a constant of integration.

**Ans. 13**

(ii) Let $$I=\int_0^\pi \frac{x\, dx}{1+\cos\alpha\sin\alpha}$$
$$=\int_0^\pi\frac{(\pi-x)\,dx}{1+\cos\alpha\sin(\pi-x)}$$
$$=\int_0^\pi\frac{\pi\, dx}{1+\cos\alpha\sin x}-\int_0^\pi\frac{x\, dx}{1+\cos a\sin\alpha}$$
$$\left[\because \int_0^a f(x)\,dx=\int_0^a f(\alpha-x)\,dx\right]$$

$\therefore$ $$I=\pi\int_0^\pi\frac{dx}{1+\cos\alpha\sin x}-I$$

or $$I=\frac{\pi}{2}\int_0^\pi\frac{dx}{1+\cos\alpha\sin x}$$

Put $$t=\tan\frac{x}{2}$$

$\therefore$ $$dt=\frac{1}{2}\sec^2\frac{\alpha}{2}dx$$

As $$\sin x=\frac{2\tan x/2}{1+\tan^2 x/2}=\frac{2t}{1+t^2}$$

when $x=0,\ t=0$

and for $x=\pi,\ t=\infty$

$\therefore$ $$I=\frac{\pi}{2}\int_0^\infty\frac{2\times dt}{\left\{1+\cos\alpha\left(\frac{2t}{1+t^2}\right)\right]\sec^2\frac{x}{2}}$$

$$=\frac{\pi}{2}\int_0^\infty \frac{2dt}{\left\{1+\cos\alpha\frac{2t}{(1+t^2)}\right\}(1+t^2)}$$

$$=\pi\int_0^\infty \frac{dt}{(1+t^2)+2t\cos\alpha}$$

$$=\pi\int_0^\infty \frac{dt}{(t+\cos\alpha)^2+\sin^2\alpha}$$

$$=\pi\times\frac{1}{\sin\alpha}\times\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)$$

$$\left[\because \int\frac{dx}{(x+a)^2+b^2}=\frac{1}{b}\tan^{-1}\frac{(x+a)}{b}\right]$$

or
$$I=\left[\pi\times\frac{1}{\sin\alpha}\times\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^\infty$$

$$=\frac{\pi}{\sin\alpha}\left[\tan^{-1}\infty-\tan^{-1}(\cot\alpha)\right]$$

$$=\frac{\pi}{\sin\alpha}\left[\frac{\pi}{2}-\tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\alpha\right)\right\}\right]$$

$$=\frac{\pi}{\sin\alpha}\left(\frac{\pi}{2}-\frac{\pi}{2}+\alpha\right)$$

or
$$I=\frac{\pi\alpha}{\sin\alpha} \quad \text{for} \quad 0<\alpha<\pi$$

Thus
$$\int_0^\pi \frac{x\,dx}{1+\cos\alpha\sin x}=\frac{\pi\alpha}{\sin\alpha},\quad 0<\alpha<\pi$$

**Ans. 14**

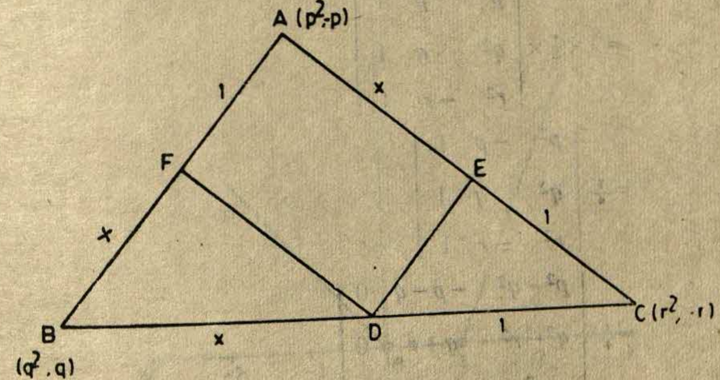



Fig. 19

In $\triangle ABC$, $E$ divides side $AC$ in the ratio $x:1$. Then,

$$\frac{AE}{EC}=\frac{x}{1}$$

Similarly, $$\frac{BD}{DC}=\frac{BF}{FA}=\frac{x}{1}$$

$\therefore$ $$\frac{BD}{DC}=\frac{BF}{FA}=\frac{AE}{EC}=\frac{x}{1} \qquad \text{(i)}$$

Area of parallelogram $AFDE = AE \times AF \times \sin A$

$$= \frac{bcx}{(1+x)^2} \sin A$$

$$= bc \sin A \left\{\frac{x}{(1+x)^2}\right\}$$

Let $$f(x) = bc \sin A \left\{\frac{x}{(1+x)^2}\right\}$$

$$= k\left\{\frac{x}{(1+x)^2}\right\}, \text{ where } k = bc \sin A$$

$$f'(x) = k\left\{\frac{(x+1)^2 \cdot 1 - x \cdot 2(x+1)}{(x+1)^3}\right\}$$

$$= k(x+1)\frac{(x+1-2x)}{(x+1)^3}$$

$$= k\frac{(1-x)}{(x+1)^2}$$

$$f'(x) = 0 \text{ gives } (1-x) = 0 \Rightarrow x = 1$$

As $f''(x) < 0$ for $x = 1$, it corresponds to the condition for $f(x)$ to be maxima at $x = 1$.

This suggests that $D$, $E$ and $F$ are the mid-points of the sides.

$\therefore$ Area of parallelogram $AEDF = \frac{1}{2}$ Area of $\triangle ABC$

$$= \frac{1}{2} \times \frac{1}{2} \times \begin{vmatrix} p^2 & -p & 1 \\ q^2 & q & 1 \\ r^2 & -r & 1 \end{vmatrix}$$

$$= \frac{1}{4}\begin{vmatrix} p^2 & -p & 1 \\ q^2 & q & 1 \\ r^2 & -r & 1 \end{vmatrix}$$

$$= \frac{1}{4}\begin{vmatrix} p^2-q^2 & -p-q & 0 \\ q^2-r^2 & q+r & 0 \\ r^2 & -r & 1 \end{vmatrix}$$

$$= \frac{1}{4}(p+q)(q+r)\begin{vmatrix} (p-q) & -1 & 0 \\ (q-r) & 1 & 0 \\ r^2 & -r & 1 \end{vmatrix}$$

$$= \tfrac{1}{4}(p+q)(q+r)\{(p-q)(1)+1(q-r)+0\}$$
$$= \tfrac{1}{4}(p+q)(q+r)(p-r)$$

Thus the maximum area of the parallelogram is equal to

$$\tfrac{1}{4}(p+q)(q+r)(p-r)$$

**Ans. 15**

The given function $f(x)$ is defined in the interval $[-2, 2]$ such that

$$f(x)=\begin{cases}-1, & -2\leqslant x\leqslant 0\\ x-1, & 0<x\leqslant 2\end{cases}$$

Given $g(x)=f(\,|\,x\,|\,)+|\,f(x)\,|$

We are required to test the differentiability of $g(x)$ in $(-2, 2)$. Let us rewrite $g(x)$ for various intervals.

(i) For $-2\leqslant x\leqslant 0$, $\quad g(x)=f(\,|\,x\,|\,)+|\,f(x)\,|$
$$=-x$$

(ii) For $0<x\leqslant 1$, $\quad g(x)=f(\,|\,x\,|\,)+|\,f(x)\,|$
$$=1-1$$
$$=0$$

(iii) For $1<x\leqslant 2$, $\quad g(x)=f(\,|\,x\,|\,)+|\,f(x)\,|$
$$=2(x-1)$$

$$\therefore \qquad g(x)=\begin{cases}-x, & -2\leqslant x\leqslant 0\\ 0, & 0<x\leqslant 1\\ 2(x-1), & 1<x\leqslant 2\end{cases}$$

Thus, the given function $g(x)$ is differentiable at all other points except at $x=0$ and $x=1$. It is shown below in the graph.

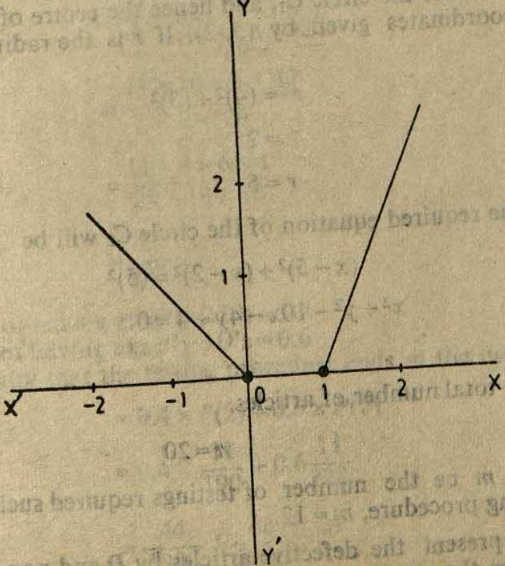



Fig. 20

**Ans. 16**

Equations of the given lines are

$$5x+12y-10=0 \qquad \text{(i)}$$

$$5x-12y-40=0 \qquad \text{(ii)}$$

Adding (i) and (ii) we have

$$10x-50=0$$

or $$x=5$$

Putting $x=5$ in (i), we get

$$y=-\frac{5}{4}$$

Thus the given lines intersect at a point whose coordinates are

$$\left(5, -\frac{5}{4}\right).$$

Equations of angle bisectors of the given lines will be given by

$$\frac{5x+12y-10}{\sqrt{(5)^2+(12)^2}}=\pm\frac{5x-12y-40}{\sqrt{(5)^2+(-12)^2}}$$

or $$5x+12y-10=\pm(5x-12y-40)$$

Taking $+$ve sign, we have, $y=-5/4$
Taking $-$ve sign, we have $x=5$

Since $C_1$ lies in the first quadrant, we can write the coordinates of $C_1$ as $(5, t)$. As radius of $C_1$ is 3, the only possible value for $t=2$, the other is not admissible as it does not lie in the first quadrant.

$\therefore$ Centre of the circle $C_1$, and hence the centre of the circle $C_2$, will have coordinates given by $(5, 2)$. If $r$ is the radius of circle $C_2$, we have

$$r^2=(4)^2+(3)^2$$

$$=25$$

or $$r=5$$

Thus the required equation of the circle $C_2$ will be

$$(x-5)^2+(y-2)^2=(5)^2$$

or $$x^2+y^2-10x-4y+4=0$$

**Ans. 17**

Let $n$ be total number of articles.

$\therefore$ $$n=20$$

Suppose $m$ be the number of testings required such that for ending the testing procedure, $m=12$.

If we represent the defective articles by $D$ and non-defected articles by $E$, then the arrangement

$$\begin{array}{cc} D & \\ \uparrow & (r-1)D\text{'s} \times (m-r)E\text{'s} \\ | & \underset{\text{for } (m-1) \text{ places}}{\longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\longrightarrow} \\ | & \\ m\text{th} & \end{array}$$

can be made in $({}^{m-1}C_{r-1} \times \underline{|r}) \times ({}^{n-r}P_{m-r})$ ways.

Total number of possible ways $= {}^{n}P_m$

$\therefore$ The probability $P(r)$ of the above arrangement will be given by

$$P(r) = \frac{{}^{m-1}C_{r-1} \times \underline{|r} \times {}^{n-r}P_{m-r}}{{}^{n}P_m}$$

Let us find out $P(2)$ and $P(3)$ for $m = 12$ and $n = 20$.

$$P(2) = \frac{{}^{12-1}C_{2-1} \times \underline{|2} \times {}^{20-2}P_{12-2}}{{}^{20}P_{12}}$$

$$= \frac{{}^{11}C_1 \times 2 \times {}^{18}P_{10}}{{}^{20}P_{12}}$$

$$= \frac{11 \times 2 \times \underline{|18} \times \underline{|8}}{\underline{|20} \times \underline{|8}}$$

$$= \frac{11 \times 2}{20 \times 19} = \frac{11}{190}$$

$$P(3) = \frac{{}^{12-1}C_{3-1} \times \underline{|3} \times {}^{20-3}P_{12-3}}{{}^{20}P_{12}}$$

$$= \frac{{}^{11}C_2 \times 3 \times 2 \times 1 \times {}^{17}P_9}{{}^{20}P_{12}}$$

$$= \frac{\underline{|11} \times 6 \times \underline{|17} \times \underline{|8}}{\underline{|9} \times \underline{|2} \times \underline{|8} \times \underline{|20}}$$

$$= \frac{11 \times 10 \times 6 \times \underline{|17}}{2 \times \underline{|20}}$$

$$= \frac{11 \times 5 \times 6 \times 1}{20 \times 19 \times 18}$$

$$= \frac{11}{228}$$

Probability of having exactly $2D$'s $= 0.4$

Probability of having exactly $3D$'s $= 0.6$

$\therefore$ Probability that the testing procedure ends at the twelfth testing

$$= 0.4 \times P(2) + 0.6 \times P(3)$$

$$= 0.4 \times \frac{11}{190} + 0.6\frac{11}{228}$$

$$= \frac{44}{1900} + \frac{66}{2280}$$

$$=\frac{44\times6+66\times5}{11400}$$

$$=\frac{264+330}{11400}$$

$$=\frac{594}{11400}$$

$$=\frac{99}{1900}$$

Thus the required probability is $\frac{99}{1900}$

## PART A

**Ans. 1**

(i) over
(ii) to
(iii) on
(iv) into
(v) for
(vi) on
(vii) around
(viii) to
(ix) beyond
(x) up

**Ans. 2**

(i) exposure
(ii) breakage
(iii) annoyance
(iv) demolition
(v) repitition
(vi) hasten
(vii) deduce
(viii) distinguished
(ix) full
(x) fright

**Ans. 3**

(i) had worked
(ii) looking
(iii) had bolted
(iv) to have discovered
(v) was still burning
(vi) asked
(vii) has been practising
(viii) would have played
(ix) being told
(x) hung

**Ans. 4**

(i) (B)
(ii) (A)
(iii) (A)
(iv) (C)
(v) (B)
(vi) (B)
(vii) (A)
(viii) (C)
(ix) (B)
(x) (B)

**Ans. 5**

(i) The good piece of news I brought made him happy.
(ii) Increasing population leads to a shaky economy.
(iii) He separated the various parts of the bicycle and verified their numbers.
(iv) Let me begin with the fundamental problem we are concerned with.
(v) Many of the difficult experiments he conducted led him nowhere.

## PART B

*Important note*: The candidate is expected to answer questions of PART B in his own words as far as possible. Marks for compositions are awarded on the criteria of (i) relĕvance of subject matter, (ii) control over language and (iii) organization, which involves the development of the subject in a logical and coherent manner. Unduly long answers (in case the word-limit is mentioned) are penalised. Serious errors of grammar and usage will also be penalised. A mere reproduction of sentences from the original passage (in case of Question 9) does not fetch any marks. For getting good marks in 'Part B' answers should be brief and relevant. Above all, the candidate must have a good control over the English language.

PART B

*Important note:* The candidate is expected to answer [illegible] his own words as far as possible. Marks [illegible] the criteria of (i) relevance of subject matter [illegible] language, and [illegible] organisation, which involves the development of [illegible] coherent [illegible]. Unduly long answers [illegible]. Serious errors of grammar and usage [illegible] reproductions of sentences [illegible]. In the case of Question 9) does not fetch any marks [illegible] answers should be brief and relevant [illegible] good control over the English language.

# 1987

# QUESTION PAPERS

## PHYSICS

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

Attempt *all* questions from Part A and any *eight* questions from Part B.

*Note*—(1) Answers must be written in English.

(2) Figures in brackets on the right hand margin indicate the marks for the corresponding questions.

(3) Answers to Part A *must* be given only on the *first six* pages of the answer book *in the order in which the questions appear in the question paper*.

(4) The answer to each question of Part B should begin on a fresh page.

(5) Answer all parts of a question at one place.

(6) Logarithmic tables will be supplied. However, use of calculators or slide rules is not permitted.

*Useful physical data*:

Acceleration due to gravity ($g$) = 9.8 m/s$^2$

Gas constant ($R$) = 8.3 J/kmol. K

Permeability of vacuum ($\mu_o$) = $4\pi \times 10^{-7}$ N/A$^2$

Boltzman constant ($k$) = $1.38 \times 10^{-23}$ J/K

Density of water = $10^3$ kg/m$^3$

1 eV = $1.6 \times 10^{-19}$ J.

### PART A

1. In each of the statements given below four alternatives are given. In some cases more than one alternative is correct. Select the correct alternative(s) in each case and write down the corresponding letter(s) (A), (B), (C) or (D) in your answer book. For each part marks will be awarded only if *all* the correct alternaives are selected and written: **(9 × 2 = 18)**

(i) Four physical quantities are listed in Column I. Their values are listed in Column II in a random order:

| Column I | Column II |
|---|---|
| (*a*) Thermal energy of air molecules at room temperature. | (*e*) 0.02 eV |
| (*b*) Binding energy of heavy nuclei per nucleon. | (*f*) 2 eV |
| (*c*) X-ray photon energy | (*g*) 1 KeV |
| (*d*) Photon energy of visible light | (*h*) 7 MeV |

The correct matching of Columns I and II is given by:

(A) $a - e, b - h, c - g, d - f$

(B) $a - e, b - g, c - f, d - h$

(C) $a - f, b - e, c - g, d - h$

(D) $a - f, b - h, c - e, d - g$

(ii) A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that:

(A) its velocity is constant

(B) its acceleration is constant

(C) its kinetic energy is constant

(D) it moves in a circular path

(iii) A particle executes simple harmonic motion with a frequency, $f$. The frequency with which its kinetic energy oscillates is:

(A) $f/2$ (B) $f$ (C) $2f$ (D) $4f$

(iv) The displacement of particles in a string stretched in the $x$-direction is represented by $y$. Among the following expressions for $y$, those describing wave motion are:

(A) $\cos kx \sin \omega t$ (B) $k^2x^2 - \omega^2t^2$ (C) $\cos^2 (kx + \omega t)$
(D) $\cos (k^2x^2 - \omega^2t^2)$

(v) A charge $q$ is placed at the centre of the line joining two equal charges $Q$. The system of the three charges will be in equilibrium if $q$ is equal to:

(A) $-\frac{Q}{2}$ (B) $-\frac{Q}{4}$ (C) $+\frac{Q}{4}$ (D) $+\frac{Q}{2}$

(vi) A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles:

(A) the charge on the capacitor increases

(A) the voltage across the plates increases

(C) the capacitance increases

(D) the electrostatic energy stored in the capacitor increases

(vii) Photoelectric effect supports quantum nature of light because:

(A) there is a minimum frequency of light below which no photoelectrons are emitted.

(B) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity.

(C) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately.
(D) electric charge of the photoelectrons is quantized.

(viii) During a negative beta decay:
(A) an atomic electron is ejected.
(B) an electron which is already present within the nucleus is ejected.
(C) a neutron in the nucleus decays emitting an electron.
(D) a part of the binding energy of the nucleus is converted into an electron.

(ix) During a nuclear fusion reaction:
(A) a heavy nucleus breaks into two fragments by itself.
(B) a light nucleus bombarded by thermal neutrons breaks up.
(C) a heavy nucleus bombarded by thermal neutrons breaks up.
(D) two light nuclei combine to give a heavier nucleus and possibly other products.

2. Fill in the blanks in the following statements. Indicate the subdivision number and write the answer corresponding to the blanks in your answer book. **($9 \times 2 = 18$)**

(i) A spotlight $S$ rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light $P$ moves along the wall at a distance of 3 m. The velocity of the spot $P$ when $\theta = 45$ degrees (*see* Fig. 1) is ......... m/s.

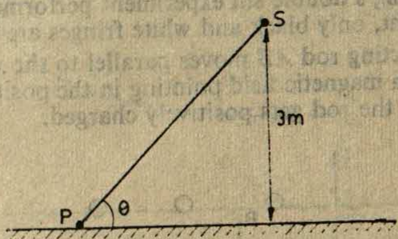



Fig. 1

(ii) A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth, where $R$ is the radius of the earth. The time period of another satellite at a height of $2.5\ R$ from the surface of the earth is ......... hours.

(iii) A particle of mass 4 m which is at rest explodes into three fragments. Two of the fragments, each of mass $m$, are found to move with a speed $v$, each in mutually perpendicular directions. The total energy released in the process of explosion is .........

(iv) A wire of length $L$ and cross sectional area $A$ is made of a material of Young's modulus $Y$. If the wire is stretched by an amount $x$, the work done is .........

(v) In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of $0.0075\ m^3$. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ......... Hz.

(vi) A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index $\frac{4}{3}$, its focal length will become ......... cm.

(vii) During an experiment, an ideal gas is found to obey an additional law $VP^2$ = constant. The gas is initially at a temperature $T$, and volume $V$. When it expands to a volume $2V$, the temperature becomes .........

(viii) An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance $R$ that one must put in series with the bulb, so that the bulb delivers 500 W is ......... $\Omega$.

(ix) A wire of length $L$ metres carrying a current $i$ amperes is bent in the form of a circle. The magnitude of its magnetic moment is ......... in MKS units.

3. State whether the following statements are *true* or *false*. Give reasons in brief in support of your answer. Marks will be awarded only if correct reasons are also given. (4 × 2 = 8)

(i) At a given temperature, the specific heat of a gas at constant pressure is always greater then its specific heat at constant volume.

(ii) The root mean square (rms) speed of oxygen molecules ($O_2$) at a certain temperature $T$ (degree absolute) is $V$. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged.

(iii) In a Young's double slit experiment performed with a source of white light, only black and white fringes are observed.

(iv) A conducting rod $AB$ moves parallel to the $x$-axis (*see* Fig. 2) in a uniform magnetic field pointing in the positive $z$-direction. The end $A$ of the rod gets positively charged.

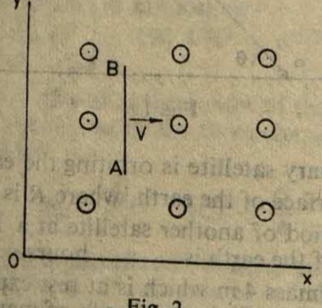



Fig. 2

## PART B

4. A particle of mass $m$ rests on a horizontal floor with which it has a coefficient of static friction $\mu$. It is desired to make the body move by applying the minimum possible force $F$. Find the magnitude of $F$ and the direction in which it has to be applied. (7)

5. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (*see*

Fig. 3) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillation becomes less than 60 degrees? (7)

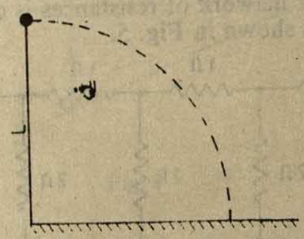



Fig. 3

6. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point *B* (which is vertically below the end of the track as shown in Fig. 4) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centre of mass? Explain. (7)

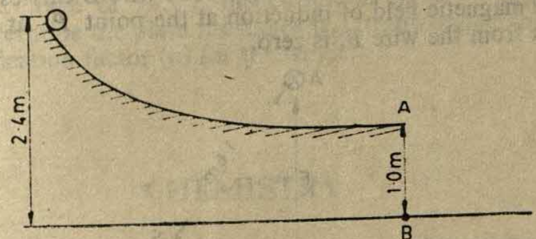



Fig. 4

7. The following equations represent transverse waves: (7)

$z_1 = A \cos (kx - \omega t)$

$z_2 = A \cos (kx + \omega t)$

$z_3 = A \cos (ky - \omega t)$

Identify the combination(s) of the waves which will produce (i) standing wave(s), (ii) a wave travelling in the direction making an angle of 45 degrees with the positive $x$ and positive $y$ axes. In each case, find the positions at which the resultant intensity is always zero.

8. An ideal gas has a specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m$^3$, at a temperature of 300 K and a pressure of $1.6 \times 10^6$ N/m$^2$. An amount of $2.49 \times 10^4$ J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

9. Three point charges $q$, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge $q$ due to the other two charges? (7)

10. An infinite ladder network of resistances is constructed with 1 $\Omega$ and 2 $\Omega$ resistances, as shown in Fig. 5. (7)

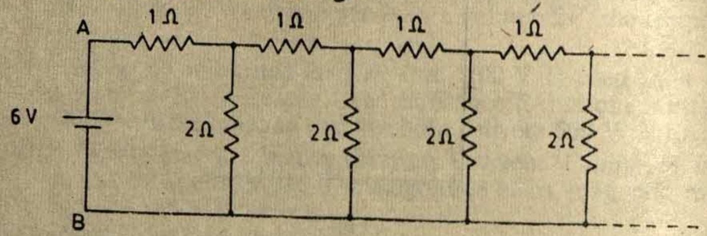



Fig. 5

The 6-V battery between $A$ and $B$ has negligible internal resistance.
(i) Show that the effective resistance between $A$ and $B$ is 2 $\Omega$.
(ii) What is the current that passes through the 2 $\Omega$ resistance nearest to the battery?

11. Two long straight parallel wires are 2 m part, perpendicular to the plane of the paper (*see* Fig. 6). The wire $A$ carries a current of 9.6 A, directed into the plane of the paper. The wire $B$ carries a current such that the magnetic field of induction at the point $P$, at a distance of 10/11 m from the wire $B$, is zero. (7)

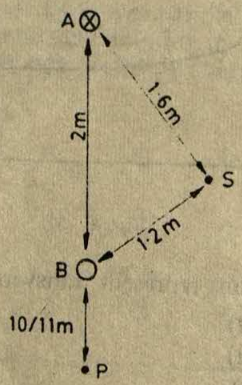



Fig. 6

Find:

(i) The magnitude and direction of the current in $B$.
(ii) The magnitude of the magnetic field of induction at the point $S$.
(iii) The force per unit length on the wire $B$.

12. A right prism is to be made by selecting a proper material and the angles $A$ and $B$ ($B \leqslant A$), as shown in Fig. 7. It is desired that a ray of light incident on the $AB$ emerges parallel to the incident direction after two internal reflections. (7)

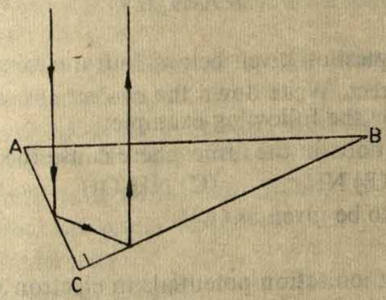



Fig. 7

(i) What should be the minimum refractive index $n$ for this to be possible?

(ii) For $n = \frac{5}{3}$ is it possible to achieve this with the angle $B$ equal to 30 degrees?

13. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of $-1$ V, the anode current $I$ (in milli-amperes) is given in terms of plate voltage $V$ (in volts) by the algebraic relation: (7)

$I = 0.125\ V - 7.5$

For grid voltage of $-3$ V, the current at anode voltage of 300 V is 5 mA. Determine the plate resistance ($r_p$), transconductance ($g_m$) and the amplification factor ($\mu$) for the triode.

# CHEMISTRY

TIME ALLOWED: 3 HOURS — MAXIMUM MARKS: 100

*Note* (1) Answers must be written in English.

(2) Figures in brackets on the right hand margin indicate the marks for the corresponding questions.

(3) Answers to Part A *must* be given only on the *first six pages* of the answer book *in the order in which the questions appear in the question paper*.

(4) Answers to each question of Part B should begin on a fresh page.

(5) Answer all parts of a question at one place.

(6) Logarithmic Tables will be supplied. But use of calculators or slide rules is not permitted.

(7) Values of some physical constants:
Atomic masses : Br = 80.0, C = 12.0, Cd = 112.4, H = 1.0
O = 16.0, Na = 23.0.
1 faraday = 96,500 coulombs.

## PART A

1. For each sub-question given below, four answers are given of which only *one* is correct. Write down the *correct answer* in the answer-book as illustrated by the following example:

[The lightning bolts in the atmosphere cause the formation of:
(A) NO (B) $NH_3$ (C) $NH_4OH$ (D) $NH_2OH$.
The answer is to be given as (A)]

(20 × 1 = 20)

(i) The first ionisation potentials in electron volts of nitrogen and oxygen atoms are respectively given by—
(A) 14.6, 13.6 (B) 13.6, 14.6 (C) 13.6, 13.6
(D) 14.6, 14.6

(ii) Atomic radii of fluorine and neon in Angstrom units are respectively given by—
(A) 0.72, 1.60 (B) 1.60, 1.60 (C) 0.72, 0.72
(D) None of these values.

(iii) When mercuric iodide is added to the aqueous solution of potassium iodide the—
(A) freezing point is raised.
(B) freezing point is lowered.
(C) freezing point does not change.
(D) boiling point does not change.

(iv) A solution of sodium sulphate in water is electrolysed using inert electrodes. The products at the cathode and anode are respectively—
(A) $H_2$, $O_2$ (B) $O_2$, $H_2$ (C) $O_2$, Na (D) $O_2$, $SO_2$

(v) The brown ring complex compound is formulated as $[Fe(H_2O)_5(NO)^+]SO_4$. The oxidation state of iron is—
(A) 1 (B) 2 (C) 3 (D) 0

(vi) The electronegativity of the following elements increases in the order—
(A) C, N, Si, P (B) N, Si, C, P (C) Si, P, C, N
(D) P, Si, N, C

(vii) Of the following compounds, which will have a zero dipole moment?
(A) 1,1-dichloroethylene (C) *trans*-1, 2-dichloroethylene
(B) *cis*-1, 2-dichloroethylene (D) None of these compounds.

(viii) The bond between carbon atom (1) and carbon atom (2) in compound—
$N \equiv \underset{1}{C} - \underset{2}{C}H = CH_2$ involves the hybrids as—
(A) $sp^2$ and $sp^2$ (B) $sp^3$ and sp (C) sp and $sp^2$
(D) sp and sp

(ix) The IUPAC name of the compound $CH_2{=}CH{-}CH(CH_3)_2$ is—
(A) 1,1-dimethyl 2-propene (C) 2-vinylpropane
(B) 3-methyl 1-butene (D) 1-isopropyl ethylene

(x) Which of the following will have least hindered rotation about carbon-carbon bond?
(A) Ethane (B) Ethylene (C) Acetylene
(D) Hexachloroethane

(xi) If two compounds have the same empirical formula but different molecular formulae they must have—
(A) different percentage composition.
(B) different molecular weight.
(C) same viscosity.
(D) same vapour density.

(xii) Which of the following oxides of nitrogen is a coloured gas?
(A) $N_2O$ (B) $NO$ (C) $N_2O_4$ (D) $NO_2$

(xiii) The number of isomers of $C_6H_{14}$ is—
(A) 4 (B) 5 (C) 6 (D) 7

(xiv) When a lead storage battery is discharged—
(A) $SO_2$ is evolved. (C) Lead sulphate is consumed.
(B) Lead is formed. (D) Sulphuric acid is consumed.

(xv) The metallic lustre exhibited by sodium is explained by—
(A) diffusion of sodium ions.
(B) oscillations of loose electrons.
(C) excitation of free protons.
(D) existence of body centred cubic lattice.

(xvi) *n*-Propyl bromide on treatment with ethanolic potassium hydroxide produces—
(A) Propane (B) Propene (C) Propyne (D) Propanol

(xvii) Amongst the trihalides of nitrogen, which one is least basic?
(A) $NF_3$ (B) $NCl_3$ (C) $NBr_3$ (D) $NI_3$

(xviii) Hydrogen bonding is maximum in—
(A) Ethanol (B) Diethylether (C) Ethyl chloride
(D) Triethylamine

(xix) Which of the following compounds is oxidised to prepare methyl ethyl ketone?
(A) 2-Propanol (B) 1-Butanol (C) 2-Butanol
(D) *t*-Butyl alcohol

(xx) Bromine can be liberated from potassium bromide solution by action of—
(A) Iodine solution (B) Chlorine water
(C) Sodium chloride (D) Potassium iodide

2. Fill in the blanks with appropriate words:

(i) Hydrogen gas is liberated by the action of aluminium with concentrated solution of...............

(ii) The electrical conductivity of a solution of acetic acid will be... ............if a solution of sodium hydroxide is added.

(iii) Silver chloride is springly soluble in water because its lattice energy is greater than..............energy.

(iv) The value of $PV$ for 5.6 litres of an ideal gas is ............ $RT$, at *NTP*.

(v) .............. phosphorus is reactive because of its highly strained tetrahedral structure.

3. State whether the following statements are *true* or *false*. If *false*, give the correct statement:

$(10 \times 1 = 10)$

(i) In benzene, carbon uses all the three $p$-orbitals for hybridisation.

(ii) Sodium when burnt in excess of oxygen gives sodium oxide.

(iii) Catalyst makes a reaction more exothermic.

(iv) Graphite is a better lubricant on the moon than on the earth.

(v) $sp^2$ hybrid orbitals have equal $s$ and $p$ character.

(vi) The reaction of methyl magnesium iodide with acetone followed by hydrolysis gives secondary butanol.

(vii) Dipositive zinc exhibits paramagnetism due to loss of two electrons from 3d-orbital of neutral atom.

(viii) An electron donating substituent in benzene orients the incoming electrophillic group to the meta position.

(ix) From the solution containing copper (+2) and zinc (+2) ions, copper can be selectively precipitated using sodium sulphide.

(x) In Group IA, of alkali metals, the ionisation potential decreases down the group. Therefore lithium is a poor reducing agent.

4. Write balanced equations for the following reactions: $(5 \times 1 = 5)$

(i) Phosphorus is reacted with boiling aqueous solution of sodium hydroxide in an inert atmosphere.

(ii) Dilute nitric acid is slowly reacted with metallic tin.

(iii) Potassium permanganate is reacted with warm solution of oxalic acid in the presence of sulphuric acid.

(iv) Gold is dissolved in *aqua regia*.

(v) Acetamide is reacted with bromine in the presence of potassium hydroxide.

5. (*a*) Explain the following in *one* or *two* sentences only: $(5 \times 1 = 5)$

(i) Magnesium oxide is used for the lining of steel making furnace.

(ii) The mixture of hydrazine and hydrogen peroxide with a copper(II) catalyst is used as a rocket propellant.

(iii) Orthophosphorous acid is not tribasic acid.

(iv) Phenol is an acid but it does not react with sodium bicarbonate.

(v) The molecule of magnesium chloride is linear whereas that of stannous chloride is angular.

(*b*) A mixture of two salts was treated as follows: (5)

(i) The mixture was heated with manganese dioxide and concentrated sulphuric acid, when yellowish green gas was liberated.

(ii) The mixture on heating with sodium hydroxide solution gave a gas which turned red litmus blue.

(iii) Its solution in water gave blue precipitate with potassium ferricyanide and red coloration with ammonium thiocyanate.

(iv) The mixture was boiled with potassium hydroxide and the liberated gas was bubbled through an alkaline solution of $K_2HgI_4$ to give brown precipitate.

Identify the two salts. Give ionic equations for reactions involved in the tests (i), (ii) and (iii).

## PART B

6. (*a*) Answer the following questions briefly: **(5×1 = 5)**
   (i) What is the actual reducing agent of haematite in blast furnace?
   (ii) Give the equations for the recovery of lead from galena by air reduction.
   (iii) Why is sodium chloride added during electrolysis of fused anhydrous magnesium chloride?
   (iv) Zinc and not copper is used for the recovery of metallic silver from complex $[Ag(CN)_2]^-$. Explain.
   (v) Why is chalcocite roasted and not calcinated during recovery of copper?

   (*b*) Answer the following with suitable equations wherever necessary: **(5×1 = 5)**
   (i) How can you prepare benzene from lime?
   (ii) How will you convert toluene to *m*-nitrobenzoic acid?
   (iii) Suggest a reagent to distinguish acetaldehyde from acetone.
   (iv) What happens when excess chlorine is passed through boiling toluene in the presence of sunlight?
   (v) What effect should the following resonance of vinylchloride have on its dipole moment?

$$CH_2 = CH - Cl \leftrightarrow \overline{CH_2} - CH = Cl^+.$$

7. (*a*) A spherical balloon of 21 cm diameter is to be filled up with hydrogen at NTP from a cylinder containing the gas at 20 atmospheres at 27°C. If the cylinder can hold 2.82 litres of water, calculate the number of balloons that can be filled up. **(5)**

   (*b*) What is the pH of the solution when 0.20 mole of hydrochloric acid is added to one litre of a solution containing **(5)**
   (i) 1 M each of acetic acid and acetate ion?
   (ii) 0.1 M each of acetic acid and acetate ion?
   Assume the total volume is one litre. $K_a$ for acetic acid = $1.8 \times 10^{-5}$.

8. (*a*) (i) What is the weight of sodium bromate and molarity of solution necessary to prepare 85.5 ml of 0.672N solution when the half-cell reaction is—

$$BrO_3^- + 6H^+ + 6e^- \rightarrow Br^- + 3H_2O$$

   (ii) What would be the weight as well as molarity if the half-cell reaction is—

$$2BrO_3^- + 12H^+ + 10e^- \rightarrow Br_2 + 6H_2O$$

   (*b*) At a certain temperature equilibrium constant ($K_c$) is 16 for the reaction— **(5)**

$$SO_2(g) + NO_2(g) \rightleftharpoons SO_3(g) + NO(g)$$

If we take one mole of each of all the four gases in a one litre container, what would be the equilibrium concentration of NO and $NO_2$?

9. (*a*) A 100-W, 110-V incandescent lamp is connected in series with an electrolyte cell containing cadmium sulphate solution. What weight of cadmium will be deposited by the current flowing for 10 hours? (5)

   (*b*) A first order gas reaction has $k = 1.5 \times 10^{-6}$ per second at 200°C. If the reaction is allowed to run for 10 hours, what percentage of the initial concentration would have changed in the product? What is the half life of this reaction?

10. (*a*) An unknown compound of carbon, hydrogen and oxygen contains 69.77% carbon and 11.63% hydrogen and has a molecular weight of 86. It does not reduce Fehling's solution, but forms a bisulphite addition compound and gives a positive iodoform test. What are the possible structures? (5)

    (*b*) An organic compound (A) on treatment with acetic acid in the presence of sulphuric acid produces an ester (B), (A) on mild oxidation gives (C), (C) with 50% potassium hydroxide followed by acidification with dilute hydrochloric acid generates (A) and (D), (D) with phosphorus pentachloride followed by reaction with ammonia gives (E), (E) on dehydration produces hydrocyanic acid.

    Identify the compounds A, B. C. D and E. (5)

# MATHEMATICS

TIME ALLOWED: 3 Hours MAXIMUM MARKS: 100

*Note* (1) Answers must be written in English.
(2) Figures in brackets on the right hand margin indicate the marks for the corresponding questions.
(3) Answers to Part A *must* be given only on the *first six pages* of the answer book *in the order in which the questions appear in the question paper.*
(4) Answer to each question of Part B should begin on a fresh page.
(5) Answer all parts of a question at one place.
(6) Use of calculator, logarithmic, trigonometric and statistical tables and graph paper is not allowed.

## PART A

*Notes* (1) Attempt *all* questions in this Part.
(2) You are advised to show the rough work used to arrive at the conclusion.

1. There are *ten* parts in this question. Each part has *one or more than one* correct answer. For each part, write the letters from (A), (B), (C), (D), (E) corresponding to the correct answers. **(10×2)**

(i) If $a, b, c, d$ and $p$ are distinct real numbers such that:

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leqslant 0,$$

then $a, b, c, d$

(A) are in A.P. (B) are in G.P.
(C) are in H.P. (D) satisfy $ab = cd$
(E) satisfy none of these.

(ii) In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P., then the length of the third side can be:

(A) $5 - \sqrt{6}$ (B) $3\sqrt{3}$ (C) 5
(D) $5 + \sqrt{6}$ (E) none of these.

(iii) The smallest positive root of the equation, $\tan x - x = 0$ lies in:

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$
(C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$
(E) none of these.

(iv) The probability that at least one of the events $A$ and $B$ occurs is 0.6. If $A$ and $B$ occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is:

(A) 0.4 (B) 0.8 (C) 1.2
(D) 1.4 (E) none of these.

(Here $\bar{A}$ and $\bar{B}$ are complements of $A$ and $B$, respectively.)

(v) If $z_1$ and $z_2$ are two nonzero complex numbers such that

$$|z_1 + z_2| = |z_1| + |z_2|,$$

then Arg $z_1 -$ Arg $z_2$ is equal to:

(A) $-\pi$ (B) $-\frac{\pi}{2}$ (C) 0
(D) $\frac{\pi}{2}$ (E) $\pi$

(vi) The set of all points where the function $f(x) = \dfrac{x}{(1 + |x|)}$ is differentiable, is:

(A) $(-\infty, \infty)$ (B) $[0, \infty)$ (C) $(-\infty, 0) \cup (0, \infty)$
(D) $(0, \infty)$ (E) none of these.

(vii) The value of $\sum_{k=1}^{6} \left(\sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7}\right)$ is:

(A) $-1$ (B) 0 (C) $-i$
(D) $i$ (E) none of these.

(viii) The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0)$ and $\vec{b} = (0, 1, 1)$ is:

(A) one (B) two (C) three
(D) infinite (E) none of these.

(ix) Let $f$ and $g$ be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is:

(A) always zero (B) always negative
(C) always positive (D) strictly increasing
(E) none of these.

(x) The number of all possible triplets $(a_1, a_2, a_3)$ such that

$$a_1 + a_2 \cos(2x) + a_3 \sin^2(x) = 0 \quad \text{for all } x \text{ is:}$$

(A) zero (B) one (C) three
(D) infinite (E) none of these.

2. This question contains *ten* incomplete statements. Fill in the blanks so that the statements are correct. Write only the answers. (10×2

(i) The set of all $x$ in the interval $[0, \pi]$ for which

$$2\sin^2 x - 3\sin x + 1 \geqslant 0, \text{ is ......}$$

(ii) The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is .........

(iii) A polygon of nine sides, each of length 2, is inscribed in a circle. The radius of the circle is .........

(iv) If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of

$$\frac{1}{(1-a)} + \frac{1}{(1-b)} + \frac{1}{(1-c)} - \text{.........}$$

(v) $$\lim_{x \to -\infty}\left[\frac{\left(x^4 \sin\left(\frac{1}{x}\right) + x^2\right)}{(1 + |x|^3)}\right] - \text{.........}$$

(vi) The sides of a triangle inscribed in a given circle subtend angles $\alpha$, $\beta$ and $\gamma$ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to .........

(vii) The set of all $x$ for which $\ln(1 + x) \leqslant x$ is equal to .........

(viii) Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the $xy$-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by .........

(ix) If the expression:

$$\frac{\left[\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) - i\tan(x)\right]}{\left[1 + 2i\sin\left(\frac{x}{2}\right)\right]}$$

is real, then the set of all possible values of $x$ is .........

(x) Let—

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

Then $\int_0^{\pi/2} f(x)\, dx = \ldots\ldots\ldots$

## PART B

*Note*: (1) Attempt *all* questions
(2) Justify your answers with mathematical arguments.

3. (*a*) A student is allowed to select atmost $n$ books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, find the value of $n$. (3)

(b) Let a given line $L_1$ intersect the $x$ and $y$ axes at $P$ and $Q$, respectively. Let another line $L_2$, perpendicular to $L_1$, cut the $x$ and $y$ axes at $R$ and $S$, respectively. Show that the locus of the point of intersection of the lines $PS$ and $QR$ is a circle passing through the origin. (3)

4. (a) Prove by mathematical induction that: (3)

$$\frac{(2n)!}{2^{2n}(n!)^2} \leqslant \frac{1}{(3n+1)^{1/2}}$$

for all positive integer $n$.

(*b*) Find the set of all $x$ for which— (3)

$$\frac{2x}{(2x^2 + 5x + 2)} > \frac{1}{(x+1)}$$

5. (*a*) The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is:

$x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find $k$. (4)

(*b*) Let $f(x)$ be a continuous function and $g(x)$ be a discontinuous function. Prove that $f(x) + g(x)$ is a discontinuous function. (2)

6. (*a*) A 2 m long object is fired vertically upwards from the midpoint of two locations $A$ and $B$, 8 m apart. The speed of the object after $t$ s is given by $ds/dt = (2t + 1)$ m/s. Let $\alpha$ and $\beta$ be the angles subtended by the object at $A$ and $B$, respectively after one and two seconds. Find the value of $\cos(\alpha - \beta)$. (3)

(*b*) Let $\alpha_1$, $\alpha_2$ and $\beta_1$, $\beta_2$ be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$, respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a nontrivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$. (3)

7. (*a*) Solve for $x$ the following equation:

$$\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9). \quad (3)$$

(*b*) A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point. (3)

8. (*a*) Find the point on the curve:

$$4x^2 + a^2y^2 = 4a^2, \qquad 4 < a^2 < 8$$

that is farthest from the point $(0, -2)$. (3)

(*b*) Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real $x$.

If $f'(0)$ exists, find its value. (3)

9. (*a*) An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. (4)

(*b*) If $A$, $B$, $C$, $D$ are any four points in space, prove that:

$$|\overrightarrow{AB}\times\overrightarrow{CD} + \overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}| = 4(\text{area of triangle } ABC). \quad (2)$$

10. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by:

$$f(x) = \begin{cases} g(x) & , \; x \leqslant 0 \\ \left[\dfrac{(1+x)}{(2+x)}\right]^{1/x} & , \; x > 0 \end{cases}$$

Find the continuous function $f(x)$ satisfying. $f'(1) = f(-1)$. (6)

11. Evaluate:

$$\int \left[\frac{(\cos 2x)^{1/2}}{\sin x}\right] dx. \quad (6)$$

12. Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x = 0$ *above* the $x$-axis. (6)

# ENGLISH

TIME ALLOWED 3 HOURS

MAXIMUM MARKS: 100

*Notes:*
(1) Answers must be written in English.
(2) Figures in brackets on the right hand margin indicate the marks for the corresponding questions.
(3) Answers to Part A *must* be given only on the *first six pages* of the answer book *in the order in which the questions papear in the question paper.*
(4) Answer to each question of Part B should begin on a fresh page.
(5) Answer *all* parts of a question at one place.

## PART A

Marks

1. Fill in the blanks in the following paragraph with appropriate articles (*a*, *an* and *the*). Write only the number of the blank and the article. *Do not reproduce the sentences.* **(5)**

(i) ......... uneasy paragraph appeared in *The Times* recently about ......... (ii) ......... serious crisis. Within ......... (iii) ......... next thirty years, not only natural gas but also other resources on which we now depend will be running low. This assumes that we shall survive......... (iv) ......... more immediate problems of land, food, population and water supply, which show every sign of compounding.........(v)......... crisis within our lifetime.

2. Fill in the blanks with suitable prepositions/adverbial particles. Write only the number of the sentence and the word. *Do not reproduce the sentences.* **(5)**

(i) The audience were swept ......... their feet by the speaker's eloquence.

(ii) Nowadays interest in grammar is ......... the decline.

(iii) Scientists hope to break ......... soon in their battle against cancer.

(iv) Motorists should be dealt ......... severely for rash driving.

(v) The minister promised to look ......... the grievances of the junior engineers.

3. Rewrite the following sentences as directed: **(5)**

(i) What on earth he cried did they say. (Punctuate.)

(ii) The visitor tried to open the gate. Two fierce-looking dogs attacked him. (Combine the two sentences beginning with 'No sooner .........'.)

(iii) People are talking about the new book everywhere. (Rewrite changing the voice.)

(iv) Gavaskar has to his credit the highest aggregate of runs in test matches. (Rewrite the sentence beginning with 'No other batsman .........' and using the comparative degree.)

(v) 'Do not write your name anywhere in the answerbook,' the teacher instructed me. (Rewrite using Reported Speech.)

4. Complete the following sentences with the correct forms of the verbs given in brackets. *Do not reproduce the sentences.* Write only the number of each sentence and the correct form of the verb. **(10)**

(i) I (know) Anita for three years, and so I can tell you a lot about her.

(ii) Next time she (visit) the library, she will find us there.

(iii) When the professor entered the classroom, the students had already (disappear).

(iv) We would not have learnt all these things if we (attend) a village school.

(v) As these soldiers (live) in this desert for three years now, they will soon be posted to a better station.

(vi) Only if these bridges (destroy) can the enemy be defeated.

(vii) Only yesterday I (realize) the gravity of the situation.

(viii) My father had hoped that I (become) an engineer, but I was not good enough at science.

(ix) I would rather you (tell) her the truth now.

(x) She's getting tired; it's time she (go) home.

5. In each of the sentences below, four parts are underlined and marked A, B, C and D. There is a mistake in only one of the underlined parts in each sentence. Write the number of the sentence, identify the incorrect part and rewrite the part in its corrected form: **(10)**

(i) I know he is having a lot of books on how to improve English.
A B C D

(ii) The theory of relativity is so complicated as we cannot describe
A B C
it in a few sentences.
D

(iii) A cell is the smallest identifiable unit of life and cannot be seen
A B C
with a naked eye.
D

(iv) If a student needs advices about careers, he or she should consult
A B C D
the Careers Officer.

(v) This machine looks good but is very badly designed and doesn't
A B C
work good.
D

(vi) He is well-known for both his kindness as well as his understanding.
A B C D

(vii) Please explain to me how is a digital computer different from
A B C
an analogue computer.
D

(viii) I'll work for you as long as you'll pay well.
A B C D

(ix) Of the two principles he put forward, the last one was the most
A B C
difficult to understand.
D

(x) All of us surprised to see an old man of sixty taking part in the
A B C D
marathon held last month.

6. Fill in the blanks with the words whose first letters are given. The meanings of the words are given in brackets. Write only the number of the sentence and the word. *Do not write the sentences.* **(10)**

(i) People often mig ......... from one part of the country to another in search of work. (move)

(ii) Her met ......... work paid off in the end. (showing attention to detail)

(iii) In Delhi, we have occ ......... showers in December. (not regular)

(iv) The chairman wou ......... up the discussion and adjourned the meeting. (brought to an end)

(v) He acted dis ......... in front of his parents. (in a shameful manner)

(vi) His pat ......... speeches have made him a well-known person. (expressing love of one's country)

(vii) An artist's moods are not easily pre ......... (which can be foretold)

(viii) We have very few san ......... in the country for the preservation of wildlife. (places of protection and safety)

(ix) Snow in Madras is an almost unknown phe ......... (unusual occurrence)

(x) He is fond of using bom ......... words in his writings. (high-sounding)

## PART B

7. Write an essay of about 300 words on any *one* of the following topics: **(25)**

   (i) I am an Indian and proud of it

   (ii) My favourite TV programme

   (iii) If I had my childhood to live over

   (iv) Clean politics : a contradiction in terms.

8. Write a paragraph of about 120 words on any *one* of the following topics: **(10)**

   (i) Something good can be said about everyone.

   (ii) If you have a grievance ......... .

   (iii) Life is becoming less human.

9. Argue *for* or *against* one of the following topics in about 120 words: **(10)**

   (i) Men and women are equal.

   (ii) Competition is necessary for growth.

   (iii) India should make the nuclear bomb.

10. Read the passages below and answer the questions that follow each of them: **(4)**

   (*a*) The evidence of history is strong, that those societies are most creative and progressive which safeguard the expression of new ideas. Societies appear to remain vigorous only so long as they are organized to receive novel and unexpected—and sometimes unpleasant—thoughts. It may seem odd that the governments of some countries give a special status to the opposition; many countries actually pay a salary to the leader of the opposition. Yet this legalization of opposition, this balance between power and dissent, is the heart of the democratic tradition.

   (i) When does a society become static?

   (ii) Why does it seem odd that some governments give a special status to the opposition parties?

   (*b*) Very often we think about the great crises in life, the mighty 'moments of truth,' and wonder how we would face them. Yet it

is the little panics, the pressure squeezes of daily life, that should gives us concern. These are the incidents that bring on those mental blackouts which make us appear to be less than we are.

**(6)**

When our word is questioned, for example, instead of stating our case more convincingly, we frequently grow dark-minded and blurt out some reply that we regret later. Or when we match wits in conversation, we may get rattled and subside into banalities. James Thurber once wrote a story based on the witty things he would have said if he had thought of them at the right time—an experience we can all understand. Sometimes, under pressure, we say something utterly different from what we intend, like the young man who became so nervous in his boss's office that he asked for a salary raise for someone else.

(iii) How often do we think of the little panics of life?

(iv) Why could James Thurber not think of witty things at the right time?

(v) What did the young man of the story really want from his boss?

# MODEL SOLUTIONS

## PHYSICS

### PART A

**Ans. 1**

(i) (A) Thermal energy of air molecules at room temperature is 0.02 eV. Binding energy of heavy nuclei per nucleon is 7 MeV. X-ray photon energy is 1 KeV. Photon energy of visible light is 2 eV.

(ii) (C) (D) A force of constant magnitude which is always perpendicular to the velocity of the particle does no work. Hence the kinetic energy of the particle is constant. As the force is perpendicular to the velocity, the particle moves in a circular path.

(iii) (C)

(iv) (A)

(v) (B) Force on charge $q$ at O is given by

$$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{a^2} - \frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{a^2} = 0$$

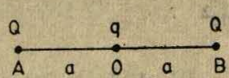



Fig. 8

Force on charge $Q$ at $A$ is given by

$$F' = \frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{a_2} + \frac{1}{4\pi\epsilon_0}\cdot\frac{Q^2}{4a^2}$$

For the system to be at equilibrium, $F'$ must be equal to zero, i.e.

$$\frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{a^2} + \frac{1}{4\pi\epsilon_0}\cdot\frac{Q^2}{4a^2} = 0$$

or $$q = -\frac{Q}{4}$$

(vi) (B) (D) When the charging battery is disconnected, the charge on the plates of the capacitor remains constant. On moving the plates of the capacitor farther apart, capacitance decreases and the voltage across the plates increases. The work done on increasing the separation between the oppositely charged plates increases the electrostatic potential energy of the capacitor.

(vii) (A) (B) (C) No photoelectrons are emitted below a minimum frequency of light. The maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity. The

quantum nature of light explains that the photoelectrons leave the metal surface immediately even when the surface is illuminated jointly.

(viii) (C)

(ix) (D)

**Ans. 2**

(i) 0.3 $$v = \frac{3}{\tan 45°} \times 0.1 = 0.3 \text{ m/s}$$

(ii) 8.485 $$\frac{T'}{T} = \left(\frac{3.5R}{7R}\right)^{3/2}$$

$$\Rightarrow T' = \frac{24}{2\sqrt{2}} = 8.485 \text{ h}$$

(iii) $\frac{3}{2}mv^2$

Let $V$ be the velocity of the fragment with mass $2m$. Then,

$$2mV = mv\sqrt{2}$$

$$\Rightarrow V = \frac{v}{\sqrt{2}}$$

Energy released in the explosion is, therefore, given by

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2} \times 2m \times \left(\frac{v}{\sqrt{2}}\right)^2$$

$$= \frac{3}{2}mv^2$$

(iv) $\frac{YAx^2}{2L}$ Work done, $W = \frac{1}{2}Fx$

Young's modulus, $Y = \frac{FL}{Ax}$

or $$F = \frac{YAx}{L}$$

$\therefore$ $$W = \frac{1}{2} \cdot \frac{YAx}{L} \cdot x$$

$$= \frac{YAx^2}{2L}$$

(v) 240

$$\frac{n'}{n} = \sqrt{\frac{(50.7 - 0.0075 \times 10^3) \times 9.8}{50.7 \times 9.8}}$$

$\Rightarrow$ $$n' = 260 \times \sqrt{\frac{43.2}{50.7}}$$

or $n' = 240$ Hz

(vi) 60

$$\frac{1}{15} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{15}\times\frac{1}{0.5} = \frac{1}{7.5}$$

$$\frac{1}{f_m} = \left(\frac{1.5}{4/3} - 1\right)\times\frac{1}{7.5} = 0.01667$$

or $$f_m = \frac{1}{0.01667} = 60 \text{ cm}$$

(vii) $\sqrt{2}\,T$

An ideal gas obeys the relation,

$$PV = RT$$

or $$P = \frac{RT}{V}$$

Also,

$$VP^2 = \text{constant}$$

or $$\frac{VT^2}{V^2} = \text{constant}$$

or $$\frac{T^2}{V} = \text{constant}$$

When it expands to $2V$, let the temperature be $T'$

Thus, $$\frac{T^2}{V} = \frac{T'^2}{2V}$$

or $$T' = \sqrt{2}\,T$$

(viii) 20

$$R_B = \frac{100\times 100}{500} = 20\ \Omega$$

$$\text{Current} = \frac{500}{100} = 5 \text{ A}$$

$\therefore$ $$5 = \frac{200}{20 + R}$$

or $$R = 20\ \Omega$$

(ix) $\dfrac{iL^2}{4\pi}$

Magnitude of magnetic moment is given by $NiA$

where $A = \pi r^2 = \pi\left(\dfrac{L}{2\pi}\right)^2$, $N = 1$.

Substituting the values, we have:

$$\text{Magnitude of magnetic moment} = 1\times i\times\pi\left(\frac{L}{2\pi}\right)^2$$

$$= \frac{iL^2}{4\pi}$$

**Ans. 3**

(i) *True*: When a gas is heated at constant pressure, some extra heat is needed compared to that at constant volume for doing work in expansion.

(ii) *False*: Let $V'$ be the rms speed on doubling the temperature. Thus

$$V' = \sqrt{\frac{3R \times 2T}{M/2}} = 2\sqrt{\frac{3RT}{M}}$$
$$= 2V$$

(iii) *False*: In a Young's double-slit experiment performed with a source of white light, the central fringe is white, and is surrounded by a few coloured and dark fringes.

(iv) *True:* Using Fleming's right-hand rule, the direction of induced current will be from $B$ to $A$. This means that the electrons move from $A$ to $B$ and the end $A$ of the rod will become positively charged.

## PART B

**Ans. 4**

The forces acting on the particle are as shown in Fig. 9. $F \cos \theta$ and $F \sin \theta$ are the forces of the applied force $F$. Equating the forces,

$$\mu R = F \cos \theta \qquad \text{(i)}$$
$$R = mg - F \sin \theta \qquad \text{(ii)}$$

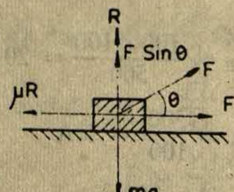



Fig. 9

From (i) and (ii),

$$\mu(mg - F \sin \theta) = F \cos \theta$$

or

$$\mu mg = F(\cos \theta + \mu \sin \theta) \qquad \text{(iii)}$$

Let $\lambda$ be the angle of friction.

Then,
$$\mu = \tan \lambda$$

Substituting in (iii),

$$\frac{\sin \lambda}{\cos \lambda} mg = F\left(\cos \theta + \frac{\sin \lambda}{\cos \lambda} \sin \theta\right)$$

$\Rightarrow$
$$F = \frac{mg \sin \lambda}{\cos (\theta - \lambda)}$$

For the force to be minimum,

$$\cos(\theta - \lambda) = 1$$

$\Rightarrow$

$$\theta = \lambda$$

Thus,

$$f_{\min} = mg \sin \lambda$$

**Ans. 5**

Let $v_1, v_2, v_3, \ldots, v_n$ be the speeds of the pendulum ball just before the first, second, third, . . ., $n$th collisions respectively. Then $v_2, v_3, v_4, \ldots$ will be the speeds of the pendulum ball just after first, second, third, . . . collisions respectively. For the first collision, we have

$$v_1 = \sqrt{2gL}$$

and

$$\frac{2}{\sqrt{5}} = \frac{v_2}{\sqrt{2gL}}$$

$\Rightarrow$

$$v^2 = \sqrt{\frac{8gL}{5}}$$

For the second collision,

$$\frac{2}{\sqrt{5}} = \frac{v_3}{\sqrt{\frac{8gL}{5}}}$$

$\Rightarrow$

$$v_3 = \sqrt{\frac{32gL}{25}}$$

For the third collision,

$$\frac{2}{\sqrt{5}} = \frac{v_4}{\sqrt{\frac{32gL}{25}}}$$

$\Rightarrow$

$$v_4 = \sqrt{\frac{128gL}{125}}$$

Similarly,

For the $n$th collision,

$$\frac{2}{\sqrt{5}} = \frac{v_{n+1}}{\left(\sqrt{\frac{4}{5}}\right)^{n-1}\sqrt{2gL}}$$

$\Rightarrow$

$$v_{n+1} = \left(\sqrt{\frac{4}{5}}\right)^{n}\sqrt{2gL} \qquad \text{(i)}$$

Let $n$ be the number of collisions after which the amplitude of oscillation becomes 60°. Then,

$$v_{n+1} = \sqrt{2g \cdot L \cos 60^\circ} = \sqrt{gL} \qquad \text{(ii)}$$

Comparing the two values of $v_{n+1}$ from (i) and (ii), and solving,

$$n = 3.1$$

Thus the minimum desired number of collisions is $(n + 1)$, i.e. $\approx 4$.

**Ans. 6**

Let the mass of the sphere be $m$ and the radius $r$. From the law of conservation of energy,

$$m\times 9.8\times 2.4 = m\times 9.8\times 1.0 + \frac{1}{2}mv_A^2 + \frac{1}{2}I\omega^2$$

or $$m\times 9.8\times 2.4 = m\times 9.8\times 1.0 + \frac{1}{2}mv_A^2 + \frac{1}{2}\times\frac{2}{5}mr^2\times\frac{v_A^2}{r^2}$$

or $$9.8\times 1.4 = \frac{1}{2}v_A^2 + \frac{1}{5}v_A^2$$

or $$v_A^2 = \frac{9.8\times 1.4\times 10}{7} = 19.6$$

or $$v_A = \sqrt{19.6}$$

Suppose the sphere falls from $A$ to the ground in time $t$. Then,

$$\frac{1}{2}\times 9.8\times t^2 = 1.0$$

or $$t = \sqrt{\frac{2}{9.8}} = 0.4518$$

Let $R$ be the distance on the ground from the point $B$ where the sphere lands. Then,

$$R = \sqrt{19.6}\times 0.4518 = 2.0 \text{ m}$$

Yes, during its slight as a projectile, it continues to rotate about its centre of mass because of the conservation of angular momentum.

**Ans. 7**

(i) The combination of transverse waves represented by the equations

$$z_1 = A\cos(kx - \omega t)$$

and $$z_2 = A\cos(kx + \omega t)$$

will produce standing waves.

The two equations represent two transverse waves of the same amplitude and frequency, but travelling in opposite directions. The resultant of these two waves is given by

$$Z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$$

$$= 2A\cos\frac{kx - \omega t + kx + \omega t}{2}\cdot\cos\frac{kx - \omega t - kx - \omega t}{2}$$

$$= 2A\cos kx\cdot\cos\omega t$$

The resultant intensity will be zero when

$$2A\cos kx = 0$$

$$\Rightarrow \quad \cos kx = 0$$

$$\Rightarrow \quad x = \frac{(2m+1)\pi}{2k}$$

where $$m = 0, 1, 2, 3, \ldots$$

(ii) The combination of transverse waves represented by the equations

$$z_1 = A \cos (kx - \omega t)$$

and $$z_3 = A \cos (ky - \omega t)$$

will produce a wave in the direction making an angle of 45° with the positive $x$ and $y$ axes. The resultant of these two waves is given by

$$Z = A \cos (kx - \omega t) + A \cos (ky - \omega t)$$

$$= 2A \cos \frac{k(x+y) - 2\omega t}{2} \cdot \cos \frac{k(x-y)}{2}$$

The resultant intensity will be zero when

$$2A \cos \frac{k(x-y)}{2} = 0$$

$$\Rightarrow \quad \cos \frac{k(x-y)}{2} = 0$$

$$\Rightarrow \quad k(x-y) = (2m+1)\pi$$

or $$(x-y) = \frac{(2m+1)\pi}{k}$$

where $$m = 0, 1, 2, 3, \ldots$$

**Ans. 8**

Let the number of moles of the ideal gas contained in the closed vessel be $n$. Then,

$$PV = nRT$$

Substituting the values, we have

$$1.6 \times 10^6 \times 0.0083 = n \times 8.3 \times 300$$

or $$n = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = 5.33$$

Also, $$C_p = \frac{5R}{2}$$

$$C_v = \frac{5R}{2} - R = \frac{3R}{2}$$

$$= \frac{3}{2} \times 8.3$$

$$= 12.45 \text{ J/mole K}$$

Heat energy supplied, $Q = nC_v\, dT$. Substituting the values, we have

$$2.49 \times 10^4 = 5.33 \times 2.45 \times dT$$

or $$dT = \frac{2.49 \times 10^4}{5.33 \times 12.45} = 375 \text{ K}$$

Final temperature is, therefore,

$$300 + 375 = 675 \text{ K}$$

Let $P_2$ be the final pressure. Then,

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$

or

$$P_2 = \frac{675 \times 1.6 \times 10^6}{300} = 3.6 \times 10^6 \text{ N/m}^2$$

**Ans. 9**

Let us place the point charges $q$, $2q$ and $8q$ on a 9 cm (0.09 m) long straight line as shown in Fig. 10.

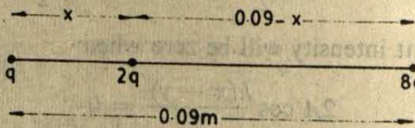



Fig. 10

Let $x$ be the distance between $q$ and $2q$. Thus, potential energy is given by

$$U = 9 \times 10^9 \left[\frac{q \cdot 2q}{x} + \frac{2q \cdot 8q}{(0.09 - x)} + \frac{q \cdot 8q}{0.09}\right]$$

$$= 18 \times 10^9 q^2 \left(\frac{1}{x} + \frac{8}{(0.09 - x)} + \frac{4}{0.09}\right)$$

For $U$ to be minimum,

$$\frac{dU}{dx} = 0$$

$$\Rightarrow \quad -\frac{1}{x^2} + \frac{8}{(0.09 - x)^2} = 0$$

$$\Rightarrow \quad x = \frac{0.09}{3.828} = 0.0235 \text{ m} = 2.35 \text{ cm}$$

Let $E_1$ be the electric field at the position of charge $q$ due to charge $2q$ and $E_2$ that due to charge $8q$. Then,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{x^2}$$

$$= 9 \times 10^9 \times \frac{2q}{(0.0235)^2}$$

Also,

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{8q}{(0.09)^2}$$

$$= 9 \times 10^9 \times \frac{8q}{(0.09)^2}$$

Thus, the electric field at $q$ due to the other two charges,

$$E = E_1 + E_2$$

or

$$E = 9 \times 10^9 \times \frac{2q}{(0.0235)^2} + 9 \times 10^9 \times \frac{8q}{(0.09)^2}$$

$$= 4.33 \times 10^{13} q$$

**Ans. 10**

(i) Let $R$ be the effective resistance between $A$ and $B$ as given in the diagram. Thus,

$$R = 1 + \frac{2R}{2 + R}$$

or $$2R + R^2 = 2 + R + 2R$$

or $$R^2 - R - 2 = 0$$

$\Rightarrow$ $$R = 2\Omega$$

(ii) Let $i$ be the current which passes through the 6-V battery. Then

$$i = \frac{6}{2} = 3 \text{ A}$$

$\because R = 2\Omega$, the current in the $2\Omega$ resistance and the resistance $R$ will be the same, i.e.

$$i' = \frac{3}{2} = 1.5 \text{ A}$$

**Ans. 11**

(i) Let $B_1$ be the magnetic field of induction at point $P$ due to current in wire $A$. Then,

$$B_1 = \frac{4\pi \times 10^{-7} \times 9.6}{2\pi \times \left(2 + \frac{10}{11}\right)}$$

$$= 6.6 \times 10^{-7} \text{ W/m}^2$$

The direction of this field is to the left as shown in Fig. 11. The magnetic field of induction at $P$ due to current in wire $B$ should be equal and opposite. Thus,

$$\frac{4\pi \times 10^{-7} \times i}{2\pi \times \frac{10}{11}} = 6.6 \times 10^{-7}$$

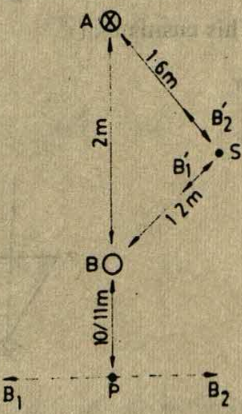


Fig. 11

which gives $\quad i = 3.0$ A

From the right-hand grip rule, it is evident that the direction of this current should be outside the plane of the paper.

(ii) Let $B_1'$ and $B_2'$ be the magnitudes of the magnetic fields of induction at point $S$ owing to currents in wires $A$ and $B$ respectively.
Then,

$$B_1' = \frac{4\pi \times 10^{-7} \times 9.6}{2\pi \times 1.6}$$

$$= 12 \times 10^{-7} \text{ Wb/m}^2 \text{ along } SB.$$

Also,

$$B_2' = \frac{4\pi \times 10^{-7} \times 3.0}{2\pi \times 1.2}$$

$$= 5 \times 10^{-7} \text{ Wb/m}^2 \text{ along } SA$$

The magnitude of the magnetic field of induction at the point $S$ is, therefore,

$$B = \sqrt{B_1'^2 + B_2'^2}$$

$$= \sqrt{(12 \times 10^{-7})^2 + (5 \times 10^{-7})^2} = 13 \times 10^{-7} \text{ Wb/m}^2$$

(iii) The force per unit length on the wire $B$ is given by

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times 9.6 \times 3.0}{2\pi \times 2}$$

$$= 28.8 \times 10^{-7} \text{ N/m directed towards } P.$$

**Ans. 12**

(i) If the incident ray at $AB$ emerges parallel to the incident direction, then

$$A > i_c \quad \text{and}$$

$$B > i_c$$

This means that $\quad A + B > 2i_c$

or $\quad i_c < \dfrac{90^\circ}{2}$

$\Rightarrow \quad i_c < 45^\circ$

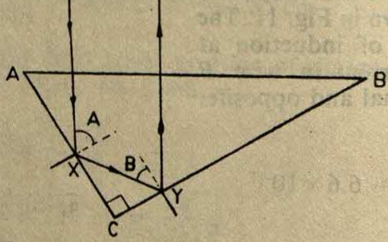



Fig. 12

The maximum value of the critical angle, $i_c$ should therefore be $45^\circ$. Thus,

$$n_{min} = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

(ii) When $n = \frac{5}{3}$, it follows that

$$\frac{5}{3} = \frac{1}{\sin i_c'}$$

$\Rightarrow \qquad \sin i'_c = 0.6$

or $\qquad i'_c = 37°$

The angle $B$ is 30° at $Y$, the angle of incidence at $X$ will be 60°. It shows that total reflection will take place at $X$ and not at $Y$. Thus, it is not possible to achieve the desired result in this case.

**Ans. 13**

It is given that

$$I = 0.125\text{ V} - 7.5$$

$$\Rightarrow \qquad dI = 0.125\,\Delta V$$

The plate resistance ($r_p$) is defined as

$$r_p = \left.\frac{\Delta V}{\Delta I}\right|_{V_g \text{ constant}}$$

$$\Rightarrow \qquad r_p = \frac{\Delta V}{0.125\,\Delta V \times 10^{-3}} = 8\times 10^3\,\Omega$$

At $V = 300$ V and $V_g = -1$ V, the anode current is given by

$$I = 0.125\times 300 \; -7.5 = 30.0 \text{ mA}$$

The transconductance ($g_m$) is defined as

$$g_m = \left.\frac{\Delta I}{\Delta V_g}\right|_{V \text{ constant}}$$

$$\Rightarrow \qquad g_m = \frac{(30 - 5)\times 10^{-3}}{-3 - (-1)} = 12.5\times 10^{-3}\,\mho$$

Amplification factor ($\mu$) is given by

$$\mu = r_p \times g_m = 8\times 10^3 \times 12.5\times 10^{-3} = 100$$

# CHEMISTRY

## PART A

**Ans. 1**

(i) (*A*) Nitrogen has more stable, fully half-filled, sub-orbitals.

(ii) (A) Electronic configurations of fluorine and neon are 2, 7 and 2, 8, respectively.

(iii) (A) Molecular association takes place due to the formation of $K_2HgI_4$.

(iv) (A) *At cathode*

$$2Na^+ + 2e^- \rightarrow 2Na$$

$$2Na + 2H_2O \rightarrow 2NaOH + H_2$$

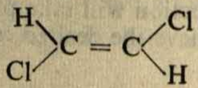
*At anode*

$$2SO_4^{2-} + 2H_2O \rightarrow 2H_2SO_4 + O_2 + 2e^-$$

(v) (A) Fe is in +1 oxidation state. Its +1 oxidation state and +1 of NO are neutralized by $SO_4^{2-}$.

(vi) (C) Electronegativity increases from left to right across a period and decreases on descending a group.

(vii) (C)



(viii) (C) Carbon bonded with two *pi* bonds is *sp* hybrid and with one *pi* bond is $sp^2$ hybrid.

(ix) (B)

$$\overset{1}{C}H_2 = \overset{2}{C}H - \underset{\displaystyle CH_3}{\overset{3}{C}H} - \overset{4}{C}H_3$$

(x) (A)

(xi) (B) Molecular formula being a multiple of empirical formula, it must have different molecular weight.

(xii) (D)

(xiii) (B)

(xiv) (D) The overall reaction taking place on discharging a lead storage battery is,

$$PbO_2 + Pb + 2H_2SO_4 \rightarrow 2PbSO_4 + 2H_2O$$

(xv) (D)

(xvi) (B)
$$CH_3 - CH_2 - CH_2Br \xrightarrow[-HBr]{\text{alc. KOH}} CH_3 - CH = CH_2$$

(xvii) (*A*) The lone pair on nitrogen is least available due to the highest electronegativity of fluorine.

(xviii) (A) In ethanol, H is bonded to highly electronegative O.

(xix) (C)
$$\overset{1}{C}H_3 - \underset{\displaystyle OH}{\overset{2}{C}H} - \overset{3}{C}H_2 - \overset{4}{C}H_3 \xrightarrow{(O)} CH_3 - \underset{\displaystyle \overset{\|}{O}}{C} - CH_2 - CH_3$$

(xx) (B) Oxidizing chlorine water will liberate bromine from KBr.

$$Cl_2 + H_2O \rightarrow 2HCl + (O)$$
$$2Br^- + 2H^+ + (O) \rightarrow Br_2 + H_2O$$

**Ans. 2**

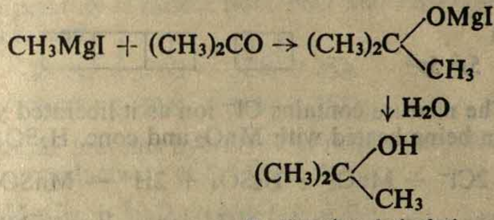
(i) sodium hydroxide
(ii) increased
(iii) hydration
(iv) 0.25
(v) White

**Ans. 3**

(i) *False*: In benzene, carbon uses only two *p*-orbitals for hybridisation.
(ii) *False*: Sodium when burnt in excess of oxygen gives sodium peroxide.
(iii) *False*: Catalyst does not make a reaction more exothermic.
(iv) *True*: Graphite is a better lubricant on the moon than on the earth. It is due to lack of gravity on moon.

(v) *False*: $sp^2$ hybrid orbitals do not have equal *s* and *p* character.

(vi) *False*:



Tertiary butyl alcohol

(vii) *False*: $Zn^{2+}$ is diamagnetic (no unpaired electron)

(viii) *True*

(ix) *True*

(x) *True*

**Ans. 4**

(i) $$P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$$

(ii) $$4Sn + \underset{(\text{dil.})}{10HNO_3} \rightarrow 4Sn(NO_3)_2 + 5H_2O + N_2O$$

(iii) $$2KMnO_4 + 3H_2SO_4 + 5H_2C_2O_4 \rightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$$

(iv) $$Au + 3HNO_3 + 4HCl \rightarrow HAuCl_4 + 3NO_2 + 3H_2O$$

(v) $$CH_3CONH_2 + 4KOH + Br_2 \rightarrow CH_3NH_2 + K_2CO_3 + 2KBr + 2H_2O$$

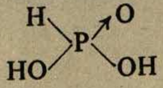
**Ans. 5**

(*a*) (i) MgO is used for the lining of steel making furnace as it facilitates the removal of impurities of silicon, phosphorus and sulphur through slag formation.

(ii) The mixture of $N_2H_4$ and $H_2O_2$ with a Cu(II) catalyst is used as a rocket propellant because the reaction is highly exothermic and is accompanied by a large increase in volume.

$$N_2H_4(l) + 2H_2O_2(l) \rightarrow N_2(g) + 4H_2O(g)$$

(iii) $H_3PO_3$ is not a tribasic acid as only two hydrogens are bonded to two oxygens.



It is a dibasic acid and dissociates according to the following equations:

$$H_3PO_3 \rightleftharpoons H^+ + H_2PO_3^-$$

$$H_2PO_3^- \rightleftharpoons H^+ + HPO_3^{2-}$$

(iv) Phenol is a very weak acid (weaker than carboxylic acid or even carbonic acid) and, therefore, does not react with $NaHCO_3$.

(v) The molecule of $MgCl_2$ is linear because valence orbital of Mg is *sp* hybridised

Mg $2s^2$ 

$SnCl_2$ is angular because valence orbital of Sn is $sp^2$ hybridised

Sn
$5s^2\ 5p^2$ $\quad \boxed{\uparrow\downarrow} \quad \boxed{\uparrow}\boxed{\uparrow}\boxed{\phantom{\uparrow}}$

(*b*) (i) The mixture contains $Cl^-$ ion as it liberated yellowish green gas on being heated with $MnO_2$ and conc. $H_2SO_4$.

$$2Cl^- + MnO_2 + H_2SO_4 + 2H^+ \rightarrow MnSO_4 + 2H_2O + Cl_2$$

(ii) The mixture contains $NH_4^+$ ion as it gave $NH_3$ on being heated with NaOH solution; the gas turned red litmus blue.

$$NH_4^+ + OH^- \rightarrow NH_3 + H_2O$$

(iii) The mixture contains $Fe^{2+}$ ion as its solution gave blue precipitate with $K_3Fe(CN)_6$

$$3Fe^{2+} + 2[Fe(CN)_6]^{3-} \rightarrow \underset{\text{(blue ppt.)}}{Fe_3[Fe(CN)_6]_2}$$

Red coloration with $NH_4SCN$ indicates that it contains $Fe^{3+}$ ion.

$$Fe^{3+} + 3SCN^- \rightarrow \underset{\text{(red coloration)}}{Fe(SCN)_3}$$

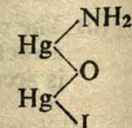
$Fe^{2+}$ present in the mixture gets oxidized to $Fe^{3+}$ by air.

$$2Fe^{2+} + 2H^+ + (O) \rightarrow 2Fe^{3+} + H_2O$$

(iv) The mixture on boiling with KOH liberated $NH_3$ which gave a brown precipitate on bubbling through an alkaline solution of $K_2HgI_4$.

$$2K_2HgI_4 + 4NH_3 + H_2O$$

$$\rightarrow 4KI + 3NH_4I +$$



(brown ppt.)

The two salts in the mixture are: $FeCl_2$ and $NH_4Cl$.

## PART B

**Ans. 6**

(*a*) (i) The actual reducing agent of haematite in blast furnace is CO.

$$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$$

(ii) Galena ore (PbS) is roasted at moderate temperature in excess of air on the hearth of a reverberatory furnace. The ore is oxidized partly to oxide and partly to sulphate.

$$2PbS + 3O_2 \rightarrow 2PbO + 2SO_2$$

$$PbS + 2O_2 \rightarrow PbSO_4$$

Now, the air supply is stopped and more of galena is added and the temperature is raised. Both PbO and $PbSO_4$ get converted to metallic lead.

$$2PbO + PbS \rightarrow 3Pb + SO_2$$

$$PbSO_4 + PbS \rightarrow 2Pb + 2SO_2$$

(ii) Sodium chloride prevents hydrolysis of magnesium chloride and increases the conductivity of the fused mass

$$MgCl_2 + H_2O \rightarrow MgO + 2HCl$$

(iv) Zinc is more reducing than copper and is also cheaper. This is why Zn and not Cu is used for the recovery of metallic Ag from complex $[Ag(CN)_2]^-$.

$$2[Ag(CN)_2]^- + Zn \rightarrow 2Ag + [Zn(CN)_4]^{2-}$$

(v) Chalcocite ($Cu_2S$) is roasted and not calcined because free supply of air is needed to convert into its oxide.

$$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$$

$$Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$$

In calcination, heating is done without free supply of air as in case of carbonates.

$$MCO_3 \xrightarrow{\text{calcination}} MO + CO_2$$

when M is an alkaline earth metal

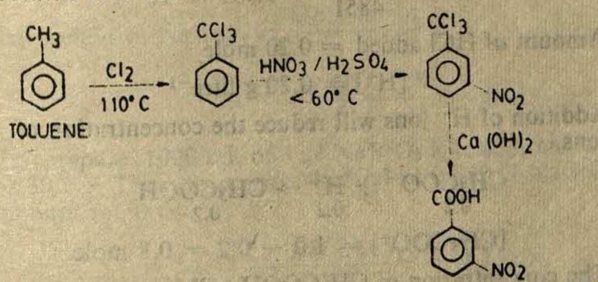
(*b*) (i) *Benzene from lime*

$$\underset{\text{lime}}{CaO} + \underset{\text{coke}}{3C} \xrightarrow[2000^\circ C]{\text{electric furnace}} CaC_2 + CO$$

$$CaC_2 + 2H_2O \rightarrow Ca(OH)_2 + \underset{\text{acetylene}}{C_2H_2}$$

$$3C_2H_2 \xrightarrow{450\text{-}500^\circ C} \underset{\text{benzene}}{C_6H_6}$$

(ii) *Toluene to m-nitrobenzoic acid*

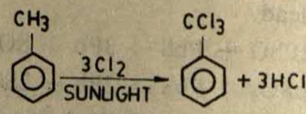



m-NITROBENZOIC ACID

(iii) Tollen's reagent (ammoniacal silver nitrate solution) may be used to distinguish acetaldehyde from acetone. Acetaldehyde reduces Tollen's reagent to silver mirror while acetone does not.

$$\underset{\substack{\text{(from}\\\text{Tollen's}\\\text{reagent)}}}{Ag_2O} + CH_3CHO \rightarrow CH_3COOH + \underset{\substack{\text{silver}\\\text{mirror}}}{2Ag}$$

(iv) Benzotrichloride is obtained when chlorine is passed through boiling toluene in presence of sunlight.




(v) Dipole moment is the product of charge and the distance between the two opposite poles ($\mu = e \times d$). Due to resonance, distance is increased and hence the dipole moment will be increased.

**Ans. 7**

(*a*) Volume of balloon $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 4851 \text{ ml}$$

Total volume of hydrogen in the cylinder at N.T.P.

$$\frac{P_0 V_0}{T_0} = \frac{P_1 V_1}{T_1}$$

$P_0 = 1$ atm $\qquad P_1 = 20$ atm

$V_0 = ?$ $\qquad V_1 = 2.82$ litres

$T_0 = 273$ $\qquad T_1 = 273 + 27 = 300$ K

$$V_0 = \frac{20 \times 2.82 \times 273}{300} = 51.324 \text{ l}$$

$$= 51324 \text{ ml}$$

Actual volume to be transferred into balloons

$$= 51324 - 2820 = 48504 \text{ ml}$$

No. of balloons that can be filled up

$$= \frac{48504}{4851} = 9.999 \approx 10$$

(*b*) (i) Amount of HCl added = 0.20 mole

$$[H^+] = 0.20 \text{ g litre}^{-1}$$

Addition of $H^+$ ions will reduce the concentration of $CH_3COO$ ions.

$$\underset{0.2}{CH_3COO^-} + \underset{0.2}{H^+} \rightarrow \underset{0.2}{CH_3COOH}$$

$$[CH_3COO^-] = 1.0 - 0.2 = 0.8 \text{ mole}$$

The concentration of $CH_3COOH$ will increase.

$$[CH_3COOH] = 1.0 + 0.2 = 1.2 \text{ mole}$$

Now, $\quad pH = -\log K_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$

$$= -\log 1.8 \times 10^{-5} + \log \frac{0.8}{1.2}$$

$$= 4.7447 + 0.3010 - 0.4771$$
$$= 4.5686$$

(*b*) (ii) Amount of HCl added = 0.20 mole
Out of 0.2 mole of $[H^+]$ added, 0.1 mole will combine with 0.1 mole of $CH_3COO^-$ to form 0.1 mole of $CH_3COOH$

$$\underset{0.1}{CH_3COO^-} + \underset{0.1}{H^+} \rightarrow \underset{0.1}{CH_3COOH}$$

Therefore, $[CH_3 \cdot COOH]$ total = 0.1 + 0.1 = 0.2 mole

In presence of $[H^+]$, $CH_3COOH$ will not ionize. Therefore, pH of the solution will be due to the presence of $H^+$ of HCl, i.e.

$$0.2 - 0.1 = 0.1 \text{ mole HCl}$$
$$pH = -\log [H^+] = -\log [0.1]$$
$$= 1$$

**Ans. 8**

(*a*) (i) From the half-cell reaction,

$$\text{Eq. weight of } NaBrO_3 = \frac{\text{Mol. weight}}{6}$$
$$= \frac{23 + 80 + 48}{6} = 25.17$$

To prepare 1000 ml of 1 N $NaBrO_3$ solution, amount required = 25.17g. Thus, to prepare 85.5 ml of 0.672 N $NaBrO_3$ solution, amount required

$$= \frac{25.17 \times 0.672 \times 85.5}{1000} = 1.446 \text{ g}$$

$$\text{Molarity} = \frac{0.672}{6} = 0.112 \text{ M}$$

(ii) From the half-cell reaction,

$$\text{Eq. weight of } NaBrO_3 = \frac{\text{Mol. weight}}{5}$$
$$= \frac{23 + 80 + 48}{5} = 30.2 \text{ g}$$

To prepare 1000 ml of 1 N $NaBrO_3$ solution, amount required = 30.2 g. Thus, to prepare 85.5 ml of 0.672 N $NaBrO_3$ solution, amount required

$$= \frac{30.2 \times 0.672 \times 85.5}{1000} = 1.7352 \text{ g}$$

$$\text{Molarity} = \frac{0.672}{5} = 0.1344 \text{ M}$$

(*b*) Initial concentration of each gas

$$= 1 \text{ mole}$$

Let $x$ moles of $NO_2$ react at equilibrium. Then,

$$\begin{array}{cccc} SO_2(g) + & NO_2(g) \rightleftharpoons & SO_3(g) + & NO(g) \\ (1-x) & (1-x) & (1+x) & (1+x) \end{array}$$

Now, $$\frac{[SO_3][NO]}{[SO_2][NO_2]} = K_e = 16$$

or $$\frac{(1+x)(1+x)}{(1-x)(1-x)} = 16$$

or $$\frac{(1+x)^2}{(1-x)^2} = 16 = (4)^2$$

or $$\frac{1+x}{1-x} = 4$$

or $$1 + x = 4 - 4x$$

or $$5x = 3$$

or $$x = \frac{3}{5} = 0.6$$

Thus, equilibrium concentration of NO

$$= 1 + x = 1 + 0.6 = 1.6 \text{ moles}$$

Equilibrium concentration of $NO_2 = 1 - x$

$$= 1 - 0.6 = 0.4 \text{ mole}$$

**Ans. 9**

(*a*) Watt = Volt × Current

100 = 110 × Current

$\therefore$ $$\text{Current, } i = \frac{100}{110} = \frac{10}{11} \text{ amp}$$

Quantity of electricity $= i \times t$

$$= \frac{10}{11} \times 10 \times 3600 \text{ coulombs}$$

No. of equivalents $= \frac{10}{11} \times 10 \times 3600 \times \frac{1}{96500} = 0.339$

$\therefore$ Weight of Cd deposited $= \frac{0.339 \times 112.4}{2} = 19.06$ g

(*b*) $$k = \frac{2.303}{t} \log \frac{1}{(1-x)}$$

Substituting the values,

$$1.5 \times 10^{-6} = \frac{2.303}{10 \times 3600} \log \frac{1}{(1-x)}$$

or $$\frac{1.5 \times 10^{-6} \times 10 \times 3600}{2.303} = \log \frac{1}{(1-x)}$$

or $$0.0234 = \log \frac{1}{(1-x)}$$

Taking antilog,

$$1.055 = \frac{1}{(1-x)}$$

or $\quad 1.055 - 1.055x = 1$

or $$x = \frac{(1.055 - 1)}{1.055} = 0.052$$

Thus, 5.2% of the initial concentration has changed into product.

Also, $$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{1.5 \times 10^{-6} s^{-1}}$$

$$= 462000 \text{ s}$$
$$= 128.33 \text{ h}$$

**Ans. 10**

(*a*) *Emp. formula*

$$C; \frac{69.77}{12} = 5.81 = 5$$

$$H; \frac{11.63}{16} = 11.63 = 1$$

$$O; \frac{18.60}{16} = 1.16 = 1$$

Emp. formula is $C_5H_{10}O$

Mol. weight is 86.

∴ Mol. formula is also $C_5H_{10}O$

Since it does not reduce Fehling's solution, —CHO (aldehyde) group is absent. It forms an addition compound with $NaHSO_3$, a keto ($>C=O$) group is present. Since it gives iodoform test, the possible formulae will be

$$CH_3-\underset{\underset{O}{\|}}{C}-CH_2-CH_2-CH_3$$

Pertanone-2

$$CH_3-\underset{\underset{O}{\|}}{C}-\underset{\underset{CH_3}{|}}{CH}-CH_3$$

3-Methylbutanone-2

(*b*) An organic compound (A) is an alcohol which on treatment with $CH_3COOH$ in presence of $H_2SO_4$ gives an ester (B).

$$\underset{(A)}{CH_3OH} + CH_3COOH \xrightarrow[-H_2O]{H_2SO_3} \underset{(B)}{CH_3COOCH_3}$$

(A) on mild oxidation gives (C). (C) with 50% KOH followed by acidification generates (A) and (D). The reactions can be written as:

$$\underset{(A)}{CH_3OH} \xrightarrow{\text{mild oxidation}} \underset{(C)}{HCHO}$$

$$\underset{(C)}{2HCHO} \xrightarrow[\text{ii. dil. HCl}]{\text{i. 50\% KOH}} \underset{(A)}{CH_3OH} + \underset{(D)}{HCOOH}$$

(D) with $PCl_5$ followed by reaction with $NH_3$ gives (E).

$$\underset{(D)}{HCOOH} \xrightarrow{PCl_5} HCOCl \xrightarrow{NH_3} \underset{(E)}{HCONH_2}$$

Dehydration of (*E*) produces HCN.

$$\underset{(E)}{HCONH_2} \xrightarrow[-H_2O]{} HCN$$

Thus, compound A is $CH_3OH$
compound B is $CH_3COOCH_3$
compound C is HCHO
compound D is HCOOH
compound E is $HCONH_2$

# MATHEMATICS

## PART A

**Ans. 1**

(i) (B) $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leqslant 0$

$\Rightarrow$ $(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leqslant 0$

$\Rightarrow$ $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leqslant 0$ (1)

As *a, b, c, d* and *p* are real numbers,

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \geqslant 0 \quad (2)$$

From (1) and (2), we get

$$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$$

$\Rightarrow$ $ap - b = 0,\ bp - c = 0,\ cp - d = 0$

$\Rightarrow$ $b/a = c/b = d/c = p$

Thus, *a, b, c* and *d* are in G.P.

(ii) A, D. Let the angles of the triangle be $\alpha + \beta, \alpha, \alpha - \beta$ where $\beta > 0$. Then $3\alpha = \pi$ which implies $\alpha = \pi/3$. By the sine formula

$$\frac{\sin(\alpha + \beta)}{10} = \frac{\sin\alpha}{9} = \frac{\sin(\alpha - \beta)}{x} \quad (1)$$

where $x$ is the third side of the triangle.
From (1), we get

$$\sin(\alpha + \beta) = 10 \sin\alpha/9 = 5\sqrt{3}/9$$

Note that $0 < \beta < \pi/3$.
We have,

$$\cos(\alpha + \beta) = \pm\sqrt{1 - \sin^2(\alpha + \beta)} = \pm\sqrt{1 - 75/81}$$
$$= \pm\sqrt{6}/9.$$

Also

$$\sin(\alpha-\beta) = \sin[2\alpha-(\alpha+\beta)]$$
$$= \sin[120^\circ-(\alpha+\beta)]$$
$$= \sin 120^\circ \cos(\alpha+\beta) - \cos 120^\circ \sin(\alpha+\beta)$$
$$\Rightarrow \sin(90^\circ+30^\circ)\cos(\alpha+\beta)$$
$$-\cos(90^\circ+30^\circ)\sin\alpha+\beta)$$
$$= \cos 30^\circ \cos(\alpha+\beta) + \sin 30^\circ \sin(\alpha+\beta)$$
$$= \frac{\sqrt{3}}{2}\left(\pm\frac{\sqrt{6}}{9}\right) + \frac{1}{2}\left(\frac{5\sqrt{3}}{9}\right)$$
$$= (5\sqrt{3} \pm \sqrt{18})/18 = (5 \pm \sqrt{6})\ \sqrt{3}/18$$

From (1),

$$x = \frac{9\sin(\alpha-\beta)}{\sin\alpha} = \frac{9(5\pm\sqrt{6})\sqrt{3}/18}{\sqrt{3}/2}$$
$$= 5 \pm \sqrt{6}.$$

**Alternative solution**

Let the angles be $\alpha+\beta$, $\alpha$, $\alpha-\beta (\beta > 0)$.

Thus, $\quad (\alpha+\beta)+\alpha+(\alpha-\beta) = \pi$

$\Rightarrow \quad \alpha = \pi/3$

Let the third side be $x$.
By the cosine formula,

$$\cos(\pi/3) = \frac{a^2+c^2-b^2}{2ac}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{10^2+x^2-9^2}{(2x)(10)}$$

$$\Rightarrow \quad 10x = x^2+19$$

$$\Leftarrow \quad x^2-10x+19 = 0$$

$$\Rightarrow \quad x = \frac{1}{2}\left[10 \pm \sqrt{100-(4\times 19)}\right]$$
$$= \frac{1}{2}(10 \pm \sqrt{24}) = 5 \pm \sqrt{6}$$

(iii) (C) The least positive root of

$$f(x) = \tan x - x = 0$$

can lie in $(0, \pi/2)$ or $(\pi, 3\pi/2)$

For $0 < x < \pi/2$, we have $f'(x) = \sec^2 x - 1 > 0$, thus $f$ is monotonically increasing in $[0, \pi/2)$. As $f(0) = 0$, we get $f(x) > 0$ for $0 < x < \pi/2$. Thus, $f(x) = 0$ does not have a root in $(0, \pi/2)$.

We have $\quad f(\pi) = \tan\pi - \pi = 0 - \pi < 0,$

and $\quad f(3\pi/2 - t) = \tan(3\pi/2 - t) - (3\pi/2 - t) > 0$

where $\quad t = 0.0001.$

Since $f(x)$ is continuous on $[\pi, 3\pi/2 - t]$ and $f(\pi) < 0$, $f(3\pi/2 - t) > 0$, $f$ vanishes at least once in $(\pi, 3\pi/2 - t) \leqslant (\pi, 3\pi/2)$.

*Remark.* This can also be proved by using graphs.

(iv) (C) We have $P(A + B) = 0.6$ and $P(AB) = 0.2$.
We know that $P(A) + P(B) = P(A + B) + P(AB) = 0.8$. Thus, $P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B) = 2 - 0.8 = 1.2$.

(v) (C) Let $Z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$
and $Z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$
where $r_1 = |Z_1|$, $r_2 = |Z_2|$, $\theta_1 = \text{Arg}\cdot Z_1$ and $\theta_2 = \text{Arg}\cdot Z_2$.
We have
$$Z_1 + Z_2 = (r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)$$
Therefore,
$$|Z_1 + Z_2|^2 = r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) \tag{1}$$
Also,
$$|Z_1| + |Z_2| = r_1 + r_2$$
$$\Rightarrow \quad (|Z_1| + |Z_2|)^2 = r_1^2 + r_2^2 + 2r_1r_2 \tag{2}$$
Since $|Z_1 + Z_2| = |Z_1| + |Z_2|$, from (1) and (2), we get
$$\cos(\theta_1 - \theta_2) = 1 \quad \text{or} \quad \theta_1 - \theta_2 = 0.$$
$$\therefore \quad \text{Arg. } Z_1 - \text{Arg. } Z_2 = 0.$$

(vi) (A) Let $g(x) = x \quad x \in (-\infty, \infty)$
and $h(x) = 1 + |x| \quad x \in (-\infty, \infty)$
Then $g$ is differentiable on $(-\infty, \infty)$ and $h$ is differentiable on $(-\infty, 0) \cup (0, \infty)$. Also $h(x) \neq 0$ for each $x \in (-\infty, \infty)$.
By the quotient rule,
$$f(x) = \frac{g(x)}{h(x)} = \frac{x}{1 + |x|}$$
is differentiable on $(-\infty, 0) \cup (0, \infty)$.
For $x = 0$, we have
$$\lim_{h\to 0} \frac{f(h) - f(o)}{h - 0} = \lim_{h\to 0} \frac{\dfrac{h}{1 + |h|} - 0}{h - 0}$$
$$= \lim_{h\to 0} \frac{1}{1 + |h|} = \frac{1}{1 + 0} = 1$$
$$(\because \quad \lim_{x\to 0} |x| = 0)$$
Thus, $f$ is differentiable at $x = 0$. Hence, $f$ is differentiable on $(-\infty, \infty)$.

(viii) (D) Let $Z = \cos\dfrac{2\pi}{7} + i\sin\dfrac{2\pi}{7}$

then for $1 \leqslant k \leqslant 7$, $Z^k = \cos\dfrac{2\pi k}{7} + i\sin\dfrac{2\pi k}{7}$

(By De-Moivre's Theorem).

Thus, $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$$= (-i)\sum_{k=1}^{6}\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$$

$$= (-i)\sum_{k=1}^{6} Z^k = (-i)\frac{Z(1-Z^6)}{1-Z}$$

$$= (-i)\frac{Z-Z^7}{1-Z}$$

But $\quad Z^7 = \cos 2\pi + i\sin 2\pi = 1 - i0 = 1$

Thus, $\quad \sum_{k\times 1}^{7}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right) = (-i)\frac{Z-1}{1-Z} = i$

(viii) (B). We have $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

$$= \hat{i} - \hat{j} + \hat{k}$$

$\therefore \quad |\vec{a}\times\vec{b}| = \sqrt{1+1+1} = \sqrt{3}.$

Two unit vectors which are perpendicular to both $\vec{a}$ and $\vec{b}$ are

$$\frac{\pm(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|} = \pm\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

(ix) (A). Let $x \geqslant y \geqslant 0$. As $g$ is decreasing, $g(x) \leqslant g(y)$. Also $g(x), g(y) \in [0, \infty)$.
Since $f$ is increasing from $[0, \infty)$ to $[0, \infty)$

$$f(g(x)) \leqslant f(g(y)) \quad \text{or} \quad h(x) \leqslant h(y).$$

Therefore, $h$ is a decreasing function from $[0, \infty)$ to $[0, \infty)$.

Thus, if $x \geqslant 0$, we have $h(x) \leqslant h(0)$. But $h(0) = 0$. Therefore, $h(x) \leqslant 0$. Also $h(x) \geqslant 0$ for all $x \geqslant 0$. It follows from here that $h(x) = 0$ for all $x \in [0, \infty)$. Hence $h(x) - h(1) = 0 - 0 = 0$ for all $x \geqslant 0$.

(x) (D). Putting $x = 0$ and $\pi/2$, we get

$$a_1 + a_2\cos(0) + a_3\sin^2 0 = 0$$

or $\quad a_1 + a_2 = 0$

and $\quad a_1 + a_2\cos(\pi) + a_3\sin^2(\pi/2) = 0$

or $\quad a_1 - a_2 + a_3 = 0$

Thus $\quad a_2 = -a_1; \quad a_3 = a_2 - a_1 = -2a_1.$

The given equation now becomes,

$$a_1 - a_1\cos(2x) - 2a_1\sin^2(x) = 0$$

or $\quad a_1[1 - \cos 2x - 2\sin^2 x] = 0$

This relation is valid for all values of $x$ irrespective of the value of $a_1$. Hence, an infinite number of such triplets is possible.

**Ans. 2.**

(i)
$$0 \leqslant x \leqslant \pi/6,\ 5\pi/6 \leqslant x \leqslant \pi \text{ and } x = \pi/2.$$

$$2\sin^2 x - 3\sin x + 1 \geqslant 0$$
$$\Rightarrow \quad (2\sin x - 1)(\sin x - 1) \geqslant 0$$
$$\Rightarrow \quad (\sin x - 1/2)(\sin x - 1) \geqslant 0$$
$$\Rightarrow \quad \sin x \geqslant 1 \quad \text{or} \quad \sin x \leqslant 1/2$$
$$\Rightarrow \quad \sin x = 1 \quad \text{or} \quad \sin x \leqslant 1/2$$

Since $x \in [0, \pi]$, $\sin x \geqslant 0$. Therefore, $\sin x \leqslant 1/2$ means that $0 \leqslant \sin x \leqslant 1/2$. The implies that $0 \leqslant x \leqslant \pi/6$ or $5\pi/6 \leqslant x \leqslant \pi$. Also, $\sin x = 1$ implies that $x = \pi/2$.

(ii)
$$\frac{192}{25} \text{ sq. units.}$$

Let $P$ be the point (4, 3). The equation of chord of contact of point $P(4, 3)$ with respect to circle

$$x^2 + y^2 = 9 \text{ is}$$
$$4x + 3y = 9$$

(equation of line $AB$).

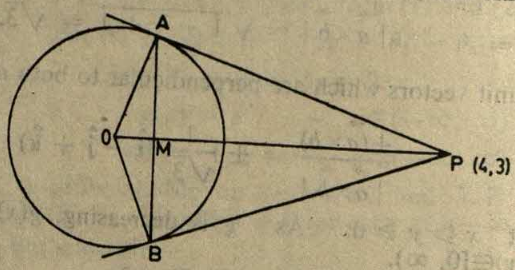



Fig. 13

We have

$$OM = \left|\frac{4(0) + 3(0) - 9}{\sqrt{4^2 + 3^2}}\right| = \frac{9}{5}$$
$$OP = \sqrt{4^2 + 3^2} = 5$$

Thus
$$PM = OP - OM = 16/5$$

We also have

$$AB = 2AM = 2\sqrt{OA^2 - OM^2}$$
$$= 2\sqrt{9 - 81/25}$$
$$= 2(12/5) = 24/5$$

Area of
$$\triangle APB = \frac{1}{2}(AB)(PM)$$
$$= \frac{1}{2}\left(\frac{24}{5}\right)\left(\frac{16}{5}\right)$$
$$= \frac{192}{25} \text{ sq. units.}$$

(iii) Cosec $(\pi/9)$.

We know that $$R = \frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$$

where $R$ is the radius of the circle, $a$ the length of the side of the polygon and $n$ the number of sides of the polygon.

$$\therefore \qquad R = \frac{2}{2}\operatorname{cosec}\left(\frac{\pi}{9}\right) = \operatorname{cosec}(\pi/9)$$

(iv) 1.

Since $a\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + b\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + c\hat{\mathbf{k}}$ are coplanar, there exist scalars $x$, $y$ and $z$ (not all zero) such that

$$x(a\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + y(\hat{\mathbf{i}} + b\hat{\mathbf{j}} + \hat{\mathbf{k}}) + z(\hat{\mathbf{i}} + \hat{\mathbf{j}} + c\hat{\mathbf{k}}) = \vec{0}$$

$$\Rightarrow \quad (ax + y + z)\hat{\mathbf{i}} + (x + by + z)\hat{\mathbf{j}} + (x + y + cz)\hat{\mathbf{k}} = \vec{0}$$

$$\Rightarrow \quad \begin{aligned} ax + y + z &= 0 \\ x + by + z &= 0 \\ x + y + cz &= 0 \end{aligned}$$

Since at least one of $x$, $y$ and $z$ is different from zero

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow \quad abc + 2 - a - b - c = 0 \qquad (1)$$

Now,

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$

$$= \frac{(1-b)(1-c) + (1-c)(1-a) + (1-a)(1-b)}{(1-a)(1-b)(1-c)}$$

$$= \frac{3 - 2(a+b+c) + bc + ca + ab}{1 - (a+b+c) + (bc+ca+ab) - abc}$$

$$= \frac{3 - 2(a+b+c) + bc + ca + ab}{1 - (a+b+c) + bc + ca + ab - (a+b+c-2)}$$

$$= 1.$$

(v) $-1$.

$$L = \lim_{x\to-\infty}\left[\frac{(x^4 \sin(1/x) + x^2)}{(1 + |x|^3)}\right]$$

$$= \lim_{x\to-\infty} \frac{x^3}{1 + |x|^3}\left[x \sin\left(\frac{1}{x}\right) + \frac{1}{x}\right]$$

$$= \lim_{x\to-\infty} \frac{x^3}{|x|^3}\left[\frac{1}{1/|x|^3 + 1}\right]\left[x \sin\left(\frac{1}{x}\right) + \frac{1}{x}\right]$$

We have, $\quad x^3/|x|^3 = -1 \quad$ for $x < 0$,

$$\lim_{x\to-\infty} \frac{1}{|x|^3} = 0$$

and $$\lim_{x \to -\infty} \left[x \sin\left(\frac{1}{x}\right)\right] = \lim_{y \to 0-} \frac{\sin(y)}{y} = 1$$

$$\therefore \quad L = (-1)\left[\frac{1}{0+1}\right][1 + 0] = -1.$$

(vi) $-\sqrt{3}/2$.

We have $$\alpha + \beta + \gamma = 360°$$

$$A.M. = \frac{1}{3}\left[\cos\left(\alpha + \frac{\pi}{2}\right) + \cos\left(\beta + \frac{\pi}{2}\right) + \cos\left(\gamma + \frac{\pi}{2}\right)\right]$$

$$= -\frac{1}{3}[\sin\alpha + \sin\beta + \sin\gamma]$$

$$= -\frac{1}{3}\left[2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}\right]$$

$$= -\frac{1}{3}\left[2\sin\left(180° - \frac{\gamma}{2}\right)\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}\right]$$

$$= -\frac{1}{3}\left[2\sin\frac{\gamma}{2}\cos\frac{\alpha-\beta}{2} + 2\sin\frac{\gamma}{2}\cos\left(180° - \frac{\alpha+\beta}{2}\right)\right]$$

$$= -\frac{2}{3}\sin\frac{\gamma}{2}\left[\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}\right]$$

$$= -\frac{2}{3}\sin\frac{\gamma}{2}\, 2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}$$

$$= -\frac{4}{3}\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\sin\left(\frac{\gamma}{2}\right)$$

$A.M.$ will be the least if $\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\sin\left(\frac{\gamma}{2}\right)$ is the greatest.

But $\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\sin\left(\frac{\gamma}{2}\right)$ is greatest when

$$\sin\left(\frac{\alpha}{2}\right) = \sin\left(\frac{\beta}{2}\right) = \sin\left(\frac{\gamma}{2}\right)$$

that is, if $$\alpha = \beta = \gamma = 120°.$$

Thus,

$$A.M. = -\frac{4}{3}\sin^3 60° = -\frac{4}{3}\cdot\frac{3\sqrt{3}}{8} = -\frac{\sqrt{3}}{2}.$$

(vii) $x \geqslant 0$.

Let $$f(x) = x - \log(1 + x) \quad \text{for } x > -1.$$

$$f'(x) = 1 - \frac{1}{1+x} \quad (x > -1)$$

$$= \frac{x}{1+x}$$

Note that $f'(x) < 0$ if $-1 < x < 0$ and

$$f'(x) > 0 \quad \text{for} \quad x > 0.$$

Therefore, $f$ decreases in $(-1, 0]$ and increases in $[0, \infty)$

Thus, if $x \geqslant 0$, we have $f(x) \geqslant f(0) = 0$ or $x \geqslant \log(1 + x)$.

(viii) $2\hat{i} - \hat{j}$

Let $\vec{c} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. Since $\vec{b}$ and $\vec{c}$ are perpendicular to each other, we have

$$\vec{b}\cdot\vec{c} = 0 \quad \text{or} \quad 4a + 3b = 0 \Rightarrow a : b = 3 : -4.$$

Thus, $\vec{c} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$

Let $\vec{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ be the required vector, then

$$\vec{b}\cdot\vec{a} = (\text{length of } \vec{b})\ (\text{projection } \vec{a} \text{ on } \vec{b})$$

$$\Rightarrow \quad 4x + 3y = \sqrt{25}(1) = 5 \tag{1}$$

Also $\vec{c}\cdot\vec{a} = (\text{length of } \vec{c})\ (\text{projection of } \vec{a} \text{ on } \vec{c})$

$$\Rightarrow \quad 3x - 4y = \sqrt{25}(2) = 10 \tag{2}$$

Solving (1) and (2) we obtain $x = 2, y = -1$.

Thus, $\vec{a} = 2\hat{i} - \hat{j}$

(ix) $x = 2n\pi \quad \text{or} \quad x = 2(n\pi + \tan^{-1}\alpha)$

where $\alpha$ satisfies the relation $2 + \alpha - \alpha^3 = 0$.

$$E = \frac{\sin(x/2) + \cos(x/2) - i\tan x}{1 + 2i\sin(x/2)}$$

$$= \frac{[\sin(x/2) + \cos(x/2) - i\tan x][1 - 2i\sin(x/2)]}{[1 + 2i\sin(x/2)][1 - 2i\sin(x/2)]}$$

$$= \frac{\sin(x/2) + \cos(x/2) + 4\tan x\sin(x/2) - i[\tan x + 2\sin^2(x/2) + 2\sin(x/2)\cos(x/2)]}{1 + 4\sin^2 x/2}$$

As $E$ is real, we must have

$$\tan x + 2\sin^2(x/2) + 2\sin(x/2)\cos(x/2) = 0$$

$$\Rightarrow \quad \frac{2\tan(x/2)}{1 - \tan^2 x/2} + \frac{[2\tan^2(x/2) + 2\tan(x/2)]}{\sec^2 x/2} = 0$$

$$\Rightarrow \quad 2\tan\frac{x}{2}\left[1 + \tan^2\frac{x}{2} + \left(1 + \tan\frac{x}{2}\right)\left(1 - \tan^2\frac{x}{2}\right)\right] = 0$$

$$\Rightarrow \quad 2\tan\frac{x}{2}\left[1 + \tan^2\frac{x}{2} + 1 + \tan\frac{x}{2} - \tan^2\frac{x}{2} - \tan^3\frac{x}{2}\right] = 0$$

$$\Rightarrow \quad 2\tan\frac{x}{2}\left[2 + \tan\frac{x}{2} - \tan^3\frac{x}{2}\right] = 0 \tag{1}$$

The equation $2 + x - x^3 = 0$ has just one real root. This can be seen from the graphs of $y = 2 + x$ and $y = x^3$. Eq. (1) implies that $\tan\frac{x}{2} = 0$ or $\tan\frac{x}{2} = \alpha$, where $\alpha$ satisfies the relation

$$2 + \alpha - \alpha^3 = 0$$

$$\therefore \quad x = 2n\pi \quad \text{or} \quad x = 2(n\pi + \tan^{-1}\alpha)$$

where $n$ is any integer and $\alpha$ satisfies the relation $2 + \alpha - \alpha^3 = 0$.

(x) $-\left[\frac{8}{15} + \frac{\pi}{4}\right]$.

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

Applying $R_1 \to R_1 - (\sec x)R_3$

$$= \begin{vmatrix} 0 & 0 & \sec^2 x - \cos x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

$$= [\sec^2 x - \cos x + (\cos x)/\sin^2 x][\cos^4 x - \cos^2 x]$$
$$= [\sec^2 x - \cos x + \cos x/\sin^2 x]\,(\cos^2 x)(-\sin^2 x)$$
$$= -[\sin^2 x - \cos^3 x \sin^2 x + \cos^3 x]$$
$$= \cos^3 x \sin^2 x - \sin^2 x - \cos^3 x$$
$$= -\cos^3 x\,(1 - \sin^2 x) - \sin^2 x$$
$$= -\cos^5 x - \sin^2 x$$

$$\therefore \quad \int_0^{\pi/2} f(x)\,dx = -\int_0^{\pi/2} [\cos^5 x + \sin^2 x]\,dx$$
$$= -\left[\frac{4}{5}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{\pi}{2}\right]$$
$$= -\left[\frac{8}{15} + \frac{\pi}{4}\right].$$

## PART B

**Ans. 3**

(*a*) Number of ways of selecting at least one and at the most $n$ books out of $(2n + 1)$ is equal to

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n$$
$$= \frac{1}{2}[2\cdot{}^{2n+1}C_1 + 2\cdot{}^{2n+1}C_2 + \ldots + 2\cdot{}^{2n+1}C_n]$$
$$= \frac{1}{2}[({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + ({}^{2n+1}C_2 + {}^{2n+1}C_{2n-1})$$
$$+ \ldots + ({}^{2n+1}C_n + {}^{2n+1}C_{n-1})] \quad (\because {}^nC_r = {}^nC_{n-r})$$
$$= \frac{1}{2}[{}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_{2n}$$
$$+ {}^{2n+1}C_{2n+1} - {}^{2n+1}C_0 - {}^{2n+1}C_{2n+1}]$$
$$= \frac{1}{2}[2^{2n+1} - 1 - 1] = 2^{2n} - 1$$

Thus,

$$2^n - 1 = 63$$

$$\Rightarrow \quad 2^{2n} = 64 = 2^6$$

$$\Rightarrow \quad 2n = 6 \quad \text{or} \quad n = 3$$

(*b*) Let the equation of the line $L_1$ be $x \cos \alpha + y \sin \alpha = p_1$
Also, let the equation of the line $L_2$ be $x \sin \alpha - y \cos \alpha = p_2$
(where $p_2$ is a variable)

The coordinates of $P, Q, R, S$ are then given by
$(p_1 \sec \alpha, 0)$, $(0, p_1 \operatorname{cosec} \alpha)$, $(p_2 \operatorname{cosec} \alpha, 0)$, $(0, -p_2 \sec \alpha)$ respectively.
Now, the equation of line PS is

$$\frac{x}{p_1 \sec \alpha} - \frac{y}{p_2 \sec \alpha} = 1$$

or

$$\frac{x}{p_1} - \frac{y}{p_2} = \sec \alpha \tag{1}$$

Also, the equation of the line $QR$ is

$$\frac{x}{p_2 \sec \alpha} + \frac{y}{p_1 \operatorname{cosec} \alpha} = 1$$

or

$$\frac{x}{p_2} + \frac{y}{p_1} = \operatorname{cosec} \alpha \tag{2}$$

By eliminating the variable $p_2$ from Eqs. (1) and (2), we can obtain locus of the point of intersection of PS and QR. Hence, the required locus is

$$\left(\frac{x}{p_1} - \sec \alpha\right) \frac{1}{y} = \left(\operatorname{cosec} \alpha - \frac{y}{p_1}\right) \frac{1}{x}$$

$$\Rightarrow \quad (x - p_1 \sec \alpha)x = (p_1 \operatorname{cosec} \alpha - y)\, y$$

$$\Rightarrow \quad x^2 + y^2 - p_1 \sec \alpha x - p_1 \operatorname{cosec} \alpha y = 0$$

which is a circle through the origin.

**Ans. 4**

(*a*) Let $P(n)$ be the statement $\dfrac{(2n)!}{2^{2n}(n!)^2} \leqslant \dfrac{1}{\sqrt{3n + 1}}$

We have

$$P(1) : \frac{2!}{2^2(!)^2} \leqslant \frac{1}{\sqrt{4}}$$

This is true as LHS $= 1/2 =$ RHS. Assume that the result is true for $n = m$, that is, assume that

$$P(m) : \frac{(2m)!}{2^{2m}(m!)^2} \leqslant \frac{1}{\sqrt{3m + 1}}$$

is true.

We shall prove that the result is true for $n = m + 1$, that is, we shall show that

$$P(m + 1) : \frac{(2m + 2)!}{2^{2m+2}[(m + 1)!]^2} \leqslant \frac{1}{\sqrt{3m + 4}} \tag{1}$$

$$\begin{aligned}
\text{LHS of } \quad & P(m+1) \\
&= \frac{(2m+2)!}{2^{2m+2}[(m+1)!]^2} \\
&= \frac{(2m)!(2m+1)(2m+2)}{2^{2m}\cdot 2^2(m!)^2(m+1)^2} \\
&= \frac{(2m)!}{2^{2m}(m!)^2}\,\frac{2m+1}{2(m+1)} \\
&\leqslant \frac{1}{\sqrt{3m+1}}\,\frac{2m+1}{2m+2} \qquad (\because\ P(m) \text{ is true})
\end{aligned}$$

To prove (1), it is sufficient to prove that

$$\frac{1}{\sqrt{3m+1}}\,\frac{2m+1}{2m+2} \leqslant \frac{1}{\sqrt{3m+4}} \tag{2}$$

Since

$$\begin{aligned}
&(2m+1)^2(3m+4) - (3m+1)(2m+2)^2 \\
&= (4m^2+4m+1)(3m+4) - (3m+1)(4m^2+8m+4) \\
&= 12m^3 + 28m^2 + 19m + 4 - (12m^3 + 28m^2 + 20m + 4) \\
&= -\,m \leqslant 0
\end{aligned}$$

$$\therefore \qquad (2m+1)^2(3m+4) < (3m+1)(2m+2)^2$$

$$\Rightarrow \qquad \frac{2m+1}{2m+2}\,\frac{1}{\sqrt{3m+1}} \leqslant \frac{1}{\sqrt{3m+4}}$$

This proves (2). Thus, $P(n)$ is true for $n = m+1$. Hence, by the principle of mathematical induction $P(n)$ is true for all $n$.

(*b*) The inequality is not defined for $x = -1/2, -1, -2$. We may write the given inequality as

$$\frac{x}{(x+1/2)(x+2)} > \frac{1}{x+1} \tag{1}$$

Let $x < -2$, then (1) can be written as

$$x^2 + x < x^2 + (5/2)x + 2 \quad (\because\ 2x^2+5x+2 > 0,\ x+1 < 0)$$

$$\Rightarrow \qquad 3x + 2 > 0 \qquad \text{or} \qquad x > -2/3$$

But $x < -2$ and $x > -2/3$ are not possible simultaneously.
Let $-2 < x < -1$, then (1) can be written as

$$x^2 + x > x^2 + (5/2)x + 1 \quad (\because\ 2x^2+5x+2 < 0,\ x+1 < 0)$$

$$\Rightarrow \qquad 3x + 2 < 0 \qquad \text{or} \qquad x < -2/3.$$

Note that $\quad -2 < x < -1 \quad$ and $\quad x < -2/3$

can hold good simultaneously.
Next, let $-1 < x < 1/2$. Then (1) implies $x > -2/3$. Note that $-1 < x < -1/2$ and $x > -2/3$ implies $-2/3 < x -1/2$.
Lastly, let $x > -1/2$. Then (1) implies $x < -2/3$. But $x > -1/2$ and $x < -2/3$ cannot hold good simultaneously.

Hence $\qquad -2 < x < -1,\ -2/3 < x < -1/2.$

**Ans. 5**

(*a*) Let the third side be $\quad (x/a) + (y/b) = 1. \quad$ (1)

Centre of the inscribed circle is (2, 2) and its radius is 2.

Since the inscribed circle touches (1), we get

$$\left|\frac{(2/a) + (2/b) - 1}{\sqrt{(1/a^2) + (1/b^2)}}\right| = 2$$

$$\Rightarrow \quad \frac{4}{a^2} + \frac{4}{b^2} + \frac{8}{ab} - \frac{4}{a} - \frac{4}{b} + 1 = \frac{4}{a^2} + \frac{4}{b^2}$$

$$\Rightarrow \quad \frac{8}{ab} - \frac{4}{a} - \frac{4}{b} + 1 = 0$$

$$\Rightarrow \quad 8 - 4b - 4a + ab = 0 \qquad (2)$$

The vertices of the triangle are (0, 0), ($a$, 0), (0, $b$).
The equation of the circumscribed circle is

$$x^2 + y^2 - ax - by = 0$$

Its centre is ($a/2$, $b/2$).
If ($\alpha$, $\beta$) is any point on the locus, then $\alpha = a/2$, $\beta = b/2$ or $a = 2\alpha$, $b = 2\beta$. Putting these values in (2) we get

$$8 - 8\beta - 8\alpha + 4\alpha\beta = 0$$

$$2\alpha + 2\beta - \alpha\beta + 2 = 0$$

Therefore, the locus of the circumcentre is

$$2x + 2y - xy + 2 = 0 \qquad (3)$$

The locus of the circumcentre is given to be

$$x + y - xy + k(x^2 + y^2)^{1/2} = 0$$

$$\Rightarrow \quad 2x + 2y - 2xy + 2k(x^2 + y^2)^{1/2} = 0 \qquad (4)$$

Subtracting (3) from (4), we get

$$-xy - 2 + 2k(x^2 + y^2)^{1/2} = 0$$

$$\Rightarrow \quad k = \frac{xy + 2}{2\sqrt{x^2 + y^2}}$$

(*b*) Let $h(x) = f(x) + g(x)$ be continuous, then $h(x) - f(x) = g(x)$ is also continuous. But this is a contradiction.

**Ans. 6**

(*a*) We have

$$\frac{ds}{dt} = 2t + 1$$

$$\Rightarrow \quad S = t^2 + t + k$$

where $k$ is a constant. At $t = 0$, the top of the object is 2 m away from $M$. Therefore,

$$2 = S(0) = k$$

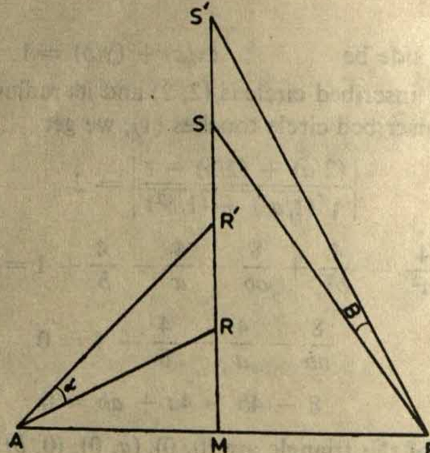



Fig. 14

Thus

$$S(t) = t^2 + t + 2$$

where $S(t)$ is the distance of the top of the object from $M$.

Let $RR'$ be the position of the object after one second and $SS'$ be its position after two seconds. Then,

$$MR' = S(1) = 1 + 1 + 2 = 4 \qquad MR = 2$$
$$MS' = S(2) = 2^2 + 2 + 2 = 8 \qquad MS = 6$$

Note that $AR^2 = 4^2 + 2^2 = 20$, $AR'^2 = 4^2 + 4^2 = 32$

$$BS^2 = 6^2 + 4^2 = 52 \qquad BS'^2 = 8^2 + 4^2 = 80$$

By the cosine formula,

$$\cos\alpha = \frac{AR^2 + AR'^2 - RR'^2}{2(AR)(AR')} = \frac{20 + 32 - 4}{2\sqrt{20}\sqrt{32}}$$
$$= \frac{48}{2\times 8\sqrt{10}} = \frac{3}{\sqrt{10}}$$

and

$$\cos\beta = \frac{BS^2 + BS'^2 - SS'^2}{2(BS)(BS')}$$
$$= \frac{52 + 80 - 4}{2\sqrt{52}\sqrt{80}} = \frac{128}{2\times 2\sqrt{13}\times 4\sqrt{5}}$$
$$= \frac{8}{\sqrt{65}}$$

$\therefore \qquad \sin\alpha = 1/\sqrt{10} \qquad \sin\beta = 1/\sqrt{65}$

Thus,

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \frac{3}{\sqrt{10}}\times\frac{8}{\sqrt{65}} + \frac{1}{\sqrt{10}}\times\frac{1}{\sqrt{65}}$$

$$= \frac{25}{\sqrt{650}} = \frac{5}{\sqrt{26}}$$

(*b*) We have

$$\alpha_1 + \alpha_2 = -b/a, \qquad \alpha_1\alpha_2 = c/a,$$
$$\beta_1 + \beta_2 = -q/p, \qquad \beta_1\beta_2 = r/d$$

The system of equations

$$\alpha_1 y + \alpha_2 z = 0$$
$$\beta_1 y + \beta_2 z = 0$$

will have a nontrivial solution, if

$$\begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \quad \text{or } \alpha_1\beta_2 - \alpha_2\beta_1 = 0$$

or $$\alpha_1/\alpha_2 = \beta_1/\beta_2$$

Applying componendo and dividendo, we get

$$\frac{\alpha_1 + \alpha_2}{\alpha_1 - \alpha_2} = \frac{\beta_1 + \beta_2}{\beta_1 - \beta_2}$$

$$\Rightarrow \quad \frac{(\alpha_1 - \alpha_2)^2}{(\alpha_1 + \alpha_2)^2} = \frac{(\beta_1 - \beta_2)^2}{(\beta_1 + \beta_2)^2}$$

$$\Rightarrow \quad \frac{(\alpha_1 + \alpha_2)^2 - 4\alpha_1\alpha_2}{(\alpha_1 + \alpha_2)^2} = \frac{(\beta_1 + \beta_2)^2 - 4\beta_1\beta_2}{(\beta_1 + \beta_2)^2}$$

$$\Rightarrow \quad 1 - \frac{4\alpha_1\alpha_2}{(\alpha_1 + \alpha_2)^2} = 1 - \frac{4\beta_1\beta_2}{(\beta_1 + \beta_2)^2}$$

$$\Rightarrow \quad \frac{\alpha_1\alpha_2}{\beta_1\beta_2} = \frac{(\alpha_1 + \alpha_2)^2}{(\beta_1 + \beta_2)^2}$$

$$\Rightarrow \quad \frac{c/a}{r/p} = \frac{b^2/a^2}{q^2/p^2}$$

$$\Rightarrow \quad \frac{ac}{pr} = \frac{b^2}{q^2}$$

**Ans. 7**

(a) We have

$$2x + 3 > 0, \qquad 2x + 3 \neq 1,$$
$$3x + 7 > 0, \qquad 3x + 7 \neq 1$$

The given equation can be written as

$$\log_{(2x+3)} (2x + 3)(3x + 7) = 4 - \log_{(3x+7)} (2x + 3)^2$$

$$\Rightarrow \quad \log_{(2x+3)} (2x + 3) + \log_{(2x+3)} (3x + 7) = 4 - 2 \log_{(3x+7)} (2x + 3)$$

$$\Rightarrow \quad 1 + \frac{\log (3x + 7)}{\log (2x + 3)} = 4 - \frac{2 \log (2x + 3)}{\log (3x + 7)}$$

$$\Rightarrow \quad \frac{2\log(2x+3)}{\log(3x+7)} + \frac{\log(3x+7)}{\log(2x+3)} - 3 = 0 \qquad (1)$$

Put $\dfrac{\log(2x+3)}{\log(3x+7)} = y$

Then (1) can be written as

$$2y + (1/y) - 3 = 0$$
$$\Rightarrow \quad 2y^2 - 3y + 1 = 0$$
$$\Rightarrow \quad (2y-1)(y-1) = 0$$
$$\Rightarrow \quad y = 1/2, \qquad y = 1$$

When $y = 1$,

$$\frac{\log(2x+3)}{\log(3x+7)} = 1$$
$$\Rightarrow \quad \log(2x+3) = \log(3x+7)$$
$$\Rightarrow \quad 2x+3 = 3x+7$$
$$\Rightarrow \quad x = -4$$

When $y = 1/2$,

$$\frac{\log(2x+3)}{\log(3x+7)} = \frac{1}{2}$$
$$\Rightarrow \quad 2\log(2x+3) = \log(3x+7)$$
$$\Rightarrow \quad (2x+3)^2 = 3x+7$$
$$\Rightarrow \quad 4x^2+12x+9 = 3x+7$$
$$\Rightarrow \quad 4x^2+9x+2 = 0$$
$$\Rightarrow \quad (4x+1)(x+2) = 0$$
$$\Rightarrow \quad x = -1/4, -2$$

Since $x > -3/2$, $x > -7/3$, $x \neq -1$, $x \neq -2$, we get $x = -1/4$.

(b) Since the man is just one step away from the starting point, he is either one step ahead or one step behind the starting point.

If he is one step ahead, then he has taken six forward and five backward steps. The man can take six forward steps in ${}^{11}C_6$ ways. Therefore, the probability that the man has taken six steps forward and five backward is

$$^{11}C_6(0.4)^6(0.6)^5 = 462(0.4)^6(0.6)^5$$

Similarly, the probability that the man has taken five forward and six backward steps is

$$^{11}C_5(0.4)^5(0.6)^6 = 462(0.4)^5(0.6)^6$$

$\therefore$ Probability of required event

$$\begin{aligned}
&= 462(0.4)^6(0.6)^5 + 462(0.4)^5(0.6)^6 \\
&= 462(0.4)^5(0.6)^5(0.4+0.6) \\
&= 462(0.4)^5(0.6)^5(1) \\
&= 462(0.24)^5
\end{aligned}$$

**Ans. 8**

(a) The given curve is $4x^2 + a^2y^2 = 4a^2$ $(4 < a^2 < 8)$. This can be written as $(x^2/a^2) + (y^2/4) = 1$. This represents an ellipse. Any point on this ellipse is given by $(a \cos \theta,\ 2 \sin \theta)$. Distance between $(a \cos \theta,\ 2 \sin \theta)$ and $(0, -2)$ is given by

$$\sqrt{a^2 \cos^2 \theta + (2 \sin \theta + 2)^2}$$

Let
$$S = a^2 \cos^2 \theta + 4 \sin^2 \theta + 8 \sin \theta + 4$$
$$= (4 - a^2) \sin^2 \theta + 8 \sin \theta + 4 + a^2$$
$$\frac{dS}{d\theta} = 2(4 - a^2) \sin \theta \cos \theta + 8 \cos \theta$$

For extreme values, $\dfrac{dS}{d\theta} = 0$

$\Rightarrow$ $$2 \cos \theta[(4 - a^2) \sin \theta + 4] = 0$$

$\Rightarrow$ $$\cos \theta = 0 \quad \text{or} \quad \sin \theta = 4/(a^2 - 4)$$

Since $\quad 4 < a^2 < 8, \qquad 0 < a^2 - 4 < 4,$

thus $4/(a^2 - 4) > 1$. This shows that there is no value of $\theta$ satisfying
$$\sin \theta = 4/(a^2 - 4).$$

Also
$$\cos \theta = 0 \Rightarrow \theta = \pi/2,\ 3\pi/2$$
$$\frac{d^2S}{d\theta^2} = 2(4 - a^2) \cos 2\theta - 8 \sin \theta$$
$$\left.\frac{d^2S}{d\theta^2}\right|_{\theta=\pi/2} = 2(4 - a^2) \cos \pi - 8 \sin (\pi/2)$$
$$= 2a^2 - 16$$

As $\quad 4 < a^2 < 8, \quad 2a^2 - 16 < 0$

$\therefore$
$$\left.\frac{d^2S}{d\theta^2}\right|_{\theta=\pi/2} < 0$$
$$\left.\frac{d^2S}{d\theta^2}\right|_{\theta=3\pi/2} = 16 - 2a^2 > 0$$

Thus, $S$ is minimum when $\theta = \pi/2$. Therefore, the required point is $(0, 2)$.

(b) Since $f'(0)$ exists, $f'(0) = Rf'(0) = Lf'(0)$.

Now,
$$f'(0) = Rf'(0) = \lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0}$$

$\Rightarrow$
$$f'(0) = \lim_{x \to 0+} \frac{f(x) - f(0)}{x} \tag{1}$$

Also,
$$f'(0) = Lf'(0) = \lim_{x \to 0+} \frac{f(-x) - f(0)}{-x - 0}$$

$\Rightarrow$
$$f'(0) = \lim_{x \to 0+} \frac{f(x) - f(0)}{-x} = - \lim_{x \to 0+} \frac{f(x) - f(0)}{x} \tag{2}$$

From (1) and (2),

$$f'(0) = \lim_{x \to 0+} \frac{f(x) - f(0)}{x} = -f'(0)$$

$$\Rightarrow \qquad 2f'(0) = 0 \Rightarrow f'(0) = 0$$

**Ans. 9**

(a) Let $E_{ww}$ denote the event that the first and second balls drawn are white. Let $E_{wb}$ denote the event that the first ball drawn is white and the second black. We similarly define $E_{bw}$ and $E_{bb}$. Let $A$ denote the event that the third ball drawn is black.
We have,

$$P(E_{ww}) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$$

$$P(E_{wb}) = \frac{2}{4} \times \frac{2}{3} = \frac{1}{3}$$

$$P(E_{bw}) = \frac{2}{4} \times \frac{2}{5} = \frac{1}{5}$$

and $$P(E_{bb}) = \frac{2}{4} \times \frac{3}{5} = \frac{3}{10}$$

Now,

$$P(A/E_{ww}) = \frac{2}{2} = 1$$

$P(A/E_{wb}) = \frac{3}{4}$ (after the completion of second experiment, there are three black and one white balls)

$$P(A/E_{bw}) = \frac{3}{4}$$

$$P(A/E_{bb}) = \frac{4}{6} = \frac{2}{3}.$$

We know that

$$P(A) = P(E_{ww})P(A/E_{ww}) + P(E_{wb})P(A/E_{wb}) + \ldots$$

$$= \left(\frac{1}{6}\right)(1) + \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{4}\right) + \left(\frac{3}{10}\right)\left(\frac{2}{3}\right)$$

$$= \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5}$$

$$= \frac{10 + 15 + 9 + 12}{60}$$

$$= \frac{46}{60} = \frac{23}{30}.$$

(b) Let the position vectors of $A$, $B$, $C$ and $D$ relative to origin $O$ be $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively. Then $\overrightarrow{AB} = \vec{b} - \vec{a}$, $\overrightarrow{BC} = \vec{c} - \vec{b}$, etc.

We have

$$\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}$$

$$=(\vec{b}-\vec{a})\times(\vec{d}-\vec{c})+(\vec{c}-\vec{b})\times(\vec{d}-\vec{a})+(\vec{a}-\vec{c})\times(\vec{d}-\vec{b})$$

$$=\vec{b}\times\vec{d}-\vec{a}\times\vec{d}-\vec{b}\times\vec{c}+\vec{a}\times\vec{c}+\vec{c}\times\vec{d}-\vec{b}\times\vec{d}-\vec{c}\times\vec{a}$$

$$+\vec{b}\times\vec{a}+\vec{a}\times\vec{d}-\vec{c}\times\vec{d}-\vec{a}\times\vec{b}+\vec{c}\times\vec{b}$$

$$=-\vec{b}\times\vec{c}+\vec{a}\times\vec{c}-\vec{c}\times\vec{a}+\vec{b}\times\vec{a}-\vec{a}\times\vec{b}+\vec{c}\times\vec{b}$$

$$=2[\vec{c}\times\vec{b}+\vec{b}\times\vec{a}+\vec{a}\times\vec{c}] \tag{1}$$

Also, area of $\Delta ABC$ is given by

$$\frac{1}{2}\,|\,\vec{BC}\times\vec{BA}\,|$$

$$=\frac{1}{2}\,|\,(\vec{c}-\vec{b})\times(\vec{a}-\vec{b})\,|$$

$$=\frac{1}{2}\,|\,\vec{c}\times\vec{a}-\vec{b}\times\vec{a}-\vec{c}\times\vec{b}+\vec{b}\times\vec{b}\,|$$

$$=\frac{1}{2}\,|-\vec{a}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{b}\,|$$

$$(\because \quad \vec{b}\times\vec{b}=\vec{0})$$

$$=\frac{1}{2}\,|\,\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{b}\,| \tag{2}$$

From (1) and (2), we get

$$|\,\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}\,|$$

$$=4\text{ (area of }\Delta ABC)$$

**Ans. 10**

Let $$g(x)=ax+b$$

Then $$f(x)=\begin{cases} ax+b, & x\leqslant 0\\ \left[\dfrac{1+x}{2+x}\right]^{1/x} & x>0\end{cases}$$

Since $f(x)$ is continuous, $f(x)$ is continuous at $x=0$ also.

$$\lim_{x\to 0-} f(x)=\lim_{x\to 0-}(ax+b)=b$$

and $$\lim_{x\to 0+} f(x)=\lim_{x\to 0+}\left[\frac{1+x}{2+x}\right]^{1/x}$$

$$=\lim_{x\to 0+}\frac{(1+x)^{1/x}}{2^{1/x}\left(1+\dfrac{x}{2}\right)^{1/x}}$$

$$= \lim_{x\to 0+} \frac{(1+x)^{1/x}}{2^{1/x}\left\{\left[1+\frac{x}{2}\right]^{2/x}\right\}^2}$$

$$= \frac{e}{(\infty)\cdot e^2} = 0$$

$(\because \lim_{x\to 0} (1+x)^{1/x} = e$ and $\lim_{x\to 0+} 2^{1/x} = \infty)$.

Since $f$ is continuous at $x = 0$,

$$b = \lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x) = 0$$

$$\therefore \qquad b = 0$$

We have $f'(-1) = a$

We shall now find $f'(1)$.

Let $\qquad y = \left[\dfrac{1+x}{2+x}\right]^{1/x} \qquad (x > 0)$

$$\log y = \frac{1}{x}[\log(1+x) - \log(2+x)]$$

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}[\log(1+x) - \log(2+x)] + \frac{1}{x}\left[\frac{1}{1+x} - \frac{1}{2+x}\right]$$

$$\Rightarrow \qquad \frac{dy}{dx} = \left[\frac{1+x}{2+x}\right]^{1/x}\left\{-\frac{1}{x^2}(\log(1+x) - \log(2+x)) + \frac{1}{x}\left(\frac{1}{1+x} - \frac{1}{2+x}\right)\right\}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \left(\frac{1+1}{2+1}\right)^{1/1}\left\{-(\log 2 - \log 3) + \frac{1}{1}\left(\frac{1}{2} - \frac{1}{3}\right)\right\}$$

$$= \frac{2}{3}\left[\log(3/2) + \frac{1}{6}\right]$$

$$= \frac{2}{3}\log\left(\frac{3}{2}\right) + \frac{1}{9}$$

Since $f'(1) = f(-1)$, we get

$$a = \frac{2}{3}\log\left(\frac{3}{2}\right) + \frac{1}{9}.$$

Hence

$$f(x) = \begin{cases} -\left[\dfrac{2}{3}\log\left(\dfrac{3}{2}\right) + \dfrac{1}{9}\right]x, & x \leqslant 0 \\ \left[\dfrac{1+x}{2+x}\right]^{1/x}, & x > 0 \end{cases}$$

**Ans. 11**

Let $\qquad I = \displaystyle\int \frac{\sqrt{\cos 2x}}{\sin x}\,dx$

$$= \int \frac{\cos 2x}{\sin x \sqrt{\cos 2x}}\, dx$$

$$= \int \frac{(2\sin^2 x - 1)}{\sin x \sqrt{\cos^2 x - \sin^2 x}}\, dx$$

$$= \int \frac{2\sin^2 x - 1}{\sin^2 x \sqrt{\cot^2 x - 1}}\, dx$$

$$= \int \frac{2 - \operatorname{cosec}^2 x}{\sqrt{\cot^2 x - 1}}\, dx$$

$$= \int \frac{2 - (1 + \cot^2 x)}{\sqrt{\cot^2 x - 1}}\, dx$$

$$= \int \frac{-(\cot^2 x - 1)}{\sqrt{\cot^2 x - 1}}\, dx$$

$$= -\int \sqrt{\cot^2 x - 1}\, dx$$

Let $\cot x = t$

$\Rightarrow$ $-\operatorname{cosec}^2 x\, dx = dt$

$\Rightarrow$ $-dx = \dfrac{dt}{(1 + t^2)}$

Thus,

$$I = \int \frac{\sqrt{t^2 - 1}}{1 + t^2}\, dt$$

Put $t = \dfrac{1}{z}$

$\Rightarrow$ $dt = \left(-\dfrac{1}{z^2}\right) dz$

$\therefore$ $$I = \int \frac{\sqrt{1 - z^2}}{z^2 + 1} \left(\frac{-z}{z^2}\right) dz$$

$$= -\int \frac{\frac{1}{z}\sqrt{1 - z^2}}{z^2 + 1}\, dz$$

Now, put $1 - z^2 = u^2$

$\Rightarrow$ $-2z\, dz = 2u\, du$

Thus,

$$I = \int \frac{u \cdot u\, du}{2 - u^2}$$

$$= \int \frac{u^2 - 2 + 2}{2 - u^2}\, du$$

$$= \int \left[ -1 + \frac{2}{2-u^2} \right] du$$

$$= -u + \frac{2}{2\sqrt{2}} \int \left[ \frac{1}{\sqrt{2}-u} + \frac{1}{\sqrt{2}+u} \right] du$$

$$= -u + \frac{1}{\sqrt{2}} \log \frac{4+\sqrt{2}}{4-\sqrt{2}} + C$$

where $u = \sqrt{1-z^2}$

$$= \sqrt{1-(1/t^2)}$$

$$= \frac{\sqrt{t^2-1}}{t}$$

$$= \frac{\sqrt{\cot^2 x - 1}}{\cot x}$$

**Ans. 12**

The curve $x^2 + y^2 = 25$ is a circle of radius 5 with centre at the origin.

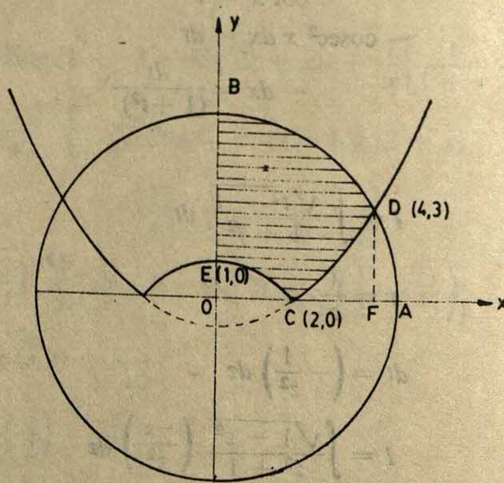



Fig. 15

Now,

$$4y = |4 - x^2|$$

$$= \begin{cases} 4 - x^2 & \text{if} \quad x^2 \leqslant 4 \\ x^2 - 4 & \text{if} \quad x^2 > 4 \end{cases}$$

Thus,

$$y = \begin{cases} 1 - x^2/4 & \text{if} \quad -2 \leqslant x \leqslant 2 \\ x^2/4 - 1 & \text{if} \quad x < -2 \quad \text{or} \quad x > 2 \end{cases}$$

$$y = 1 - x^2/4 \qquad (-2 \leqslant x \leqslant 2)$$

represents part of the parabola with the vertex at (0, 1) and above the $x$-axis.

$$y = x^2/4 - 1 \qquad (x < -2 \quad \text{or} \quad x > 2)$$

represents part of the parabola with the vertex at (0, −1) and above the $x$-axis.

Coordinates of point $D$ can be obtained by solving

$$x^2 + y^2 = 25 \qquad \text{and} \qquad y = (x^2/4) - 1$$

Simultaneously. Putting $x^2 = 4y + 4$ in $x^2 + y^2 = 25$, we obtain

$$y^2 + 4y + 4 = 25$$

or
$$y^2 + 4y - 21 = 0 \Rightarrow (y + 7)(y - 3) = 0$$

As the point lies in the first quadrant, we get $y = 3$. Therefore, coordinates of point $D$ are (4, 3).

$$\text{Required area} = \text{Area}\,(OBAO) - \text{Area}\,(OECO) - \text{Area}\,(CDFC) - \text{Area}\,(FDAF)$$

$$\text{Area}\,(OBAO) = \frac{1}{4}(\pi \cdot 5^2) = \frac{25\pi}{4}$$

$$\text{Area}\,(OECO) = \int_0^2 \left[1 - \frac{x^2}{4}\right] dx$$

$$= \left| x - \frac{x^3}{12} \right|_0^2 = 2 - \frac{8}{12} = \frac{4}{3}$$

$$\text{Area}\,(CDFC) = \int_2^4 \left[\frac{x^2}{4} - 1\right] dx$$

$$= \left| \frac{x^3}{12} - x \right|_2^4$$

$$= \frac{64}{12} - 4 - \frac{8}{12} + 2$$

$$= \frac{16}{3} - 4 + \frac{4}{3} = \frac{8}{3}$$

and
$$\text{Area}\,(FDAF) = \int_4^5 \sqrt{25 - x^2}\, dx$$

$$= \int_\alpha^{\pi/2} 5\cos\theta \cdot 5\cos\theta\, d\theta \qquad \begin{bmatrix} x = 5\sin\theta \\ \alpha = \sin^{-1}(4/5) \end{bmatrix}$$

$$= \frac{25}{2}\int_\alpha^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]\Big|_\alpha^{\pi/2}$$

$$= \frac{25}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin(\pi) - \alpha - \frac{1}{2}\sin(2\alpha)\right]$$

$$= \frac{25\pi}{4} - \frac{1}{2}(0) - \frac{25}{2}[\alpha + \sin\alpha\cos\alpha]$$

$$= \frac{25\pi}{4} - \frac{25}{2}[\alpha + \sin\alpha\sqrt{1-\sin^2\alpha}]$$

$$= \frac{25\pi}{4} - \frac{25}{2}\left[\sin^{-1}(4/5) + \frac{4}{5} \times \frac{3}{5}\right]$$

$$= \frac{25\pi}{4} - \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) - 6.$$

Hence, required area

$$= \frac{25\pi}{4} - \frac{4}{3} - \frac{8}{3} - \frac{25\pi}{4} + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) + 6$$

$$= \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) + 2$$

# ENGLISH

## PART A

**Ans. 1**

(i) An
(ii) a
(iii) the
(iv) the
(v) the

**Ans. 2**

(i) off
(ii) on
(iii) through
(iv) with
(v) into

**Ans. 3**

(i) "What on earth," he carried, "did they say?"
(ii) No sooner did the visitor try to open the gate, two fierce-looking dogs attacked him.
(iii) The new book is being talked about everywhere by the people.
(iv) No other batsman has more runs to his credit in test matches than Gavaskar.
(v) I was instructed by the teacher not to write my name anywhere in the answer book.

**Ans. 4**

(i) have known
(ii) visits
(iii) disappeared

(iv) had attended
(v) have lived
(vi) are destroyed
(vii) realized
(viii) would become
(ix) told
(x) went

**Ans. 5**

(i) B; he has
(ii) C; that
(iii) D; the naked eye
(iv) A; advice
(v) D; well
(vi) C; and
(vii) B; how a digital computer is
(viii) C; you pay
(ix) B; the second
(x) B; were surprised

**Ans. 6**

(i) migrate
(ii) meticulous
(iii) occasional
(iv) wound
(v) disgracefully
(vi) patriotic
(vii) predicted
(viii) sanctuaries
(ix) phenomenon
(x) bombastic

## PART B

*Important Note*

The candidate is expected to answer questions of PART B in his own words as far as possible. Marks for compositions are awarded on the criteria of (i) relevance of subject matter, (ii) control over language and (iii) organization, which involves the development of the subject in a logical and coherent manner. Unduly long answers (in case the word-limit is mentioned) are penalised. Serious errors of grammar and usage will also be penalised. A mere reproduction of sentences from the original passage (in case of Question 10 does not fetch any marks. For getting good marks in 'Part B' answers should be brief and relevant. Above all, the candidate must have a good control over the English language.

# 1988

# QUESTION PAPERS

## PHYSICS

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

*Note*: (1) Attempt *all* questions.
(2) Answers must be written in English.
(3) Figures in the brackets on the right-hand margin indicate the marks for the corresponding questions.
(4) There will be *no negative marking*.
(5) There are 13 questions in the paper. The answer to each question should begin on a fresh page.
(6) Answer all parts of a given question at one place.
(7) Logarithmic tables will be supplied. The use of *calculators* or *slide rules* or *graph papers* is not permitted.
(8) Useful Physical Data:

| | |
|---|---|
| Speed of light in vacuum | $= 3 \times 10^8$ m/s |
| Planck's constant | $= 6.6 \times 10^{-34}$ J s |
| Charge of the electron | $= 1.6 \times 10^{-19}$ C |
| 1 atmosphere | $= 10^5$ N/m$^2$ |
| Gas constant $R$ | $= 8.3$ J/mole K |
| Mass of the electron | $= 9.1 \times 10^{-31}$ kg |

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2/\text{C}^2$$

### PART A

1. A bullet of mass $M$ is fired with a velocity 50 m/s at an angle $\theta$ with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass $3M$ suspended by a massless string of length 10/3 m and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find

(i) the angle $\theta$, and
(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet.

(Take $g = 10$ m/s$^2$) **(6)**

2. Three particles, each of mass $m$, are situated at the vertices of an equilateral triangle of side length $a$. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle move in a circle while maintaining the original mutual separation $a$. Find the initial velocity that should be given to each particle and also the time period of the circular motion. (5)

3. A train approaching a hill at a speed of 40 km/h sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find

(i) the frequency of the whistle as heard by an observer on the hill, and

(ii) the distance from the hill at which the echo from the hill is heard by the driver, and its frequency.

(Velocity of sound in air = 1,200 km/h) (5)

4. Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.

(i) Sketch the process on a $p$–$V$ diagram.

(ii) What are the final volume and pressure of the gas?

(iii) What is the work done by the gas? (6)

5. Three particles, each of mass 1 g and carrying a charge $q$, are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side length 3 cm, calculate the charge $q$ on each particle. (Take $g = 10$ m/s²) (5)

6. In the given circuit

$$E_1 = 3E_2 = 2E_3 = 6 \text{ V}$$
$$R_1 = 2R_4 = 6\ \Omega$$
$$R_3 = 2R_2 = 4\ \Omega$$
$$C = 5\mu f.$$

Find the current in $R_3$ and the energy stored in the capacitor. (5)

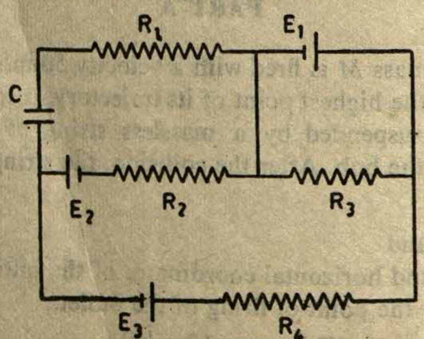



Fig. 1

7. Two long parallel horizontal rails, a distance $d$ apart and each having a resistance $\lambda$ per unit length, are joined at one end by a resistance $R$. A perfectly conducting rod $MN$ of mass $m$ is free to slide along the rails without friction (see Fig. 2). There is a uniform magnetic field of induction $B$ normal to the plane of the paper and directed into the paper. A variable force $F$ is applied to the rod $MN$ such that, as the rod moves, a constant current flows through $R$.

(i) Find the velocity of the rod and the applied force $F$ as functions of the distance $x$ of the rod from $R$.

(ii) What fraction of the work done per second by $F$ is converted into heat? **(6)**

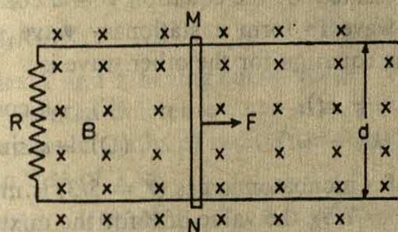



Fig. 2

8. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial,

(i) find the position of the image due to refraction at the first surface and the position of the final image, and

(ii) draw a ray diagram showing the positions of both the images. **(6)**

9. A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite.) Assuming that the Bohr model of the atom is applicable to this system,

(i) derive an expression for the radius of the $n$ th Bohr-orbit,

(ii) find the value of $n$ for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom, and

(iii) find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit to the first orbit.

(Rydberg's constant = $1.097 \times 10^7\ m^{-1}$) **(6)**

## PART B

10. Select the correct answer from among those provided for the following and write in your answer-book the corresponding letter (A), (B), (C) or (D) given in the bracket. **($6 \times 1 = 6$)**

(i) Two bodies $M$ and $N$ of equal mass are suspended from two separate massless springs of spring constants $k_1$ and $k_2$ respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of $M$ to that of $N$ is

(A) $k_1/k_2$ (B) $\sqrt{k_1/k_2}$ (C) $k_2/k_1$ (D) $\sqrt{k_2/k_1}$

(ii) A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in kilometres per hour is

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

(iii) A wave represented by the equation $y = a \cos (kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for the other wave is

(A) $a \sin (kx + \omega t)$ (C) $-a \cos (kx + \omega t)$
(B) $-a \cos (kx - \omega t)$ (D) $-a \sin (kx - \omega t)$

(iv) If one mole of a monoatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$), the value of $\gamma$ for the mixture is

(A) 1.40 (B) 1.50 (C) 1.53 (D) 3.07

(v) Two coherent monochromatic light beams of intensities $I$ and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are

(A) $5I$ and $I$ (C) $9I$ and $I$
(B) $5I$ and $3I$ (D) $9I$ and $3I$

(vi) A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of

(A) each of them increases
(B) each of them decreases
(C) copper increases and germanium decreases
(D) copper decreases and germanium increases

11. In each of the statements given below, several alternatives are given. In some cases more than one alternative is correct. Select the correct alternative(s) in each case and write down the corresponding letter(s) (A), (B), (C) or (D) in your answer-book. For each part, marks will be awarded only if all the correct alternatives are written. **($8 \times 2 = 16$)**

(i) A vessel contains oil (density = 0.8 g/cm$^3$) over mercury (density = 13.6 g/cm$^3$). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm$^3$ is

(A) 3.3 (B) 6.4 (C) 7.2 (D) 12.8

(ii) An organ pipe $P_1$, closed at one end and vibrating in its first harmonic, and another pipe $P_2$, open at both ends and vibrating in its third harmonic,

are in resonance with a given tuning fork. The ratio of the length of $P_1$ to that of $P_2$ is

(A) 8/3 (B) 3/8 (C) 1/2 (D) 1/3

(iii) A cylinder of radius $R$ made of a material of thermal conductivity $K_1$ is surrounded by a cylindrical shell of inner radius $R$ and outer radius $2R$ made of a material of thermal conductivity $K_2$. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in a steady state. The effective thermal conductivity of the system is

(A) $K_1 + K_2$ (C) $(K_1 + 3K_2)/4$
(B) $K_1K_2/(K_1 + K_2)$ (D) $(3K_1 + K_2)/4$

(iv) Two particles $X$ and $Y$ having equal charge, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii $R_1$ and $R_2$ respectively. The ratio of the mass of $X$ to that of $Y$ is

(A) $(R_1/R_2)^{1/2}$ (B) $R_2/R_1$ (C) $(R_1/R_2)^2$ (D) $R_1/R_2$

(v) A short linear object of length $b$ lies along the axis of a concave mirror of focal length $f$ at a distance $u$ from the pole of the mirror. The size of the image is approximately equal to

(A) $b\left(\frac{u-f}{f}\right)^{1/2}$ (C) $b\left(\frac{u-f}{f}\right)$

(B) $b\left(\frac{f}{u-f}\right)^{1/2}$ (D) $b\left(\frac{f}{u-f}\right)^2$

(vi) The potential difference applied to an x-ray tube is increased. As a result, in the emitted radiation,

(A) the intensity increases
(B) the minimum wavelength increases
(C) the intensity remains unchanged
(D) the minimum wavelength decreases

(vii) A freshly prepared radioactive source of half life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safety with this source is

(A) 6 h (B) 12 h (C) 24 h (D) 128 h

(viii) The impurity atoms with which pure silicon should be doped to make a $p$-type semiconductor are those of

(A) phosphorus (C) anitmony
(B) boron (D) aluminium

12. Read the following statements carefully. Indicate the subdivision number and write down your answer corresponding to each blank strictly in the order in which it appears in the statement. **(10×2 = 20)**

(i) In the formula $X = 3YZ^2$, $X$ and $Z$ have dimensions of capacitance and magnetic induction respectively. The dimensions of $Y$ in MKSQ system are . . . . . . . . . .

(ii) The masses and radii on the earth and the moon are $M_1$, $R_1$ and $M_2$, $R_2$ respectively. Their centres are a distance $d$ apart. The minimum speed with which a particle of mass $m$ should be projected from a point midway between the two centres as so to escape to infinity is . . . . . . . . . .

(iii) A smooth uniform rod of length $L$ and mass $M$ has two identical beads of negligible size, each of mass $m$, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with an angular velocity $\omega_0$ about an axis perpendicular to the rod and passing through the midpoint of the rod (see Fig. 3). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is . . . . . . . . . .

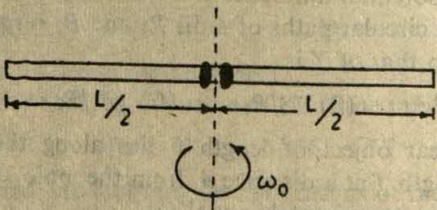



Fig. 3

(iv) A cylinder of mass $M$ and radius $R$ is resting on a horizontal platform (which is parallel to the $x$–$y$ plane) with its axis fixed along the $y$-axis and free to rotate about its axis. The platform is given a motion in the $x$-direction given by $x = A \cos (\omega t)$. There is no slipping between the cylinder and platform. The maximum torque acting on the cylinder during its motion is . . . . . . . . . .

(v) A solid sphere of radius $R$ made of a material of bulk modulus $K$ is surrounded by a liquid in a cylindrical container. A massless piston of area $A$ floats on the surface of the liquid. When a mass $M$ is placed on the piston to compress the liquid, the fractional change in the radius of the sphere, $\delta R/R$, is . . . . . . . . . .

(vi) Two parallel-plate capacitors of capacitances $C$ and $2C$ are connected in parallel and charged to a potential difference $V$. The battery is then disconnected and the region between the plates of the capacitor $C$ is completely filled with a material of dielectric constant $K$. The potential difference across the capacitors now becomes . . . . . . . . . .

(vii) In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making $10^{16}$ revolutions per second. The magnetic moment associated with the orbital motion of the electron is . . . . . . . . . .

(viii) The wire loop $PQRSP$ formed by joining two semicircular wires of radii $R_1$ and $R_2$ carries a current $I$ as shown. The magnitude of the magnetic induction at the centre $C$ is . . . . . . . . . .

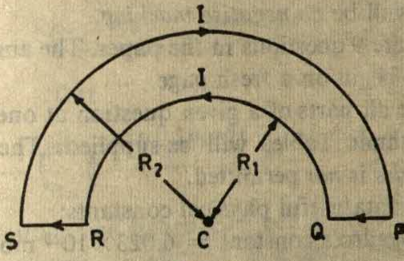



Fig. 4

(ix) The binding energies per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus ($_2He^4$) is . . . . . . . . . .

(x) In the forward bias arrangement of a *p–n* junction rectifier, the *p* end is connected to the . . . . . . . . . . terminal of the battery and the direction of the current is from . . . . . . . . . . to . . . . . . . . . . in the rectifier.

13. State whether the following statements are *true* or *false*. Give very *brief* reasons in support of your answers. Marks will be awarded only if correct reasons are also given. $(4 \times 2 = 8)$

(i) Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The energy radiated per second by the first sphere is greater than that by the second.

(ii) An electric line of force in the *x–y* plane is given by the equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1, y = 0$ in the *x–y* plane, will move along the circular line of force.

(iii) A ring of radius $R$ carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring.

(iv) A parallel beam of white light falls on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet at the outer edge.

# CHEMISTRY

TIME ALLOWED: 3 HOURS MAXIMUM MARKS: 100

*Note*: (1) Attempt *all* questions.
(2) Answers must be written in English.
(3) Figures in brackets on the right hand margin indicate the marks for the corresponding questions.

(4) There will be *no negative marking*.

(5) There are 9 questions in the paper. The answer to each question should begin on a fresh page.

(6) Answer all parts of a given question at one place.

(7) Logarithmic Tables will be supplied. The use of *calculators* or *slide rules* is *not* permitted.

(8) Useful data/useful physical constants:

Avogadro's constant $= 6.023 \times 10^{23}\ \text{mol}^{-1}$

Gas constant $R = 2\ \text{cal deg}^{-1}\ \text{mol}^{-1}$;

$8.314\ \text{J deg}^{-1}\ \text{mol}^{-1}$;

$0.082$ litre-atm $\text{deg}^{-1}\ \text{mol}^{-1}$.

1 Faraday = 96,500 Coulombs.

Atomic masses: Ag = 108, Al = 27, C = 12, Cl = 35.5, Cu = 63.5, Fe = 56, H = 1, Mg = 24, N = 14, Na = 23, O = 16, S = 32, Si = 28.

Atomic numbers: Al = 13, C = 6, Cu = 29, Fe = 26, N = 7, Mg = 12, Mn = 25, Na = 11, Ni = 28, P = 15, Sb = 51, Si = 14.

## PART A

1. (a) A sugar syrup of weight 214.2 g contains 34.2 g of sugar ($C_{12}H_{22}O_{11}$). Calculate (i) molal concentration and (ii) mole fraction of sugar in the syrup. (2)

(b) Give reasons for the following:

(i) The colour of mercurous chloride, $Hg_2Cl_2$, changes from white to black when treated with ammonia.

(ii) The carbon-oxygen bond lengths in formic acid are 1.23 Å and 1.36 Å and both the carbon-oxygen bonds in sodium formate have the same value, i.e. 1.27 Å.

(iii) The valency of oxygen is generally two whereas sulphur shows valencies of two, four and six. $(3 \times 1 = 3)$

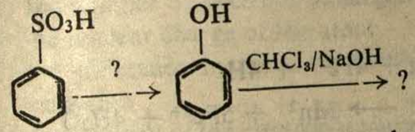
(c) Write the structural formula of 4-chloro-2-pentene. (1)

(d) Complete the following reactions:

(i) $H_3C{-}CHO \xrightarrow{CH_3MgBr} ? \xrightarrow{?} H_3C{-}\underset{\displaystyle H}{\overset{\displaystyle OH}{C}}{-}CH_3$

(ii) $CH_3COOH \xrightarrow{?} ClCH_2COOH \xrightarrow{\text{excess ammonia}} ?$

(iii) $2CH_3\overset{\overset{\displaystyle O}{\|}}{C}CH_3 \xrightarrow{\text{base catalyst}} ? \xrightarrow{\text{acid catalyst}} ?$

(iv) 

(4×1 = 4)

2. (a) A cell contains two hydrogen electrodes. The negative electrode is in contact with a solution of $10^{-6}$ M hydrogen ions. The emf of the cell is 0.118 V at 25°C. Calculate the concentration of hydrogen ions at the positive electrode. (2)

(b) $N_2O_4$ is 25% dissociated at 37°C and one atmosphere pressure. Calculate: (i) $K_p$, and (ii) the percentage dissociation at 0.1 atmosphere and 37°C. (4)

(c) How many gram-moles of HCl will be required to prepare 1 litre of a buffer solution (containing NaCN and HCl) of pH 8.5 using 0·01 gram formula weight of NaCN? (4)

$$K_{\text{dissociation}}\,(\text{HCN}) = 4.1 \times 10^{-10}$$

3. (a) A hydrated metallic salt *A*, light green in colour, gives a white anhydrous residue *B* after being heated gradually. *B* is soluble in water and its aqueous solution reacts with NO to give a dark brown compound *C*. *B* on strong heating gives a brown residue *D* and a mixture of two gases *E* and *F*. The gaseous mixture, when passed through acidified permanganate, discharges the pink colour and when passed through acidified $BaCl_2$ solution, gives a white precipitate. Identify *A*, *B*, *C*, *D*, *E* and *F*.

(b) A first order reaction is 50% complete in 30 minutes at 27°C and in 10 minutes at 47°C. Calculate the reaction rate constant at 27°C and the energy of activation of the reaction in kJ/mole. (3)

(c) In a fuel cell hydrogen and oxygen react to produce electricity. In the process hydrogen gas is oxidised at the anode and oxygen at the cathode. If 67.2 litres of $H_2$ at STP react in 15 minutes, what is the average current produced? If the entire current is used for electrodeposition of copper from copper(II) solution, how many grams of copper will be deposited?

Anode reaction: $H_2 + 2OH^- \longrightarrow 2H_2O + 2e^-$

Cathode reaction: $O_2 + 2H_2O + 2e^- \longrightarrow 4OH^-$ (4)

4. (a) An intimate mixture of ferric oxide, $Fe_2O_3$ and aluminium, Al, is used in solid fuel sockets. Calculate the fuel value per gram and fuel value per cc of the mixture. Heats of formation and densities are as follows:

$H_f(Al_2O_3) = 399$ kcal/mole; $H_f(Fe_2O_3) = 199$ kcal/mole;

Density of $Fe_2O_3 = 5.2$ g/cc; Density of Al $= 2.7$ g/cc. (2)

(b) A sample of hydrazine sulphate ($N_2H_6SO_4$) was dissolved in 100 ml of water. 10 ml of this solution was made to react with excess of ferric chloride solution and warmed to complete the reaction. Ferrous ion formed

was estimated and it required 20 ml of M/50 potassium permanganate solution. Estimate the amount of hydrazine sulphate in one litre of the solution.

Reactions:

$$4Fe^{3+} + N_2H_4 \longrightarrow N_2 + 4Fe^{2+} + 4H^+$$

$$MnO_4^- + 5Fe^{2+} + 8H^+ \longrightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O$$ (4)

(c) Give balanced equations for the following:

(i) Iodate ion reacts with bisulphite ion to liberate iodine.

(ii) Extraction of silver from silver glance by cyanide process.

(iii) Phosphorus reacts with nitric acid to give equimolar ratio of nitric oxide and nitrogen dioxide.

(iv) Carbon dioxide is passed through a concentrated aqueous solution of sodium chloride saturated with ammonia. $(4\times1=4)$

5. (a) Give a chemical test and the reagents used to distinguish between the following pairs of compounds:

(i) Cyclohexane and cyclohexene

(ii) Ethylamine and diethylamine. $(2\times1=2)$

(b) An organic compound *A*, containing C, H, N and O, on analysis gives 49.32% carbon, 9.59% hydrogen and 19.18% nitrogen. *A* on boiling with NaOH gives off $NH_3$ and a salt which on acidification gives a monobasic nitrogen free acid *B*. The silver salt of *B* contains 59.67% silver. Deduce the structures of *A* and *B*. (3)

(c) A hydrocarbon *A* (molecular formula $C_5H_{10}$) yields 2-methyl butane on catalytic hydrogenation. *A* adds HBr (in accordance with Markownikoff's rule) to form a compound *B* which on reaction with silver hydroxide forms an alcohol *C*, $C_5H_{12}O$. Alcohol *C* on oxidation gives a ketone *D*. Deduce the structures of *A*, *B*, *C*, and *D* and show the reactions involved. (5)

## PART B

6. For each subquestion given below, four answers are given of which only one is correct. Write down the correct answer in the answer-book as illustrated by the following example: $(20\times1=20)$

[The lightning bolts in the atmosphere cause the formation of

(A) NO (B) $NH_3$ (C) $NH_4OH$ (D) $NH_2OH$

The answer is to be given as (A).]

(i) The triad of nuclei that is isotonic is

(A) $^{14}_{6}C$, $^{15}_{7}N$, $^{17}_{9}F$

(B) $^{12}_{6}C$, $^{14}_{7}N$, $^{19}_{9}F$

(C) $^{14}_{6}C$, $^{14}_{7}N$, $^{17}_{9}F$

(D) $^{14}_{6}C$, $^{14}_{7}N$, $^{19}_{9}F$

(ii) The wavelength of a spectral line for an electronic transition is inversely related to

(A) the number of electrons undergoing the transition
(B) the nuclear charge of the atom
(C) the difference in the energy of the energy levels involved in the transition
(D) the velocity of the electron undergoing the transition

(iii) The orbital diagram in which the Aufbau principle is violated is

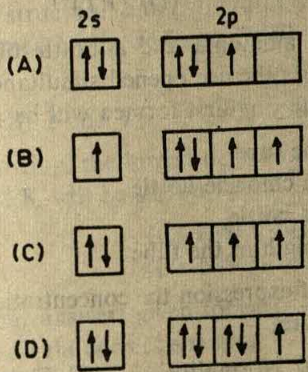



Fig. 5

(iv) The outermost electronic configuration of the most electronegative element is

(A) $ns^2np^3$ (B) $ns^2np^4$ (C) $ns^2np^5$ (D) $ns^2np^6$

(v) The first ionisation potential of Na, Mg, Al and Si are in the order

(A) Na < Mg > Al < Si (C) Na < Mg < Al > Si
(B) Na > Mg > Al > Si (D) Na > Mg > Al < Si

(vi) The species in which the central atom uses $sp^2$ hybrid orbitals in its bonding is

(A) $PH_3$ (B) $NH_3$ (C) $CH_3^+$ (D) $SbH_3$

(vii) Polarisation of electrons in acrolein may be written as

(A) $\overset{\delta-}{CH_2}=CH-\overset{\delta+}{CH}=O$
(B) $\overset{\delta-}{CH_2}=CH-CH=\overset{\delta+}{O}$
(C) $\overset{\delta-}{CH_2}=\overset{\delta+}{CH}-CH=O$
(D) $CH_2=CH-\overset{\delta+}{CH}=\overset{\delta-}{O}$

(viii) The molecule that has linear structure is

(A) $CO_2$ (B) $NO_2$ (C) $SO_2$ (D) $SiO_2$

(ix) The Cl—C—Cl angle in 1,1,2,2-tetracholoroethene and tetrachloromethane respectively will be about

(A) 120° and 109.5° (C) 109.5° and 90°
(B) 90° and 109.5° (D) 109.5° and 120°

(x) The equivalent weight of $MnSO_4$ is half its molecular weight when it is converted to

(A) $Mn_2O_3$ (B) $MnO_2$ (C) $MnO_4^-$ (D) $MnO_4^{2-}$

(xi) In van der Waals equation of state for a non-ideal gas the term that accounts for intermolecular forces is

(A) $(V - b)$
(B) $RT$
(C) $\left(P - \frac{a}{V^2}\right)$
(D) $(RT)^{-1}$

(xii) A bottle of dry ammonia and a bottle of dry hydrogen chloride connected through a long tube are opened simultaneously at both ends, the white ammonium chloride ring first formed will be

(A) at the centre of the tube
(B) near the hydrogen chloride bottle
(C) near the ammonia bottle
(D) throughout the length of the tube

(xiii) In which mode of expression the concentration of a solution remains independent of temperature?

(A) Molarity (B) Normality (C) Formality (D) Molality

(xiv) When equal volumes of the following solutions are mixed precipitation of AgCl ($K_{sp} = 1.8 \times 10^{-10}$) will occur only with

(A) $10^{-4}$ M ($Ag^+$) and $10^{-4}$ M ($Cl^-$)
(B) $10^{-5}$ M ($Ag^+$) and $10^{-5}$ M ($Cl^-$)
(C) $10^{-6}$ M ($Ag^+$) and $10^{-6}$ M ($Cl^-$)
(D) $10^{-10}$ M ($Ag^+$) and $10^{-10}$ M ($Cl^-$)

(xv) The $pK_a$ of acetylsalicylic acid (aspirin) is 3.5. The pH of gastric juice in the human stomach is about 2–3 and the pH in small intestine is about 8. Aspirin will be

(A) unionised in the small intestine and in the stomach
(B) completely ionised in the small intestine and in the stomach
(C) ionised in the stomach and almost unionised in the small intestine
(D) ionised in the small intestine and almost unionised in the stomach

(xvi) The standard reduction potentials, $E°$, for the half reactions are as

$$Zn = Zn^{2+} + 2e^-; \quad E° = +0.76 \text{ V}$$
$$Fe = Fe^{2+} + 2e^-; \quad E° = +0.41 \text{ V}$$

The emf for the cell reaction $Fe^{2+} + Zn \rightarrow Zn^{2} + Fe$ is

(A) $-0.35$ V
(B) $+0.35$ V
(C) $+1.17$ V
(D) $-1.17$ V

(xvii) In $CH_3CH_2OH$, the bond that undergoes heterolytic cleavage most readily is

(A) C—C
(B) C—O
(C) C—H
(D) O—H

(xviii) Phenol reacts with bromine in carbon disulphide at low temperature to give

(A) *m*-bromophenol
(B) *o*- and *p*-bromophenols
(C) *p*-bromophenol
(D) 2,4,6-tribromophenol

(xix) The number of structural and configurational isomers of a bromo-compound, $C_5H_9Br$, formed by the addition of HBr to 2-pentyne respectively are

(A) 1 and 2 (B) 2 and 4 (C) 4 and 2 (D) 2 and 1

(xx) Among the following, the lowest degree of paramagnetism per mole of the compound at 298 K will be shown by

(A) $MnSO_4 \cdot 4H_2O$
(B) $CuSO_4 \cdot 5H_2O$
(C) $FeSO_4 \cdot 6H_2O$
(D) $NiSO_4 \cdot 6H_2O$

7. (a) Among the four answers given for each subquestion, one or more is correct. Write down all the correct answers. **(5×1 = 5)**

[*Example*: The atomic nucleus contains

(A) protons
(B) neutrons
(C) electrons
(D) photons

*Answer*: (A) and (B) are correct.]

(i) Potassium manganate ($K_2MnO_4$) is formed when

(A) chlorine is passed into an aqueous $KMnO_4$ solution
(B) manganese dioxide is fused with potassium hydroxide in air
(C) formaldehyde reacts with potassium permanganate in presence of a strong alkali
(D) potassium permanganate reacts with conc. sulphuric acid

(ii) Keto-enol tautomerism is observed in

(A) $H_5C_6-\overset{\overset{O}{\|}}{C}-H$
(B) $H_5C_6-\overset{\overset{O}{\|}}{C}-CH_3$
(C) $H_5C_6-\overset{\overset{O}{\|}}{C}-C_6H_5$
(D) $H_5C_6-\overset{\overset{O}{\|}}{C}-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

(iii) The rate law for the reaction

$$RCl + NaOH\,(aq.) \rightarrow ROH + NaCl$$

is given by, Rate = $k_1$[RCl]. The rate of the reaction will be

(A) doubled on doubling the concentration of sodium hydroxide
(B) halved on reducing the concentration of alkyl halide to one half

(C) increased on increasing the temperature of the reaction
(D) unaffected by increasing the temperature of the reaction

(iv) Nuclear reactions accompanied with emission of neutron(s) are

(A) $^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{15}P$
(B) $^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N$
(C) $^{30}_{15}P \rightarrow ^{30}_{14}Si + ^{0}_{1}e$
(D) $^{241}_{96}Am + ^{4}_{2}He \rightarrow ^{244}_{97}Bk + ^{0}_{1}e$

(v) The statements that are true for the long form of the periodic table are:

(A) It reflects the sequence of filling the electrons in the order of sub-energy levels *s*, *p*, *d* and *f*.
(B) It helps predict the stable valency states of the elements.
(C) It reflects trends in physical and chemical properties of the elements.
(D) It helps predict the relative ionicity of the bond between any two elements.

(b) Fill in the blanks choosing appropriate words given within brackets against each statement **(5×1 = 5)**

(i) The uncertainty principle and the concept of wave nature of matter were proposed by .......... and .......... respectively.
(Heisenberg, Schodinger, Maxwell, de Broglie)

(ii) In extractive metallurgy of zinc partial fusion of ZnO with coke is called .......... and reduction of the ore to the molten metal is called ..........
(smelting, calcining, roasting, sintering)

(iii) Isomers which are .......... mirror images are known as ..........
(superimposable, non-superimposable, enantiomers, diastereomers, epimers)

(iv) The salts .......... and .......... are isostructural.
($FeSO_4.7H_2O$; $CuSO_4.5H_2O$; $MnSO_4.4H_2O$; $ZnSO_4.7H_2O$)

(v) .......... acid gives hypo .......... ion.
(hydrobromic, hypobromous, perbromic, bromide, bromite, perbromate)

. Each entry in column X is in some way related to the entries in columns Y and Z. Match the appropriate entries. **(10×1 = 10)**

[*Example*: Animal charcoal; C, $Ca_3(PO_4)_2$; sugar refining]

| X | Y | Z |
|---|---|---|
| (i) Animal charcoal | kJ $deg^{-1}$ | watch spring |
| Invar | $cm^{-1}$ | $1.3805 \times 10^{-26}$ |
| Nichrome | Co, Ni | sugar refining |
| Rydberg | Fe, Ni | cutlery |
| Stainless steel | Fe, Cr, Ni, C | 109677 |
| Boltzmann | C, $Ca_3(PO_4)_2$ | heating element |

| | | |
|---|---|---|
| (ii) Friedel-Crafts | oil | alkenes |
| Fermentation | Lewis acid | soap |
| Dehydrohalogenation | cuprous chloride | anhydrous $AlCl_3$ |
| Sandmeyer | yeast | chlorobenzene |
| Saponification | alcoholic alkali | ethanol |

9.(a) Arrange the following in each subquestion as indicated. **($6\times1 = 6$)**

(i) $HOCl$, $HOClO_2$, $HOClO_3$, $HOClO$
in increasing order of thermal stability.

(ii) *n*-butane, *n*-butanol, *n*-butyl chloride, isobutane
in increasing order of boiling point.

(iii) $N_2$, $O_2$, $F_2$, $Cl_2$
in increasing order of bond dissociation energy

(iv) benzene, toluene, methoxybenzene, chlorobenzene
in increasing order of reactivity towards sulphonation with fuming sulphuric acid.

(v) $CO_2$, $N_2O_5$, $SiO_2$, $SO_3$
in the order of increasing acidic character.

(vi) methylamine, dimethylamine, aniline, *N*-methylaniline
in increasing order of base strength.

(b) In each subquestion below a statement (*S*) is given which may be correct or wrong.

For each statement an explanation (*E*) is given which may be correct or wrong.

Choose the correct answer from the code A, B, C, D given for each subquestion. **($2\times2 = 4$)**

(i) Statement (*S*): If a strong acid is added to a solution of potassium chromate it changes its colour from yellow to orange.

Explanation (*E*): The colour change is due to the oxidation of potassium chromate.

Code:

(A) Both (*S*) and (*E*) are correct and (*E*) is the correct explanation of (*S*)

(B) Both (*S*) and (*E*) are correct but (*E*) is not the correct explanation of (*S*).

(C) (*S*) is correct but (*E*) is wrong

(D) (*S*) is wrong but (*E*) is correct.

(ii) Statement (*S*): Solubility of *n*-alcohols in water decreases with increase in molecular weight.

Explanation (*E*): The relative proportion of the hydrocarbon part in alcohols increases with increasing molecular weight which permits enhanced hydrogen bonding with water.

Code:

(A) Both ($S$) and ($E$) are correct and ($E$) is the correct explanation of ($S$)

(B) Both ($S$) and ($E$) are correct but ($E$) is not the correct explanation of ($S$)

(C) ($S$) is correct but ($E$) is wrong

(D) ($S$) is wrong but ($E$) is correct.

# MATHEMATICS

TIME ALLOWED : 3 HOURS — MAXIMUM MARKS : 100

*Note*: (1) Attempt *all* questions.
(2) Answers must be written in English.
(3) Figures in brackets on the right-hand margin indicate the marks for the corresponding questions.
(4) There will be *no negative marking*.
(5) There are 14 questions in the paper. The answer to each question should begin on a fresh page.
(6) Answer all parts of a given question at one place.
(7) The use of calculators, slide rules, logarithmic, trigonometric and statistical tables, or graph papers is not permitted.

## PART A

1. (a) Let $R$ be the set of real numbers and $f: R \to R$ be such that for all $x$ and $y$ in $R$ $|f(x) - f(y)| \leqslant |x - y|^3$. Prove that $f(x)$ is a constant. (2)

(b) A box contains 2 fifty paise coins, 5 twentyfive paise coins and a certain fixed number $N$ ($\geqslant 2$) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. (3)

2. (a) Prove that

$$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha \qquad (2)$$

(b) Let $OACB$ be a parallelogram with $O$ at the origin and $OC$ a diagonal. Let $D$ be the midpoint of $OA$. Using vector methods prove that $BD$ and $CO$ intersect in the same ratio. Determine this ratio. (3)

3. Solve:

$$|x^2 + 4x + 3| + 2x + 5 = 0 \qquad (5)$$

4. Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point $P$ and make an angle $\theta$ with each other. Find the equation of a line $L$ different from $L_2$ which passes through $P$ and makes the same angle $\theta$ with $L_1$. (5)

5. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of $S$ which subtends a right angle at the origin. **(5)**

6. Investigate for maxima and minima the function

$$f(x) = \int_1^x [2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2]\, dt \qquad \textbf{(5)}$$

7. Find the area of the region bounded by the curve $C: y = \tan x$, tangent drawn to $C$ at $x = \pi/4$ and the $x$ axis. **(5)**

8. Evaluate:

$$\int_0^1 \log[\sqrt{(1-x)} + \sqrt{(1+x)}]\, dx \qquad \textbf{(5)}$$

9. A sign-post in the form of an isosceles triangle $ABC$ is mounted on a pole of height $h$ fixed to the ground. The base $BC$ of the triangle is parallel to the ground. A man standing on the ground at a distance $d$ from the sign-post finds that the top vertex $A$ of the triangle subtends an angle $\beta$ and either of the other two vertices subtends the same angle $\alpha$ at his feet. Find the area of the triangle. **(5)**

10. Let $R = (5\sqrt{5} + 11)^{2n+1}$ and $f = R - [R]$, where [ ] denotes the greatest integer function. Prove that $Rf = 4^{2n+1}$. **(5)**

## PART B

11. This question contains four statements, each of which is either true or false. Indicate your choice of the answer in the answer-book by writing *true* or *false* for each statement. **(4×1 = 4)**

(i) If $f_1(x)$ and $f_2(x)$ are defined on domains $D_1$ and $D_2$, respectively, then $f_1(x) + f_2(x)$ is defined on $D_1 \cup D_2$.

(ii) The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle.

(iii) The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes in concyclic points.

(iv) The value of the integral

$$\int_0^{2a} [f(x)/\{f(x) + f(2a - x)\}]\, dx$$

is equal to $a$.

12. There are eight parts in this question. Four choices are given for each part and one of them is correct. Indicate your choice of the correct answer for each part in your answer-book by writing one of the letters A, B, C, D, whichever is appropriate. **(8×2 = 16)**

(i) The value of the expression $\sqrt{3}\operatorname{cosec} 20° - \sec 20°$ is equal to

(A) 2
(B) $2 \sin 20°/\sin 40°$
(C) 4
(D) $4 \sin 20°/\sin 40°$

(ii) The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, for

(A) $x = n\pi$ (C) $x = 0$

(B) $x = (n + 1/2)\pi$ (D) no value of $x$

(iii) If $P = (1, 0)$ $Q = (-1, 0)$ and $R = (2, 0)$ are three given points, then the locus of the point $S$ satisfying the relation $SQ^2 + SR^2 = 2SP^2$, is

(A) a straight line parallel to the $x$-axis
(B) a circle passing through the origin
(C) a circle with the centre at the origin
(D) a straight line parallel to $y$-axis

(iv) If a circle passes through the point $(a, b)$ and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

(A) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(B) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(C) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(D) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$

(v) The sum of the first $n$ terms of the series

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \cdots$$

is equal to

(A) $2^n - n + 1$ (C) $n + 2^{-n} - 1$
(B) $1 - 2^{-n}$ (D) $2^n - 1$

(vi) If $y^2 = P(x)$, a polynomial of degree 3, then

$$2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$$

equals

(A) $P'''(x) + P'(x)$ (C) $P(x)P'''(x)$
(B) $P''(x)P'''(x)$ (D) a constant

(vii) Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations

$$\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \quad \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \quad \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$$

Then the value of the expression

$$(\vec{a} + \vec{b})\cdot\vec{p} + (\vec{b} + \vec{c})\cdot\vec{q} + (\vec{c} + \vec{a})\cdot\vec{r}$$

is equal to

(A) 0 (B) 1 (C) 2 (D) 3

(viii) One hundred identical coins, each with probability, $p$, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of $p$ is

(A) 1/2 (B) 49/101 (C) 50/101 (D) 51/101

13. There are five parts in this question. Each part has one or more than one correct answer. Indicate all correct answers for each part by writing the corresponding letters A, B, C, D in the answer-book. **($5 \times 2 = 10$)**

(i) If the first and the $(2n-1)$st terms of an A.P., a G.P. and an H.P. are equal and their $n$th terms are $a$, $b$ and $c$ respectively, then

(A) $a = b = c$ (C) $a + c = b$
(B) $a \geqslant b \geqslant c$ (D) $ac - b^2 = 0$

(ii) The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are

(A) $x = 0$ (C) $(h^2 - r^2)x - 2rhy = 0$
(B) $y = 0$ (D) $(h^2 - r^2)x + 2rhy = 0$

(iii) The values of $\theta$ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+\sin 4\theta \end{vmatrix} = 0,$$

are

(A) $7\pi/24$ (B) $5\pi/24$ (C) $11\pi/24$ (D) $\pi/24$

(iv) The function

$$f(x) = \begin{cases} |x-3|, & x \geqslant 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$$

is

(A) continuous at $x = 1$ (C) continuous at $x = 3$
(B) differentiable at $x = 1$ (D) differentiable at $x = 3$

(v) For two given events $A$ and $B$, $P(A \cap B)$ is

(A) not less than $P(A) + P(B) - 1$
(B) not greater than $P(A) + P(B)$
(C) equal to $P(A) + P(B) - P(A \cup B)$
(D) equal to $P(A) + P(B) + P(A \cup B)$

14. This question contains ten incomplete statements. Determine your answers to be inserted in the blanks so that the statements are complete. Write those answers only in your answer-book, strictly in the order in which the statements appear below. **($10 \times 2 = 20$)**

(i) If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3} + 1)$ cm, then the area of the triangle is . . . . . . . . . .

(ii) The value of the determinant

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$$

is . . . . . . . . . .

(iii) The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are . . . . . . . . . . and . . . . . . . . . . respectively.

(iv) For any two complex numbers $z_1$, $z_2$ and any real numbers $a$ and $b$.

$$| az_1 - bz_2 |^2 + | bz_1 + az_2 |^2 = \ldots\ldots\ldots\ldots$$

(v) The sum of the first $n$ terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$$

is $n(n + 1)^2/2$, when $n$ is even. When $n$ is odd, the sum is . . . . . . . . . .

(vi) Total number of ways in which six '+' and four '−' signs can be arranged in a line such that no two '−' signs occur together is . . . . . . . . . .

(vii) If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle $C_2$ of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of $C_2$ are . . . . . . . . . .

(viii) If $f(9) = 9, f'(9) = 4$, then

$$\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$$

equals . . . . . . . . . .

(ix) The integral

$$\int_0^{1.5} [x^2]\, dx$$

where [ ] denotes the greatest integer function, equals . . . . . . . . . .

(x) Urn A contains 6 red and 4 black balls and urn $B$ contains 4 red and 6 black balls. One ball is drawn at random from urn $A$ and placed in urn $B$. Then one ball is drawn at random from urn $B$ and placed in urn $A$. If one ball is now drawn at random from urn $A$, the probability that it is found to be red is . . . . . . . . . .

# MODEL SOLUTIONS

## PHYSICS

### PART A

**Ans. 1**

(i) Referring to Fig. 6, when the bullet reaches the highest point $A$, its horizontal velocity component is

$$v_x = v \cos \theta \qquad \text{(where } v = 50 \text{ m/s)}$$

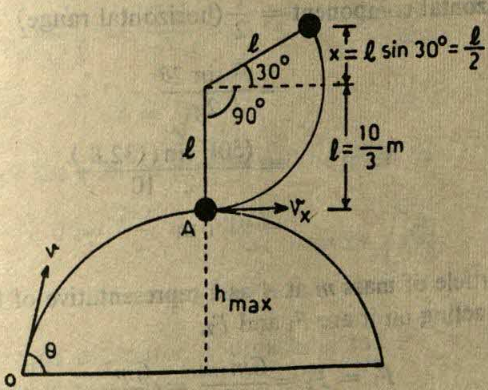



Fig. 6

$\therefore$ Kinetic energy imparted $= \dfrac{1}{2} M v_x^2 = \dfrac{1}{2} M v^2 \cos^2 \theta$

As a result, the composite body (i.e. bob and bullet) of mass $M + 3M = 4M$ is raised to a vertical height,

$$h = l + \frac{l}{2} \quad \text{(Fig. 6)}$$

$$= \frac{3l}{2} = \frac{3}{2} \times \frac{10}{3} = 5 \text{ m}$$

$\therefore$ Potential energy gained $= 4Mgh$

$$= 4\,Mg \times 5 = 20\,Mg$$

From the law of conservation of energy,

$$\frac{1}{2} Mv^2 \cos^2 \theta = 20\, Mg$$

$$\Rightarrow \qquad \cos \theta = \frac{\sqrt{40\, g}}{v}$$

$$= \frac{\sqrt{40 \times 10}}{50} = \frac{2}{5} = 0.4$$

$$\therefore \qquad \theta = 66.4^\circ$$

(ii) Vertical component of the initial position $A$ of the bob with respect to the point $O$ of projection

$$= h_{max} = \frac{v^2 \sin \theta}{2g}$$

$$= \frac{(50)^2 \sin^2 66.4^\circ}{2 \times 10}$$

$$\simeq 105 \text{ m}$$

$$\text{Horizontal component} = \frac{1}{2}(\text{horizontal range})$$

$$= \frac{v^2 \sin 2\theta}{2g}$$

$$= \frac{(50)^2 \sin (132.8^\circ)}{2 \times 10}$$

$$= 91.7 \text{ m}$$

**Ans. 2**

Consider the particle of mass $m$ at $A$ as a representative of the three masses (Fig. 7). Forces acting on it are $F_1$ and $F_2$.

$$F_1 = F_2 = \frac{Gm \cdot m}{a^2} = \frac{Gm^2}{a^2}$$

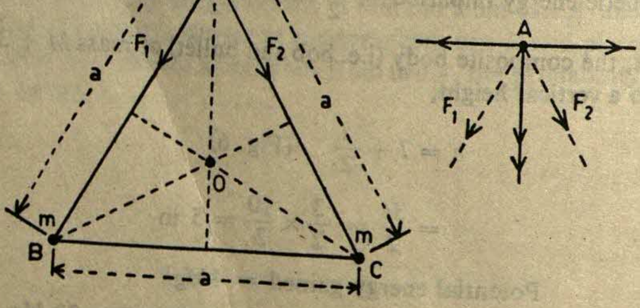



**Fig. 7**

These can be broken up into rectangular components along $AO$ and perpendicular to the direction of $AO$. The perpendicular components will cancel out and those along $AO$ will unite together giving a net force equal to

$$F = \frac{Gm^2}{a^2}\cos 30^\circ + \frac{Gm^2}{a^2}\cos 30^\circ$$

$$= \frac{Gm^2}{a^2}\times 2\times\frac{\sqrt{3}}{2} = \frac{Gm^2}{a^2}\times\sqrt{3}$$

For a circular motion with $O$ as centre, the initial velocity, $v$ given to the mass $m$ should be such that the corresponding centripetal force must become equal to the net force. Thus,

$$\frac{mv^2}{r} = \frac{Gm^2}{a^2}\times\sqrt{3}$$

The radius $r$ of the circular path is given by

$$\frac{2}{3}\sqrt{\left(a^2-\frac{a^2}{4}\right)} = \frac{a}{\sqrt{3}}$$

$$\therefore \qquad \frac{mv^2}{a/\sqrt{3}} = \frac{Gm^2}{a^2}\times\sqrt{3}$$

$$\Rightarrow \qquad v = \sqrt{\frac{Gm}{a}}$$

The time period of the circular motion will be given by

$$T = \frac{2\pi r}{v} = \frac{2\pi\, a/\sqrt{3}}{\sqrt{\frac{Gm}{a}}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$$

**Ans. 3**

Considering the case of source approaching and listener stationary, the observed frequency is given by

$$\eta' = \left(\frac{v}{v-v'}\right)\eta$$

where $\eta$ and $\eta'$ are the real and observed frequencies respectively, and $v$ and $v'$ are the wave and source velocities respectively.

If the wind also moves in the direction of the waves (source to listener), $v$ changes to $(v+v_m)$ where $v_m$ is the velocity of the medium (wind). Thus, in this case,

$$\eta' = \left(\frac{v+v_m}{v+v_m-v'}\right)\eta$$

Here,

$$v = 1200\ \text{km/h}$$

$$v_m = 40\ \text{km/h}$$

$$v' = 40\ \text{km/h}$$

$$\eta = 580\ \text{Hz}$$

Substituting the values,

$$\eta' = \left(\frac{1200 + 40}{1200 + 40 - 40}\right) \times 580$$

$$= \frac{1240}{1200} \times 580 = 599 \text{ Hz}$$

For the receipt of the echo by the driver, we have to take 599 Hz as the frequency given out by the source at the hill, the listener approaching with a speed of 40 km/h and the wind blowing at a speed of 40 km/h opposite to the direction of the waves. The frequency $\eta''$ will be given by

$$\eta'' = \left(\frac{1200 - 40 + 40}{1200 - 40}\right)\eta'$$

$$= \left(\frac{1200}{1160}\right) \times 599 = 620 \text{ Hz}$$

The time taken by the waves in their upward journey to the hill is given by

$$t_1 = \frac{1 \text{ km}}{(1200 + 40) \text{ km/h}} = \frac{1}{1240} \text{ h}$$

In this time, train moves closer to the hill by a distance

$$40 \times \frac{1}{1240} = \frac{1}{31} \text{ km}$$

Therefore, the distance between the train and the hill becomes

$$\left(1 - \frac{1}{31}\right) = \frac{30}{31} \text{ km}$$

Suppose the echo is heard $x$ hours after this instant. The distance travelled by the train in this interval is given by

$$d = 40 \times x \text{ km}$$

Also, the distance travelled by the waves in the same time is given by

$$d' = (1200 - 40) \times x \text{ km}$$

$$= 1160 \times x \text{ km}$$

The sum of $d$ and $d'$ is equal to 30/31 km

$$\therefore \quad 40 \times x + 1160 \times x = \frac{30}{31}$$

$$\Rightarrow \quad x = \frac{1}{1240} \text{ h}$$

The distance travelled by the train in this time will be

$$D = \frac{1}{1240} \times 40 = \frac{1}{31} \text{ km}$$

The echo is heard by the driver when the train has moved overall a distance of $\frac{1}{31} + \frac{1}{31} = \frac{2}{31}$ km from the start. Thus, the echo will be heard when

the distance between the driver and the hill is

$$1-\frac{2}{31}=\frac{29}{31}=0.935 \text{ km}$$

**Ans. 4**

(ii) $PV = nRT$

Here,
$$V = 20 \text{ litres} = 0.02 \text{ m}^3$$
$$n = 2 \text{ moles}$$
$$R = 8.3 \text{ J/mole K}$$
$$T = 27 + 273 = 300 \text{ K}$$

Substituting the values,

$$P = \frac{2\times 8.3\times 300}{0.02} \doteq 24.9\times 10^4 \text{ N/m}^2$$

For expansion and constant pressure,

$$V \propto T$$

or
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Here,

$$V_1 = 0.02 \text{ m}^3$$
$$T_1 = 300 \text{ K}$$
$$V_2 = 2\times V_1 = 2\times 0.02 \text{ m}^3$$
$$T_2 = ?$$

Thus

$$T_2 = \frac{T_1\times V_2}{V_1} = \frac{300\times 2\times 0.02}{0.02}$$
$$= 600 \text{ K}$$

After expansion and constant pressure, we have

$$P = 24.9\times 10^4 \text{ N/m}^2$$
$$V = 0.04 \text{ m}^3$$
$$T = 600 \text{ K}$$

For adiabatic change,

$$TV^{\gamma-1} = \text{constant}$$

or

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

Here,

$$T_1 = 600 \text{ K}$$
$$V_1 = 0.04 \text{ m}^3$$

$$\gamma = 5/3$$
$$T_2 = 300 \text{ K}$$
$$V_2 = ?$$

We can write,

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_1}{T_2}\right)$$

or $$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{1/\gamma-1}$$

or $$V_2 = V_1\left(\frac{T_1}{T_2}\right)^{1/\gamma-1}$$

Substituting the values,

$$V_2 = 0.04\left(\frac{600}{300}\right)^{\frac{1}{5/3-1}}$$
$$= 0.04 \times 2^{3/2} = 0.11312 \text{ m}^3$$

The final volume, therefore, is 0.11312 $m^3$

Also, $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$\therefore$ Final pressure,

$$P_2 = \frac{P_1V_1T_2}{T_1V_2}$$
$$= \frac{(24.9 \times 10^4) \times 0.04 \times 300}{600 \times (0.04 \times 2^{3/2})}$$
$$= 4.4 \times 10^4 \text{ N/m}^2$$

(i) The process on a $P$–$V$ diagram can now be sketched as shown in Fig. 8.

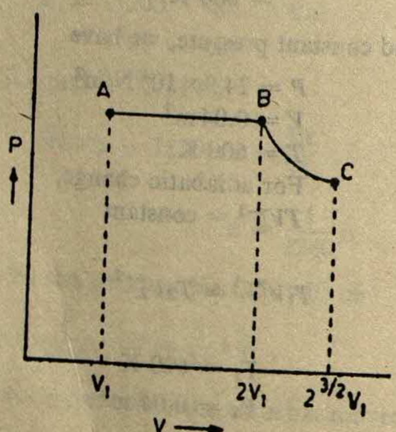



Fig. 8

(iii) The work done by the gas is the work done during the adiabatic process. It is given by the expression

$$W = \frac{nR(T_1 - T_2)}{(\gamma - 1)}$$

$$= \frac{2 \times 8.3 \times (600 - 300)}{\left(\frac{5}{3} - 1\right)}$$

$$= 7470 \text{ J}$$

The work done on the gas is the work done during the first expansion. It is given by the expression

$$W' = P(V_{\text{final}} - V_{\text{initial}})$$

$$= 24.9 \times 10^4 \times (0.04 - 0.02)$$

$$= 4980 \text{ J}$$

Thus, net work done by the gas is

$$(W - W') = 7470 - 4980$$

$$= 2490 \text{ J}$$

**Ans. 5**

Consider one of the masses at the corner $A$ of an equilateral triangle of side length 3 cm (Fig. 9).

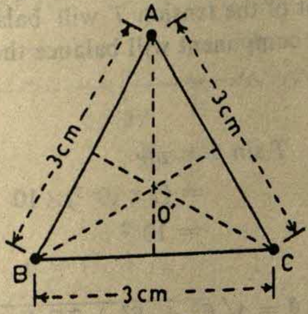



Fig. 9

The mass is in equilibrium under the action of following forces: $F_B$ and $F_C$ in the horizontal plane at 60° to each other, the weight acting vertically downward ($mg$) and the tension $T$ of the string acting at an angle $\theta$ to the horizontal direction (Fig. 10). The angle $\theta$ can be calculated as in Fig. 11.

From trigonometry,

$$AO' = \frac{2}{3} \times \sqrt{3^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{2}{3} \times \sqrt{9 - \frac{9}{4}} = \frac{2}{3} \times \sqrt{\frac{27}{4}}$$

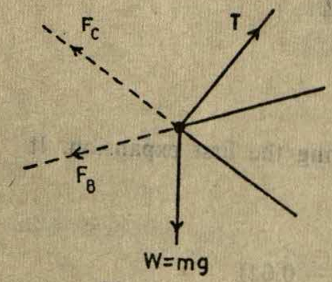


Fig. 10

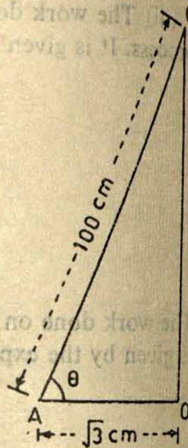


Fig. 11

$$= \frac{2}{3} \times \frac{3}{2} \times \sqrt{3} = \sqrt{3}\,\text{cm}$$

Also,
$$\frac{AO'}{AO} = \frac{\sqrt{3}}{100} = \cos\theta$$

$\therefore$
$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{100}\right)$$

The vertical component of the tension $T$ will balance the weight of the bob and the horizontal component will balance the resultant of the forces $F_B$ and $F_C$.

Vertical component,

$$T\sin\theta = mg$$
$$= (1 \times 10^{-3}) \times 10$$
$$= 10^{-2}$$

Horizontal component,

$$T\cos\theta = \sqrt{F_B^2 + F_C^2 + 2F_BF_C\cos 60^\circ}$$

But
$$F_B = F_C$$
$$= \frac{9 \times 10^9 \times q \times q}{(3 \times 10^{-2})^2}$$
$$= q^2 \times 10^{13}\,\text{N}$$

$\therefore$
$$T\cos\theta = q^2 \times 10^{13}\sqrt{2 + 2\cos 60^\circ}$$
$$= q^2 \times 10^{13}\sqrt{3}$$

Thus,

$$\frac{T\sin\theta}{T\cos\theta} = \frac{10^{-2}}{q^2 \times 10^{13}\sqrt{3}}$$

or
$$\tan\theta = \frac{1}{\sqrt{3}\, q^2 \times 10^{15}}$$

But
$$\tan\theta = \frac{\sqrt{10000-3}}{\sqrt{3}} \simeq \sqrt{\frac{10000}{3}} = \frac{100}{\sqrt{3}}$$

$\therefore$
$$\frac{100}{\sqrt{3}} = \frac{1}{\sqrt{3}\, q^2 \times 10^{15}}$$

$\Rightarrow$
$$q = 3.16 \times 10^{-9}$$

**Ans. 6**

It is given that current $I_1$ starts from $E_1$ and moves up to $B$ where it meets $I_2$ coming from $E_3$. On reaching $G$, the current $I_3$ splits into $I_1$ and $I_2$ (Fig. 12). There is no flow of current through $R_1$ or $C$. Due to potential difference between $F$ and $G$, a change will develop across $C$. As $E_3$ is dominant over $E_2$, the current coming out of $E_2$ can be ignored.

$$I_3 = I_1 + I_2$$

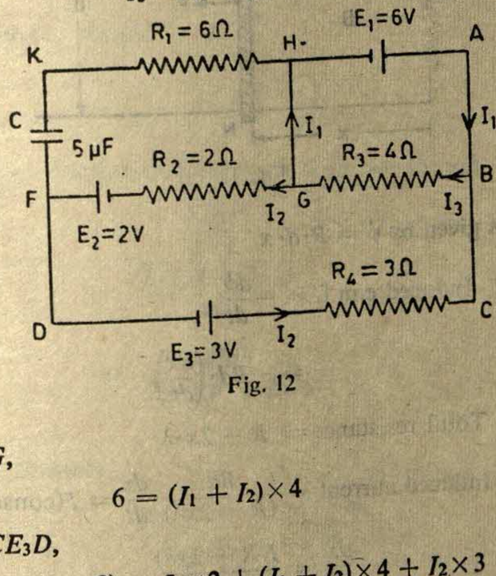



Fig. 12

From $HE_1ABG$, (1)

$$6 = (I_1 + I_2) \times 4$$

From $FE_2GBCE_3D$,

$$(3-2) = I_2 \times 2 + (I_1 + I_2) \times 4 + I_2 \times 3 \quad (2)$$

or
$$1 = 4I_1 + 9I_2$$

From Eqs. (1) and (2), we get

$$I_1 = \frac{10}{4} = 2.5 \text{ A}$$

$$I_2 = -\frac{5}{5} = -1 \text{ A}$$

The negative sign of $I_2$ indicates that $F$ is at a higher potential than $G$.

$\therefore$ Potential difference between $F$ and $G = 2 \times 1 = 2$ V

The energy developed by the potential difference is given by

$$E_r = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\times(5\times10^{-6})\times(2)^2$$

$$= 10^{-5}\text{ J}$$

Current is $R_3 = I_3 = I_1 + I_2$

$$= 2.5 - 1$$

$$= 1.5\text{ A}$$

**Ans. 7**

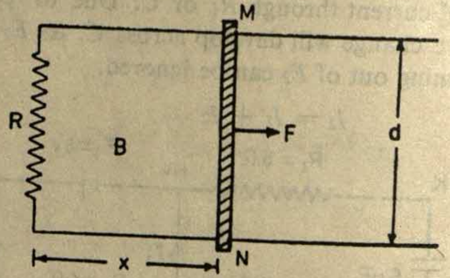



Fig. 13

**Flux at time $t$ is given by $\phi = B\cdot d\cdot x$**

$$\therefore \quad \text{Induced e.m.f.} = -\frac{d\phi}{dt}$$

$$= -Bd\cdot\left(\frac{dx}{dt}\right)$$

$$\text{Total resistance} = R + 2x\cdot\lambda$$

$$\text{Induced current} = \frac{Bd}{(R+2x\lambda)}\cdot\frac{dx}{dt} = I(\text{constant})$$

$$\therefore \quad \frac{dx}{dt} = \frac{I(R+2x\lambda)}{Bd} \qquad (1)$$

(i) Differentiation gives the velocity of the rod as

$$a = \frac{d^2x}{dt^2} = \frac{I}{Bd}\cdot 2\lambda\,\frac{dx}{dt}$$

Substituting the value of $\frac{dx}{dt}$ from Eq. (1), we get

$$a = \frac{I}{Bd}\cdot 2\lambda\cdot\frac{I(R+2x\lambda)}{Bd}$$

$$= \left(\frac{I}{Bd}\right)^2\cdot 2\lambda(R+2x\lambda)$$

Applied force is given by

$$F = ma$$

$$= m\left(\frac{I}{Bd}\right)^2 \cdot 2\lambda(R + 2x\lambda) \qquad (2)$$

(ii) From Eq. (2),

$$I = \sqrt{\frac{F}{2\lambda(R + 2x\lambda)m}} \cdot Bd$$

Heat produced per second is given by

$$I^2 \times (R + 2x\lambda)$$

$$= \frac{F}{2\lambda(R + 2x\lambda)m} B^2d^2(R + 2x\lambda)$$

$$= \frac{FB^2d^2}{2\lambda m}$$

Total work done per second $= F \cdot \dfrac{dx}{dt}$

$$= F \cdot \frac{I(R + 2x\lambda)}{Bd}$$

Heat produced as fraction of total work done per second

$$= \frac{\dfrac{FB^2d^2}{2\lambda m}}{\dfrac{F \cdot I(R + 2x\lambda)}{Bd}}$$

$$= \frac{B^3d^3}{2\lambda mI(R + 2x\lambda)}$$

**Ans. 8**

(i) Using the formula,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

we have, for first refraction

$$\mu_1 = \frac{4}{3}, \qquad \mu_2 = 1, \qquad u = -\infty, \qquad v = ?$$

$$R = 2 \text{ mm} = 0.2 \text{ cm}$$

Substituting the values,

$$\frac{1}{v} - \frac{(4/3)}{\infty} = \frac{1 - (4/3)}{0.2}$$

$$\Rightarrow \qquad v = -0.6 \text{ cm}$$

Therefore, first image will appear to be formed on the left of $P_1$ (Fig. 14) at a distance of 0.6 cm

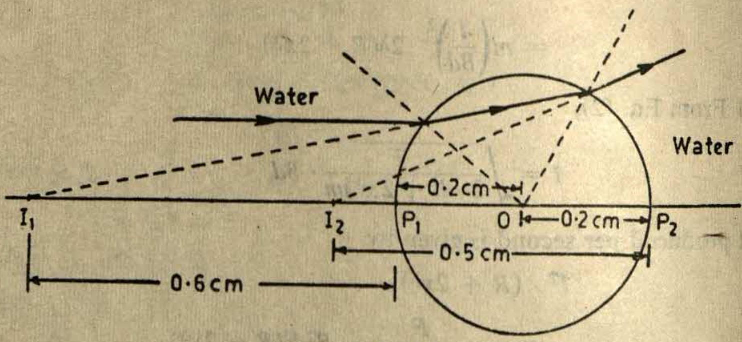



Fig. 14

For final image,

$$\mu_1 = 1, \qquad \mu_2 = \frac{4}{3}, \qquad u = -(0.6 + 0.4) = -1.0 \text{ cm}$$

$$v = ?, \qquad R = -0.2 \text{ cm}$$

$$\frac{4/3}{v} - \frac{1}{-1.0} = \frac{(4/3) - 1}{(-0.2)}$$

$$\Rightarrow \qquad v = -0.5 \text{ cm}$$

Thus, final image is formed at a distance of 0.5 cm on the left of $P_2$, i.e. at a distance of 0.1 cm to the left of $P_1$ (Fig. 14).

(ii) The ray diagram is shown in Fig. 14.

**Ans. 9**

(i) For centripetal force, we have

$$\frac{k(3e)e}{r_n^2} = \frac{(208m)v_n^2}{r_n} \tag{1}$$

For angular momentum, we have

$$(208m)v_n r_n = \frac{nh}{2\pi} \tag{2}$$

Substituting the value of $v_n$ from (2) in (1),

$$\frac{k(3e)e}{r_n^2} = \frac{208m}{r_n} \times \frac{n^2h^2}{4\pi^2 r_n^2 (208m)^2}$$

This gives the radius of the $n$th Bohr orbit

$$r_n = \frac{n^2h^2}{4\pi^2 \cdot k3e^2 \cdot (208m)}$$

$$= \frac{n^2(6.625 \times 10^{-34})^2}{4\pi \times 9 \times 10^9 \times 3(1.6 \times 10^{-19})^2 \times 208 \times 9.1 \times 10^{-31}}$$

$$= n^2 \times 8.426 \times 10^{-14} \text{ m}$$

(ii) The first hydrogen orbit has a radius of $0.51 \times 10^{-10}$ m. To find the value of $n$ for which the radius of the orbit is approximetly the same as that of the first Bohr orbit for the hydrogen atom, we can write

$$n^2 \times 8.426 \times 10^{-14} = 0.51 \times 10^{-10}$$

or
$$n^2 = \frac{0.51 \times 10^{-10}}{8.426 \times 10^{-14}}$$

$\Rightarrow$
$$n = 24.6$$

As $n$ can be a whole number,

therefore $\quad n = 25$

(iii)
$$E_n = -\frac{k(3e)e}{r_n} + \frac{1}{2} m v_n^2$$

$$= -\frac{3ke^2}{r_n} + \frac{1}{2}\frac{k3e^2}{r_n} \times \frac{1}{208}$$

$$= \frac{3ke^2}{r_n}\left(\frac{1}{416} - 1\right)$$

$$= -\frac{415 \times 3}{416}\frac{ke^2}{r_n}$$

$$= -\frac{415 \times 3}{416} ke^2 \frac{4\pi^2 \cdot k \cdot 3e^2 \cdot (208m)}{n^2h^2}$$

$$= -\frac{2\pi^2 mk^2e^4}{n^2h^2} \times 3735$$

$\therefore$
$$E_{n1} - E_{n2} = \frac{2\pi^2 mk^2e^4}{h^2} \times 3735 \times \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

or
$$\frac{hc}{\lambda} = \frac{2\pi^2 mk^2e^4}{h^2} \times 3735\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

or
$$\frac{1}{\lambda} = \frac{2\pi^2 mk^2e^4}{ch^3} \times 3735\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$= R \times 3735\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

(when $R$ is the Rydberg constant, $1.97 \times 10^7\ m^{-1}$).

For the jump from the third to the first orbit,

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times 3735\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 1.079 \times 10^7 \times 3735 \times \frac{8}{9}$$

$$= 3582 \times 10^7\ \text{m}^{-1}$$

or
$$\lambda = 2.792 \times 10^{-11}\ \text{m}$$

## PART B

**Ans. 10**

(i) (D) Let the maximum velocity be $a\omega$

Now $$a_1\omega_1 = a_2\omega_2$$

or $$\frac{\omega_1}{\omega_2} = \frac{a_2}{a_1} \qquad \text{(i)}$$

Also $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k}}$$

or $$\frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{M}{k_1}}$$

$$\frac{2\pi}{\omega_2} = 2\pi\sqrt{\frac{M}{k_2}}$$

$\therefore$ $$\frac{\omega_1}{\omega_2} = \sqrt{\frac{k_1}{k_2}} \qquad \text{(ii)}$$

From (i) and (ii),

$$\frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

(ii) (B)

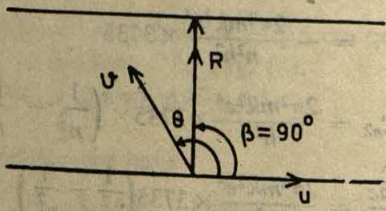



Fig. 15

$v = 5$ km/h, $\qquad R = \dfrac{1 \text{ km}}{15 \text{ min}} = 4$ km/h

$u = ?$

$$R^2 = u^2 + v^2 = 2uv\cos\theta \qquad \text{(i)}$$

$$\tan\beta = \frac{v\sin\theta}{u + v\cos\theta}$$

or $$u + v\cos\theta = 0 \qquad (\because \quad \beta = 90°) \qquad \text{(ii)}$$

From (i) and (ii),

$$R^2 = u^2 + v^2 + 2u(-u) = v^2 - u^2$$

or $$(4)^2 = (5)^2 - u^2$$

or $$u^2 = (5)^2 - (4)^2 = 9$$

$\therefore$ $$u = 3 \text{ km/h}$$

(iii) (C) $$y_1 = a\cos(kx - \omega t)$$

$$y_2 = ?$$

For the superposition of $y_1$ on $y_2$ to form a stationary wave, $y_2$ should move in the $(-x)$ direction. Choices (B) and (D) are ruled out. Also, since it is difficult to combine a cos wave with a sin wave, choice (A) is also not possible. (C) is, therefore, the correct choice.

$$\begin{aligned} y &= a\cos(kx - \omega t) - a\cos(kx + \omega t) \\ &= a[\cos(kx - \omega t) - \cos(kx + \omega t)] \\ &= a[2\sin(kx)\sin(\omega t)] \\ &= A\sin\omega t \end{aligned}$$

where amplitude $A = 2a\sin kx$; $A = 0$ at $x = 0$. Thus there is a node at $x = 0$ as desired in choice (C).

(iv) (C) $\gamma_1 = 5/3$, $\gamma_2 = 7/5$

$$\text{Ratio of the two gases} = 1:1$$

$$\therefore \quad \text{Resultant} = \frac{(5/3) + (7/5)}{2} = \frac{25 + 21}{30} = \frac{46}{30} = 1.53$$

(v) (C) Intensity $\propto$ (Amplitude)$^2$

Suppose $$I_1 = I = kA^2$$

Thus $$I_2 = 4I = k(2A)^2$$

The amplitudes of the two combining beams are $A$ and $2A$ respectively.

$$A_{max} = 2A + A = 3A$$

$$A_{min} = 2A - A = A$$

$$\therefore \quad I_{max} = k(3A)^2 = 9kA^2$$

$$I_{min} = kA^2$$

$$\frac{I_{max}}{I_{min}} = \frac{9kA^2}{kA^2} = \frac{9}{1}$$

(vi) (D) Copper is a good conductor while germanium is a semiconductor. On cooling, the resistance of copper decreases while that of germanium increases.

**Ans. 11**

(i) (B) Suppose the volume of homogeneous sphere is $V$. It floats half immersed in mercury and half in oil. The weight of mercury displaced will act upwards while weight of oil displaced will act downwards.

$$\text{Resultant upthrust} = \frac{V}{2}\times 13.6\times g - \frac{V}{2}\times 0.8\times g$$

$$= \frac{V}{2}\times 12.8\times g$$

This must be equal to the weight of the sphere.

Thus,

$$\frac{V}{2}\times 12.8\times g = V\times d\times g$$

(where $d$ is the density of the sphere)

or
$$g = 6.4 \text{ g/cm}^3$$

(ii) (B) For organ pipe $P_1$ closed at one end, the fundamental note $= \frac{c}{4l_1}$ (where $l_1$ is the length of $P_1$)

Higher nodes = 3 : 5 : 7, etc.

For organ pipe $P_2$ open at both ends, fundamental note $=\frac{c}{2l_2}$ (where $l_2$ is the length of $P_2$)

Higher nodes = 2 : 3 : 4 : 5, etc.

$$\text{First harmonic of } P_1 = \frac{3c}{4l_1}$$

$$\text{Third harmonic of } P_2 = \frac{4c}{2l_2}$$

$$\text{For resonance } \frac{3c}{4l_1} = \frac{4c}{2l_2}$$

or
$$\frac{l_1}{l_2} = \frac{3}{8}$$

(iii) (C) Heat flowing per second across the cylinder of radius $R$ is given by

$$\frac{K_1\times\pi R^2\times(\theta_1 - \theta_2)}{x} \tag{i}$$

where $\theta_1$ and $\theta_2$ are the temperatures at the two ends ($\theta_1 > \theta_2$) and $x$ is the distance between them.

Heat flowing per second across the cylinderical shell of outer radius $2R$ is given by

$$\frac{K_2\times\pi[(2R)^2 - R^2]\times(\theta_1 - \theta_2)}{x} \tag{ii}$$

Total heat flowing per second

$$= \frac{K_1\times\pi R^2\times(\theta_1 - \theta_2)}{x} + \frac{K_2\times\pi[(2R)^2 - R^2]\times(\theta_1 - \theta_2)}{x}$$

$$= \frac{(\theta_1 - \theta_2)}{x}\times\pi(R^2K_1 + 3R^2K_2) \tag{iii}$$

Also, total heat flowing per second across the system is given by

$$\frac{K\pi(2R)^2\times(\theta_1 - \theta_2)}{x} \tag{iv}$$

where $K$ is the equivalent thermal conductivity of the system.

From (iii) and (iv),

$$\frac{(\theta_1 - \theta_2)}{x} \times \pi(R^2K_1 + 3R^2K_2) = \frac{K\pi(2R)^2 \times (\theta_1 - \theta_2)}{x}$$

or $$4K = K_1 + 3K_2$$

or $$K = \frac{K_1 + 3K_2}{4}$$

(iv) (C) The charge $q$ and potential difference $V$ being the same for the two particles $x$ and $y$, they possess equal energies.

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 \quad \text{(i)}$$

On entering the magnetic field, particles describe circular paths of radii $R_1$ and $R_2$ given as

$$\frac{m_1v_1^2}{R_1} = Bqv_1 \qquad \frac{m_2v_2^2}{R_2} = Bqv_2 \quad \text{(ii)}$$

From (i) and (ii),

$$\frac{R_1}{R_2} = \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}}$$

or $$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

(v) (D)

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Differentiation gives

$$0 = -\frac{dv}{v^2} = -\frac{du}{u^2}$$

$\Rightarrow$ $$dv = -\frac{v^2}{u^2}du$$

But $$\frac{v}{u} = \frac{f}{u-f}$$

$\therefore$ $$dv = -\left(\frac{f}{u-f}\right)^2 b$$

(vi) (D) From the relation

$$\lambda_{min} = \frac{hC}{V \times e}$$

it is clear that the minimum wavelength decreases when the potential difference is increased.

(vi) (B)

$$t_{1/2} = 2 \text{ h}$$

In one half life (i.e. 2 h), intensity is $\frac{64}{2} = 32$ times the safe level.

Therefore, in six half-lives (i.e. $2 \times 6 = 12$ h), the intensity reaches the safe level.

(viii) (B), (D)

$p$ − type semiconductors are obtained by adding trivalent impurities to pure silicon.

**Ans. 12**

(i) − 3 in mass, − 2 in length, + 4 in time and + 4 in charge

$$X = 3YZ^2$$

$$\text{Dimensions of } X = [C] = \frac{[Q]}{[V]}$$

$$= \frac{Q}{[\text{Energy/charge}]}$$

$$= \frac{Q \times Q}{ML^2T^{-2}}$$

$$= M^{-1}\,L^{-2}\,T^{+2}\,Q^{+2}$$

$$\text{Dimensions of } Z = [B] = \frac{[F]}{[I \cdot L]}$$

$$= \frac{MLT^{-2}}{QT^{-1}\,[L]} = MT^{-1}Q^{-1}$$

$$\therefore \text{ Dimensions of } Y = \frac{[X]}{[Z]^2}$$

$$= \frac{M^{-1}L^{-2}T^{+2}Q^{+2}}{M^{+2}T^{-2}Q^{-2}}$$

$$= M^{-3}L^{-2}T^{+4}Q^{+4}$$

(ii) $$2\sqrt{\frac{G}{d}(M_1 + M_2)}$$

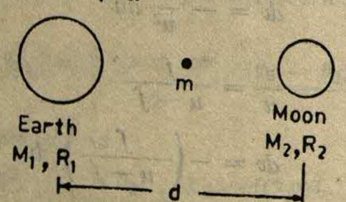



Fig. 16

Gravitational potential energy due to the earth

$$= -\frac{GM_1\,m}{d/2} = -\frac{2GM_1m}{d}$$

Gravitational potential energy due to the moon

$$= -\frac{GM_2m}{d/2} = -\frac{2GM_2m}{d}$$

Total potential energy due to both the earth and the moon

$$= -\frac{2Gm\,(M_1 + M_2)}{d}$$

Total potential energy at infinity = 0

Difference $$= 0 - \left[\frac{-2GM\,(M_1 + M_2)}{d}\right]$$

$$= \frac{2Gm}{d}\,(M_1 + M_2)$$

This much work is to be done to shift the particle to infinity. For this, the particle should have the velocity $v$ given by the relation

$$\frac{1}{2}\,mv^2 = \frac{2Gm}{d}\,(M_1 + M_2)$$

$\Rightarrow$ $$v = 2\sqrt{\frac{G}{d}\,(M_1 + M_2)}$$

(iii) $$\omega_0\sqrt{\frac{M}{(M + 6m)}}$$

Initial moment of inertia of the rod about the axis of rotation

$$= \frac{ML^2}{12}$$

Initial angular velocity $= \omega_0$

$\therefore$ Kinetic energy of rotation $$= \frac{1}{2} \times \frac{ML^2}{12} \times \omega_0^2$$

$$= \frac{ML^2\omega_0^2}{24}$$

Final moment of inertia when the beads reach the ends of rod

$$= \frac{ML^2}{12} + 2 \times m \times \left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{2}$$

If the final angular velocity is $x$, the final kinetic energy of rotation will be

$$\frac{1}{2}\left(\frac{ML^2}{12} + \frac{ML^2}{2}\right)x^2$$

From the law of conservation of energy,

$$\frac{1}{2}\left(\frac{ML^2}{12} + \frac{mL^2}{2}\right)x^2 = \frac{ML^2\omega_0^2}{24}$$

$\Rightarrow$ $$x = L\omega_0\sqrt{\frac{M}{12} \times \frac{1}{\left(\frac{ML^2}{12} + \frac{mL^2}{2}\right)}}$$

$$= \omega_0 \sqrt{\frac{M}{(M + 6m)}}$$

(iv) $$\frac{1}{2} MR^3 A \omega^2$$

$x = A \cos(\omega t)$ discribes a simple harmonic motion.
Maximum acceleration $= A\omega^2$
Maximum angular acceleration $= R \cdot A\omega^2$
$\therefore$ Maximum torque acting on the cylinder during its motion

$$= I \cdot R \cdot A\omega^2$$

$$= \left(\frac{1}{2} MR^2\right) \cdot RA\omega^2$$

$$= \frac{1}{2} MR^3 A\omega^2$$

(v) $$\frac{Mg}{3AK}$$

Downward force due to mass $M = Mg$

Downward pressure $= \dfrac{Mg}{A}$

From Pascal's law, the pressure of the liquid is transmitted equally in all directions. Therefore, excess pressure on the surface $= \dfrac{Mg}{A}$

Bulk modulus $$K = \frac{\delta P}{-\delta V/V}$$

$$= \frac{\dfrac{Mg}{A}}{\dfrac{\frac{4}{3}\pi[R^3 - (R - \delta R)^3]}{\frac{4}{3}\pi R^3}}$$

$$= \frac{Mg}{A} \frac{R^3}{R^3\left[1 - \left(1 - \frac{\delta R}{R}\right)^3\right]}$$

$$= \frac{Mg}{A} \frac{1}{\left(1 - 1 + 3\frac{\delta R}{R}\right)}$$

$$= \frac{Mg}{A} \times \frac{1}{3} \times \frac{R}{\delta R}$$

or $$\frac{\delta R}{R} = \frac{Mg}{3AK}$$

(vi) $$\frac{3V}{(K+2)}$$

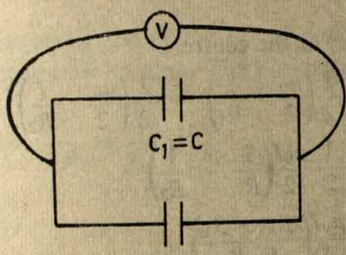



Fig. 17

$$q_1 = CV$$

$$q_2 = 2CV$$

Total charge $= q_1 + q_2$

$$= CV + 2CV = 3CV$$

On completely filling the capacitor $C_1$ with a material of dielectric constant $K$, its capacitance becomes $KC$

Total capacitance $= KC + 2C = C(K+2)$

Total charge $= 3CV$

$\therefore$ Potential difference across the capacitors

$$= \frac{3CV}{C(K+2)} = \frac{3V}{(K+2)}$$

(vii) 20.1 T

Current = charge flowing per second

$$= 10^{16} \times 1.6 \times 10^{-19} \text{ A}$$

$$= 1.6 \times 10^{-3} \text{ A}$$

Magnetic moment associated with the orbital motion of the electron

$$= B = \frac{\mu_0}{2} \times \frac{I}{r} \text{ T}$$

$$= \frac{4\pi \times 10^{-7}}{2} \times \frac{1.6 \times 10^{-3}}{0.5 \times 10^{-10}}$$

$$= 20.1 \text{ T}$$

(viii) $\frac{\mu_0 I}{4} \left( \frac{R_2 - R_1}{R_1 R_2} \right)$

Magnetic field at $C$ due to bigger semicircular coil

$$= \frac{1}{2} \left( \frac{\mu_0}{2} \times \frac{I}{R_2} \right)$$

(increased from the plane of the coils)

Magnetic field at $C$ due to smaller semicircular coil

$$= \frac{1}{2} \left( \frac{\mu_0}{2} \times \frac{I}{R_1} \right)$$

(outward from the plane of coils)

∴ Net field (outward) at the centre $C$

$$= \frac{1}{2}\left(\frac{\mu_0}{2} \times \frac{I}{R_1}\right) - \frac{1}{2}\left(\frac{\mu_0}{2} \times \frac{I}{R_2}\right)$$

$$= \frac{1}{2}\,\frac{\mu_0 I}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{\mu_0 I}{4}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

(ix) 23.6 $MeV$

Total binding energy of one deuteron ($_1H^2$) nucleus

$$= 2 \times 1.1 = 2.2 \text{ MeV}$$

Total binding energy of one helium ($_2He^4$) nucleus $= 4 \times 7.0 = 28.0$ MeV

∴ Energy released when two deuterons fuse to form a helium nucleus

$$= 28.0 - (2 \times 2.2) = 23.6 \text{ MeV}$$

(x) In the forward bias arrangement of a $p$–$n$ junction rectifier, the $p$ end is connected to the *positive* terminal of the battery and the direction of the current is from *p-part* to *n-part* in the rectifier.

**Ans. 13** (i) *False*

$$E = (K\sigma)\, AT^4$$

where $E$ is the energy radiated per second, $K$ is relative emissivity, $\sigma$ is Stefan's constant, $A$ in the area of the body and $T$ is the temperature in kelvin.

Energy radiated per second by first sphere

$$= E_1 = (K\sigma)\, 4\pi\, (1)^2 \times (4000)^4 = (K\sigma)\, 1024\pi \times 10^{12}$$

Energy radiated per second by the second sphere

$$= E_2 = (K\sigma)\, 4\pi\, (4)^2 \times (2000)^4 = (K\sigma)\, 1024\pi \times 10^{12}$$

$$\therefore \quad E_1 = E_2$$

Statement is *false*. Energy radiated by the first sphere is equal to that by the second sphere.

(ii) *True*

The particle at $A(1, 0)$ can move along the circle without loss or gain of any energy as the circle forms an equipotential surface and $A$ lies on it.

The given statement is *true* as the force acts as a centripetal force.

(iii) *False*

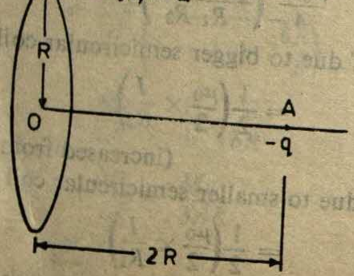



Fig. 18

The force on $A$ is inversely proportional to the distance $OA$. So it is not a case of simple harmonic motion. The given statement in *false*.

(iv) *True*
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$=-\frac{1}{15}+\frac{1}{30}=-\frac{1}{30}$$

or
$$F=-30 \text{ cm}$$

Thus, the combination acts as a diverging system. As the deviation for violet is always more than that for red, one will see coloured patterns with violet colour at the outer edge. The statement is *true*.

## CHEMISTRY

### PART A

**Ans. 1**

(a) Amount of water in syrup $= 214.2 - 34.2$

$= 180.0$ g

(i) $\therefore$ Molal concentration $=\dfrac{34.2\times 1000}{180\times 342}=0.556$

($\because$ mol. wt. of $C_{12}H_{22}O_{11} = 144 + 22 + 176 = 342$)

(ii) Mole fraction $\dfrac{34.2/342}{(34.2/342)+(180/18)}$

($\because$ mol. wt. of $H_2O = 2 + 16 = 18$)

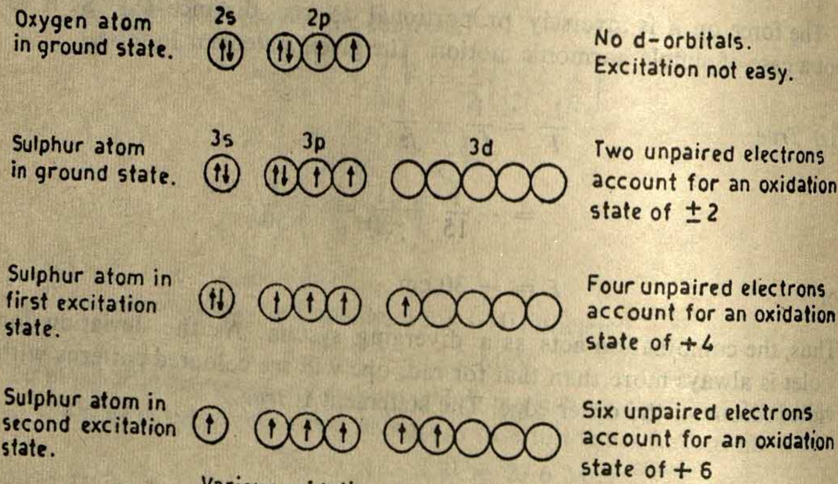
$$=\frac{0.1}{0.1+10}=0.0099$$

(b) (i) The colour of $Hg_2Cl_2$ changes from white to black when treated with ammonia due to the formation of metallic mercury.

$$Hg_2Cl_2 + 2NH_3 \rightarrow Hg(NH_2)Cl + Hg + NH_4Cl$$

(ii) Unlike in formic acid, formate ion is a resonance hybrid of two equivalent structures having equal bond lengths.

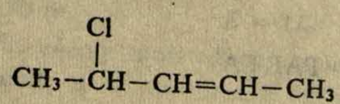
(iii) In contrast to oxygen, the octet can be expanded in sulphur using *d* orbitals.



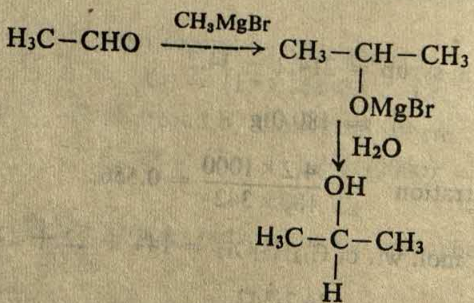

Various oxidation states exhibited by sulphur

Fig. 19

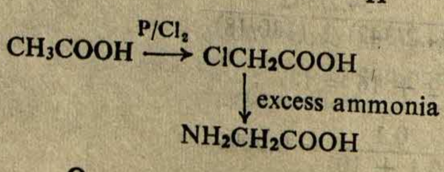
(c)



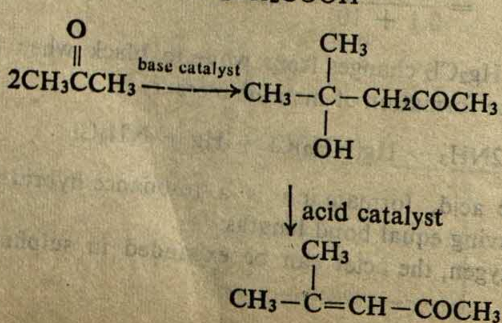
(d) (i)

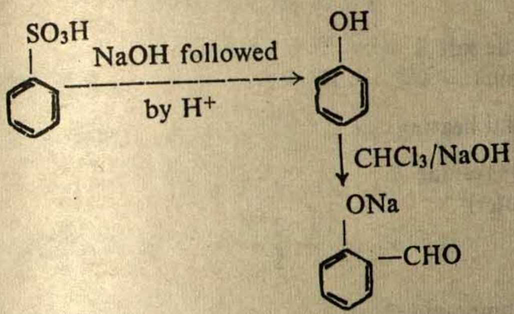


(ii)



(iii)



(iv) 

**Ans. 2**

(a) Let the concentration of the $H^+$ ions at the positive electrode be $x$ mol litre$^{-1}$.

$$E_2 = E^\circ + 0.0591 \log x$$
$$E_1 = E^\circ + 0.0591 \log 10^{-6}$$
$$E_2 - E_1 = 0.0591 (\log x - \log 10^{-6})$$
$$0.118 = 0.0591 (\log x - \log 10^{-6})$$

or
$$x = 10^{-4}$$

Concentration of $H^+$ ions $= 10^{-4}$ mol litre$^{-1}$

(b) $N_2O_4 \rightleftharpoons 2NO_2$

Let $\alpha$ be the degree of dissociation at equilibrium,

$$K_p = \frac{(2\alpha)^2}{(1-\alpha)}$$

(i) At 37°C and 1 atm pressure,

$$K_p = \frac{(0.5/1.25)^2}{(0.75/1.25)} = 0.267$$

(ii) $K_p = 0.267 = \dfrac{[2\alpha/(1+\alpha)\times 0.1]^2}{(1-\alpha)/(1+\alpha)\times 0.1}$

This gives $\alpha = \pm 0.65$ at 0.1 atm. and at 37°C

$N_2O_4$ is, therefore, 65% dissociated.

(c)
$$[H^+] = 10^{-pH}$$
$$= 10^{-8.5} = 3.2\times 10^{-9} \text{ mol litre}^{-1}$$

$$HCN \rightleftharpoons H^+ + CN^-$$

$$K_{diss} = \frac{[H^+][CN^-]}{[HCN]} = 4.1\times 10^{-10} \quad \text{(given)}$$

$\therefore$
$$\frac{[CN^-]}{[HCN]} = \frac{4.1\times 10^{-10}}{3.2\times 10^{-9}} = 0.128$$

Let [HCN] be $x$.

$\therefore$
$$[CN^-] = 0.01 - x$$
$$\frac{[CN^-]}{[HCN]} = \frac{(0.01 - x)}{x} = 0.128$$

or
$$x = 0.0089$$

Gram-mole of HCl required $= 0.0089$

**Ans. 3**

(a) Hydrated metallic salt $A$
(light green colour)

$\downarrow$ careful heating

White anhydrous residue $B$
(soluble in water)

$\downarrow$ NO

Dark brown compound C

$\downarrow$ Strong heating

Brown residue $D$ + $E\uparrow + F\uparrow$
(mixture of gases)

$\downarrow$ acidified $KMnO_4$ — Pink colour discharged

$\downarrow$ acidified $BaCl_2$ — White ppt.

The sequence of reactions shows:

$A = FeSO{\cdot}nH_2O$ or $FeSO_4{\cdot}7H_2O$

$B = FeSO_4$

$C = FeSO_4{\cdot}NO$ or $[Fe(H_2O)_5NO]SO_4$

$D = Fe_2O_3$

$E = SO_2$

$F = SO_3$

(b) For first order reaction,

$$k = 0.693/t_{1/2}$$

Reaction rate constant at 27°C,

$$k_1 = 0.693/30$$
$$= 0.0231\ \text{min}^{-1}$$

Reaction rate constant at 47°C,

$$k_2 = 0.693/10$$
$$= 0.0693\ \text{min}^{-1}$$

Energy of activation, $E$ can be evaluated using the relationship.

$$\log k_1/k_2 = \frac{-E}{2.303 \times R}\left(\frac{T_2 - T_1}{T_1 T_2}\right)$$

$$\log \frac{0.0231}{0.0693} = \frac{-E}{2.303 \times 8.314}\left(\frac{320 - 300}{320 \times 300}\right)$$

$$-0.477 = \frac{-E}{2.303 \times 8.314}\left(\frac{320 - 300}{320 \times 300}\right)$$

This gives $E = 43.839\ \text{kJ mol}^{-1}$

(c) No. of moles of $H_2$ reacted $= \dfrac{67.2}{22.4} = 3.0$

$$1 \text{ mole of } H_2 \equiv 2 \text{ F}$$

$\therefore$ $3 \text{ moles of } H_2 \equiv 3\times 2 = 6 \text{ F}$

No. of coulombs $= 6\times 96500 = 579000$

Current produced $= \dfrac{579000}{15\times 60} = 643.33 \text{ A}$

$$2\text{F} \equiv 63.5 \text{ g Cu}$$

$\therefore$ Cu deposited $= \dfrac{63.5\times 6}{2} = 190.5 \text{ g}$

**Ans. 4**

(a) Reaction involved is

$$2Al + Fe_2O_3 \rightarrow Al_2O_3 + 2Fe + \text{Heat}$$

Heat of rection $= 399 - 199$

$= 200 \text{ kcal}$

Total weight of reactants

$= \text{Mol}\cdot\text{wt}\cdot \text{ of } Fe_2O_3 + (2\times \text{At}\cdot\text{wt}\cdot \text{ of Al})$

$= 160 + (2\times 27) = 214$

Heat evolved (fuel value) per gram of reactants

$= \dfrac{200}{214} = 0.935 \text{ kcal}$

Total volume of reactants $= \dfrac{160}{5.2} + \dfrac{54}{2.7}$

$= 50.77 \text{ cc}$

Heat evolved (fuel value) per cc

$= \dfrac{200}{50.77} = 3.94 \text{ kcal}$

(b)
$$4Fe^{3+} + N_2H_4 \rightarrow N_2 + 4Fe^{2+} + 4H^+]\times 5$$
$$MnO_4^- + 5Fe^{2+} + 8H^+ \rightarrow Mn^{2+} + 5Fe^{3+} + 4H_2O]\times 4$$

---

$$4MnO_4^- + 5N_2H_4 + 32H^+ \rightarrow 4Mn^{2+} + 20H^+ + 16H_2O$$

$4 \text{ moles } MnO_4^- \equiv 5 \text{ moles } N_2H_4 \equiv 5 \text{ moles } N_2H_6SO_4$

$\equiv 5\times 130 \text{ g } N_2H_6SO_4$

(mol. wt. of $N_2H_6SO_4 = 130$)

$= 650 \text{ g } N_2H_6SO_4$

$4000 \text{ ml of 1M } MnO_4^- \equiv 650 \text{ g } N_2H_6SO_4$

$20 \text{ ml of M/50 } MnO_4^- \equiv \dfrac{650\times 20}{4000\times 50} = 0.065 \text{ g}$

$\therefore$ $N_2H_6SO_4/\text{litre} = \dfrac{0.065\times 1000}{10} = 6.5 \text{ g}$

(c) (i) $2IO_3^- + 5HSO_3^- \rightarrow 2SO_4^{2-} + 3HSO_4^- + I_2 + H_2O$

(ii) $2Ag_2S + 2Zn + 8CN^- + O_2 + 2H_2O$

$\rightarrow 4Ag + 2[Zn(CN)_4]^{2-} + 4OH^- + 2S$

(iii) $4P + 10HNO_3 + H_2O \rightarrow 4H_3PO_4 + 5NO + 5N_2O_2$

(iv) $NaCl + NH_4OH + CO_2 \rightarrow NaHCO_3 + NH_4Cl$

**Ans. 5**

(a) (i) Baeyer's unsaturation test with $KMnO_4$ solution: Cyclohexene will decolorise $KMnO_4$ solution while cyclohexene will not.

Or

Bromine-water test: Cyclohexene will decolorise bromine-water while cyclohexane will not.

(ii) Hinsberg test: Ethylamine reacts with benzene sulphonyl chloride to form an alkali *soluble* product whereas diethylamine reacts with benzene sulphonyl chloride to form an alkali *insoluble* product.

(b) *Empirical formula of A*

| | | | *Gram atom ratio* |
|---|---|---|---|
| C | 49.32 | 49.32/12 = 4.11 | 4.11/1.37 = 3 |
| H | 9.59 | 9.59/1 = 9.59 | 9.59/1.37 = 7 |
| N | 19.18 | 19.18/14 = 1.37 | 1.37/1.37 = 1 |
| O | 21.91 | 21.91/16 = 1.37 | 1.37/1.37 = 1 |

Therefore, the empirical formula of *A* is $C_3H_7NO$

$$\underset{(A)}{C_3H_7NO} \xrightarrow{NaOH} NH_3\uparrow + \text{Salt} \xrightarrow{H^+} \underset{(B)}{\text{Nitrogen free acid}}$$

Since *A* gives $NH_3$ on boiling with NaOH, it may be an amide. The silver salt of *B* contains 59.67% Ag

$$\text{Equivalent weight of acid} = \frac{100 \times 108}{59.67} - 107$$

$$= 74$$

$$\text{Mol. wt. of the monobasic acid} = 74 \times 1$$

$$= 74$$

Mol. formula of acid $= C_3H_6O_2$

Mol. formula of amide $= C_3H_7NO$

the structure of amide (*A*) is $C_2H_5{-}\underset{\underset{O}{\|}}{C}{-}NH_2$ and

that of acid (*B*) is $C_2H_5{-}\underset{\underset{O}{\|}}{C}{-}OH$

(c) Hydrocarbon *A* ($C_5H_{10}$) $\xrightarrow[\text{hydrogenation}]{\text{catalytic}}$ $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-CH_3$

2—Methyl butane

$$\downarrow HB_2$$

$$B \xrightarrow{Ag(OH)} C,\ C_5H_{12}O,\ \text{alcohol}$$

$$\downarrow \text{oxidation}$$

$$D,\ \text{ketone}$$

Hydrogenation of *A* ($C_5H_{10}$) gives 2-methyl butane. Hence *A* should contain one double bond. The possible double bond isomers for *A* are:

$$H_2C=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_2-CH_3 \qquad H_3C-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-CH_3 \qquad H_3C-\overset{\displaystyle H}{\overset{|}{\underset{\displaystyle CH_3}{\underset{|}{C}}}}-CH=CH_2$$

(i) (ii) (iii)

Since the alcohol *C* on oxidation gives a ketone, it must be a secondary alcohol. Therefore, the corresponding bromide *B* should also be a secondary bromide. Since a secondary bromide is formed by the addition of HBr according to Markownikoff's rule, it can arise only from structure (iii). Therefore,

*A* is $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH=CH_2$

*B* is $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle Br}{\underset{|}{CH}}-CH_3$

*C* is $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$

*D* is $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3$

Reactions involved are:

$$\underset{(A)}{CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH=CH_2} \xrightarrow{H_2} CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-CH_3$$

$$\downarrow HBr$$

$$\underset{(B)}{CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle Br}{\underset{|}{CH}}-CH} \xrightarrow{Ag(OH)} \underset{(C)}{CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3}$$

$$\downarrow \text{oxidation}$$

$$\underset{(D)}{CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3}$$

## PART B

**Ans. 6**

| | | | |
|---|---|---|---|
| (i) A | (ii) C | (iii) B | (iv) C |
| (v) A | (vi) C | (vii) D | (viii) A |
| (ix) A | (x) B | (xi) C | (xii) B |
| (xiii) D | (xiv) A | (xv) D | (xvi) A |
| (xvii) D | (xviii) B | (xix) B | (xx) B |

**Ans. 7**

(a) (i) B, C (ii) B, D (iii) B, C
(iv) A, D (v) A, B, C, D

(b) (i) Heisenberg, de Broglie
(ii) sintering, smelting
(iii) non-superimposable, enantiomers
(iv) $FeSO_4 \cdot 7H_2O$, $ZnSO_4 \cdot 7H_2O$
(v) Hypobromous, bromite

**Ans. 8**

(i) Animal charcoal/C, $Ca_3(PO_4)_2$/sugar refining
Invar/Fe, Ni/watch spring
Nichrome/Co, Ni/heating element
Rydberg/$cm^{-1}$/109677
Stainless steel/Fe, Cr, Ni, C/cutlery
Boltzmann/kJ $deg^{-1}$/$1.3805 \times 10^{-26}$
(ii) Friedel-Crafts/Lewis acid/anhydrous $AlCl_3$
Fermentation/yeast/ethanol
Dehydrohalogenation/alcoholic alkali/alkenes
Sandmeyer/cuprous chloride/chlorobenzene
Saponification/oil/soap

**Ans. 9**

(a) (i) $HOCl < HOClO < HOClO_2 < HOClO_3$
(ii) isobutane < n-butane < n-butyl chloride < n-butanol
(iii) $F_2 < Cl_2 < O_2 < N_2$
(iv) chlorobenze < benzene < toluene < methoxybenzene
(v) $SiO_2 < CO_2 < N_2O_5 < SO_3$
(vi) aniline < N-methylaniline < methylaniline < dimethylaniline

(b) (i) C
(ii) C

# MATHEMATICS

## PART A

**Ans.**

1. (a) $$|f(x) - f(y)| \leqslant |x - y|^3$$

$$\Rightarrow \quad \left|\frac{f(x) - f(y)}{x - y}\right| \leqslant |x - y|^2$$

$$\Rightarrow \quad \lim_{y \to x} \left|\frac{f(x) - f(y)}{x - y}\right| \leqslant \lim_{y \to x} |x - y|^2$$

$$\Rightarrow \quad |f'(x)| \leqslant 0 \Rightarrow |f'(x)| = 0$$

$$\Rightarrow \quad f(x) \text{ is a constant}$$

(b) Required probability

$= 1 - p$ (value of coins is greater than or equal to Rs. 1.50)

$$= 1 - \frac{2C_2 \times 5C_3 \times NC_0 + 2C_2 \times 5C_2 \times NC_1 + 2C_1 \times 5C_4 \times NC_0}{{}^{n+7}C_5}$$

$$= 1 - \frac{20\, NC_0 + 10\, NC_1}{{}^{n+7}C_5}$$

**Ans.**

2. (a) We have $\cot\alpha - \tan\alpha = \dfrac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha} = 2\cot 2\alpha$

Similarly, $\cot 2\alpha - \tan 2\alpha = 2\cot 4\alpha$

$\cot 4\alpha - \tan 4\alpha = 2\cot 8\alpha$

so that
$$\begin{aligned}\cot\alpha &= \tan\alpha + 2(\tan 2\alpha + 2\cot 4\alpha) \\ &= \tan\alpha + 2\tan 2\alpha + 4(\tan 4\alpha + 2\cot 8\alpha) \\ &= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha\end{aligned}$$

(b)

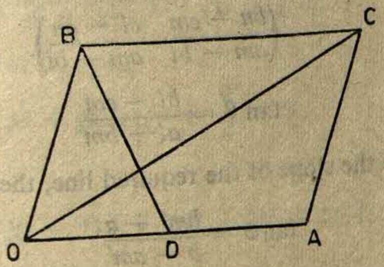



Fig. 20

Suppose $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of $A$, $B$ and $C$ respectively w.r.t. O so that $\vec{c} = \vec{a} + \vec{b}$.
Suppose $BD$ and $CO$ divide each other in the ratio $m : 1$ and $n : 1$.
Equating the position vectors of the point of intersection

$$\frac{m\frac{\vec{a}}{2} + \vec{b}}{m+1} = \frac{n\vec{o} + \vec{c}}{n+1}$$

$$\Rightarrow \quad \frac{m\frac{\vec{a}}{2} + \vec{b}}{m+1} = \frac{\vec{a} + \vec{b}}{n+1}$$

$$\Rightarrow \quad \frac{m}{2(m+1)} = \frac{1}{n+1} \text{ and } \frac{1}{m+1} = \frac{1}{n+1}$$

$$\Rightarrow \quad m = n = 2$$

Hance, the required ratio is 2 : 1

**Ans. 3.**

If
$$x^2 + 4x + 3 \geqslant 0 \qquad \text{(i)}$$
then the given equation is
$$x^2 + 6x + 8 = 0$$
$$\Rightarrow \quad x = -2 \text{ or } x = -4$$
But only $x = -4$ satisfies (i)
If
$$x^2 + 4x + 3 < 0 \qquad \text{(ii)}$$
then the given equation is
$$x^2 + 2x - 2 = 0$$
$$\Rightarrow \quad x = -1 - \sqrt{3} \quad \text{or} \quad x = -1 + \sqrt{3}$$
But only $x = -1 - \sqrt{3}$ satisfies (ii)
Hence $x = -4$ or $x = -1 - \sqrt{3}$ is the required solution.

**Ans. 4.**

The point of intersection of $L_1$ and $L_2$ is
$$\left(\frac{bn - cm}{am - bl}, \frac{cl - an}{am - bl}\right)$$
and
$$\tan\theta = \frac{bl - am}{al + bm} \qquad \text{(i)}$$
Let $m'$ be the slope of the required line, then
$$\tan\theta = \frac{bm' + a}{b - am'} \qquad \text{(ii)}$$
Equating (i) and (ii) we get

$$(bl - am)(b - am') = (bm' + a)(al + bm)$$

$$\Rightarrow \qquad m' = \frac{(a^2 - b^2)l + 2abm}{(a^2 - b^2)m - 2abl}$$

So the equation of the required line is

$$y - \frac{cl - an}{am - bl} = \frac{(a^2 - b^2)l + 2abm}{(a^2 - b^2)m - 2abl}\left(x - \frac{bn - cm}{am - bl}\right)$$

**Ans. 5.**

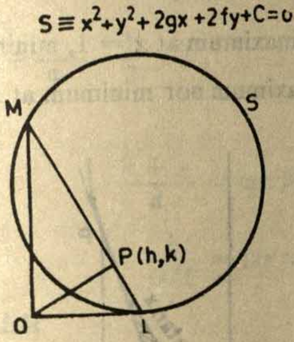



Fig. 21

Let $LM$ be the chord of the circle $S = 0$ which subtends a right angle at O and $(h, k)$ be the foot of the perpendicular from O on this chord. Then equation of $LM$ is

$$y - k = -\frac{h}{k}(x - h)$$

or $$hk + ky = h^2 + k^2$$

Joint equation of the lines $OL$ and $OM$ is, therefore

$$x^2 + y^2 + 2gx \cdot \frac{hx + ky}{h^2 + k^2} + 2fy\,\frac{hx + ky}{h^2 + k^2} + c\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

Since $$OL \perp OM,$$

$$\therefore \qquad 2 + \frac{2(gh + fk)}{h^2 + k^2} + C\,\frac{(h^2 + k^2)}{(h^2 + k^2)^2} = 0$$

so that the locus of $(h, k)$ is

$$x^2 + y^2 + gx + fy + \frac{C}{2} = 0$$

**Ans. 6.**

$$f(x) = \int_1^x [2(t - 1)(t - 2)^3 + 3(t - 1)^2 (t - 2)^2]\, dt$$

$$\Rightarrow \quad f'(x) = 2(x - 1)(x - 2)^3 + 3(x - 1)^2 (x - 2)^2$$
$$= (x - 1)(x - 2)^2 (5x - 7)$$

Now $f'(x) = 0 \Rightarrow x = 1, 2, \frac{7}{5}$

Again $f''(x) = (x-2)^2(5x-7) + 2(x-1)(x-2)(5x-7) + 5(x-1)(x-2)^2$

$$\therefore \quad f''(1) = -2 < 0$$

$$f''(2) = 0$$

$$f''\left(\frac{7}{5}\right) = \frac{18}{25} > 0$$

Hence $f(x)$ is maximum at $x = 1$, minimum at $x = \frac{7}{5}$

but neither maximum nor minimum at $x = 2$.

**Ans. 7.**

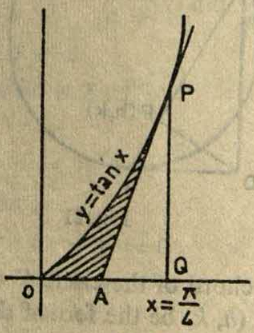



Fig. 22

Equation of the curve is $y = \tan x$

$$\frac{dy}{dx} (\text{at } x = \pi/4) = 2$$

The equation of the tangent at $P$ is

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

Required area

$$= \text{Ar. } OAP$$

$$= \text{Ar. } OPQ - \text{Ar. } APQ$$

$$= \int_0^{\pi/4} \tan x dx - \frac{1}{2} AQ \cdot PQ$$

$$= \log \sqrt{2} - \frac{1}{4}$$

**Ans. 8.**

$$= \int_0^1 \log [\sqrt{1-x} + \sqrt{1+x}]\, dx$$

$$= x \log [\sqrt{1-x} + \sqrt{1+x}] \Big|_0^1$$

$$-\int_0^1 \frac{x}{\sqrt{1-x}+\sqrt{1+x}}\left[\frac{-1}{2\sqrt{1-x}}+\frac{1}{2\sqrt{1+x}}\right]dx$$

$$= \log\sqrt{2} + \frac{1}{2}\int_0^1 \frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx$$

$$= \log\sqrt{2} + \frac{1}{2}\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\int_0^1 dx$$

$$= \log\sqrt{2} + \frac{1}{2}\sin^{-1} x\,\Big|_0^1 - \frac{1}{2}$$

$$= \log\sqrt{2} + \frac{\pi}{4} - \frac{1}{2}$$

**Ans. 9.**

We have $$\frac{h+x}{d} = \tan\beta \qquad \text{(i)}$$

and $$\frac{h}{OM} = \text{tax}\,\alpha$$

From rt. angle triangle $OLM$

$$LM = \sqrt{OM^2 - d^2}$$
$$= \sqrt{h^2\cot^2\alpha - d^2} = BD$$

From (i) $$x = \alpha\tan\beta - h$$

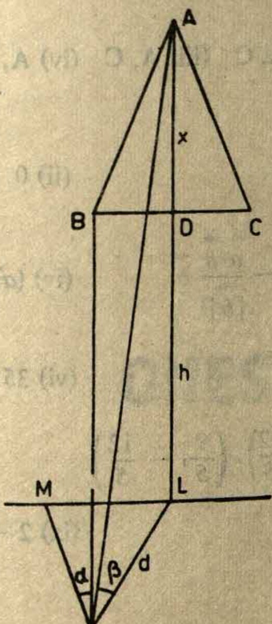



Fig. 23

Therefore area of the $\triangle ABC$

$$= \tfrac{1}{2}\text{ base} \times \text{height}$$
$$= BD \times x$$
$$= \sqrt{h^2 \cot^2 \alpha - d^2}\ (d \tan \beta - h)$$

**Ans. 10.**

Let us consider $\qquad 5\sqrt{5} - 11 = \dfrac{4}{5\sqrt{5} + 11}$

then $\qquad h = (5\sqrt{5} - 11)^{2n+1} < 1$

$\Rightarrow \qquad (5\sqrt{5} + 11)^{2n+1} - (5\sqrt{5} - 11)^{2n+1} = R - h$

$$= [R] + f - h$$

L.H.S. being clearly an integer, R.H.S must also be an integer.

$\Rightarrow \qquad f = h$

so that $\qquad Rf = (5\sqrt{5} + 11)^{2n+1}\,(5\sqrt{5} - 11)^{2n+1}$

$$= 4^{2n+1}$$

**Ans. 11.**

(i) *Flase* (ii) *True* (iii) *True* (iv) *True*

**Ans. 12.**

(i) C (ii) D (iii) D (iv) A (v) C (vi) C (vii) D (viii) D

**Ans. 13.**

(i) B, D (ii) A, C (iii) A, C (iv) A, B, C (v) A, B, C

**Ans. 14.**

(i) $\dfrac{\sqrt{3} + 1}{2}$ $\qquad$ (ii) 0

(iii) $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}\ ;\ \vec{a} - \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}$ $\qquad$ (iv) $(a^2 + b^2)\,(|z_1|^2 + |z_2|^2)$

(v) $\dfrac{n^2(n+1)}{2}$ $\qquad$ (vi) 35

(vii) $\left(-\dfrac{9}{5}, \dfrac{12}{5}\right), \left(\dfrac{9}{5}, -\dfrac{12}{5}\right)$

(viii) 4 $\qquad$ (ix) $2 - \sqrt{2}$

(x) 32/55

# QUESTION BANK

# PHYSICS

## PART A

**A-1**

Choose the correct alternative from the following:

A bomb dropped from an aeroplane explodes in air. Its total

(*a*) momentum increases
(*b*) momentum decreases
(*c*) kinetic energy increases
(*d*) kinetic energy decreases

**A-2**

Which of the following pairs do not have identical dimensions?

(*a*) work and pressure energy
(*b*) angular momentum and Planck's constant
(*c*) pressure and stress
(*d*) moment of a force and momentum

**A-3**

The dimensional formula for Planck's constant is:

(*a*) $ML^2T^{-1}$
(*b*) $ML^2T^{-2}$
(*c*) $ML^{-2}T^{-2}$
(*d*) $ML^{-2}T^{-3}$

**A-4**

Choose the physical quantity that is different from the others:

(*a*) moment of inertia
(*b*) electric current
(*c*) pressure energy
(*d*) rate of change of velocity

**A-5**

Of the following gases, velocity of sound at 30°C will be least through:

(*a*) $N_2$
(*b*) $O_2$
(*c*) $SO_2$
(*d*) $CO_2$

**A-6**

If the earth (radius $R$) stops rotating about its axis, the value of $g$ at the equator will:

(*a*) decreases by $\omega^2 R$
(*b*) increase by $\omega^2 R$
(*c*) remain the same
(*d*) increase by $\omega R$

**A-7**

A pipe closed at one end and open at the other will give:

(*a*) all the harmonics
(*b*) all even harmonics
(*c*) all odd harmonics
(*d*) none of the harmonics

**A-8**

A red postage stamp when viewed in green light appears:

(*a*) nearly black
(*b*) fully green
(*c*) blue
(*d*) none of these is correct

**A-9**

In a dc generator, the current in the armature is:

(*a*) dc
(*b*) pulsating dc
(*c*) ac
(*d*) none of these is correct

**A-10**

Choose the incorrect relationship:

(*a*) 1 MeV = 931 amu
(*b*) 1kWh = $3.6 \times 10^6$ J
(*c*) 1 V = 1J/C
(*d*) 1 dyne = $10^{-5}$ N

**A-11**

A copper disc with a hole at its centre is uniformly heated so as to produce 100°C rise in temperature. The diameter of the hole will:

(*a*) increase
(*b*) decrease
(*c*) remain unchanged
(*d*) sufficient data not given

**A-12**

The time period of a simple pendulum is $T$ s when the experiment is performed inside a stationary lift. If the lift accelerates upwards with an acceleration of $g/8$, then the period of the simple pendulum will:

(*a*) decrease
(*b*) increase
(*c*) remain the same
(*d*) first increase and then decrease

**A-13**

If you float in water with just your nose out, the average density of your body must be:

(*a*) same as that of the water
(*b*) greater than that of the water
(*c*) less than that of the water
(*d*) infinity

**A-14**

At what temperature, pressure remaining constant, will the root mean square velocity of a gas be half of its value at 0°C?

(*a*) −204°C
(*b*) −273°C
(*c*) 0°C
(*d*) 32°C

**A-15**

X-rays do not show:
(*a*) reflection and refraction
(*b*) polarisation
(*c*) diffraction
(*d*) longitudinal character

**A-16**

An air bubble in water shines because of the phenomenon of:

(*a*) dispersion
(*b*) diffraction
(*c*) reflection
(*d*) total internal reflection

**A-17**

If the momentum of a body increases by 20%, the percentage increase in its K.E is equal to:

(*a*) 44
(*b*) 66
(*c*) 88
(*d*) 20

**A-18**

A body goes from *A* to *B* with a velocity of 40 m/s and returns to *A* with a velocity of 60 m/s. The average velocity for the whole journey is:

(*a*) zero
(*b*) 50 m/s
(*c*) 48 m/s
(*d*) none of these

**A-19**

The shortest wavelength emitted from an X-ray tube depends upon:

(*a*) the current in the tube
(*b*) the voltage applied to the tube
(*c*) the nature of the gas in the tube
(*d*) the atomic number of the target material

**A-20**

If the elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be:

(*a*) 60
(*b*) 32
(*c*) 4
(*d*) 64

**A-21**

The dimensional formula for the modulus of rigidity is:

(*a*) $ML^{-1}T^{-2}$
(*b*) $ML^{-1}T^{-3}$
(*c*) $ML^{-2}T^{-2}$
(*d*) $ML^{2}T^{-2}$

**A-22**

When a spiral spring is stretched by a weight attached to it, the strain is:

(*a*) tensile
(*b*) shear
(*c*) bulk
(*d*) all the above

**A-23**

The dimensions of electromotive force in terms of charge $Q$ are:

(*a*) $ML^{2}T^{-2}Q^{-1}$
(*b*) $ML^{2}T^{-2}Q^{-2}$
(*c*) $ML^{2}T^{-2}Q^{2}$
(*d*) $MT^{-2}Q^{-2}$

**A-24**

If a graph is plotted taking temperature in °C along the *Y*-axis and in °F along the *X*-axis, which of the graphs in Fig. 1 is approximately correct?

(*a*) (A)
(*b*) (B)
(*c*) (C)
(*d*) (D)

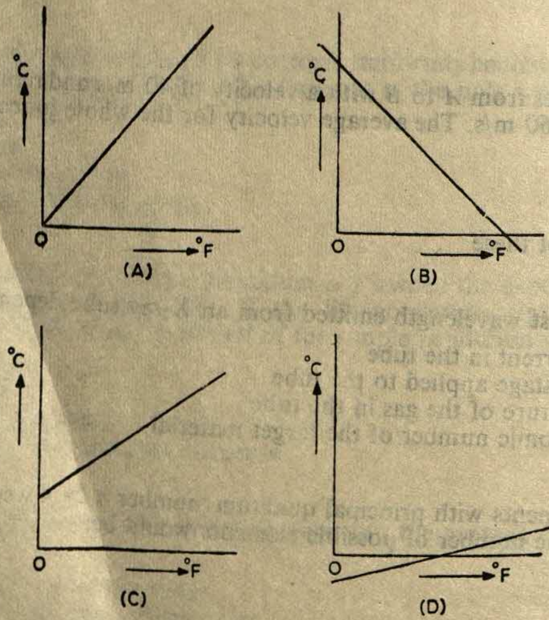



Fig. 1

A-25

The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a:

(*a*) 50 W infrared lamp
(*b*) 10 W infrared lamp
(*c*) 1 W infrared lamp
(*d*) 50 W ultraviolet lamp

A-26

An alpha particle of energy 5 Me V is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of:

(*a*) 1 Å
(*b*) $10^{-10}$ cm
(*c*) $10^{-13}$ cm
(*d*) $10^{-16}$ cm

A-27

$\beta$-rays emitted from a radioactive material are:

(*a*) electromagnetic radiations
(*b*) the electrons orbiting around the nucleus
(*c*) charged particles emitted by the nucleus
(*d*) neutral particles

A-28

What is the number of alpha particles and $\beta$-rays emitted in the following radioactive decay?

$$_{90}X^{200} \rightarrow {}_{80}Y^{168}$$

(*a*) 8 and 6
(*b*) 6 and 8
(*c*) 8 and 8
(*d*) 6 and 6

A-29

The half-life of radon is 3.8 days. The time at the end of which $(1/20)^{th}$ of the radon sample will remain undecayed is ($\log e = 0.4343$).

(*a*) 3.8 days
(*b*) 36 days
(*c*) 16.5 days
(*d*) 20 days

A-30

The plate resistance of a triode is 3 k $\Omega$. What is its mutual conductance if its amplification factor is 45?

(*a*) 1.5 A $V^{-1}$
(*b*) $1.5 \times 10^{-2}$ A $V^{-1}$
(*c*) $1.5 \times 10^{-3}$ A $V^{-1}$
(*d*) $1.5 \times 10^{2}$ A $V^{-1}$

A-31

The number of electrons in one coulomb of charge is:

(*a*) $6.25 \times 10^{18}$
(*b*) $6.25 \times 10^{20}$

(*c*) $1.6 \times 10^{19}$
(*d*) $1.6 \times 10^{20}$

**A-32**

A wire has a resistance of 10 $\Omega$. What will be its resistance if it is stretched by one-tenth of its original length?

(*a*) 12.1 $\Omega$
(*b*) 14.2 $\Omega$
(*c*) 10 $\Omega$
(*d*) 10.1 $\Omega$

**A-33**

What is current $i$ in the circuit in Fig. 2.

(*a*) 2 A

(*b*) 1 A

(*c*) 0.5 A

(*d*) 1.2 A

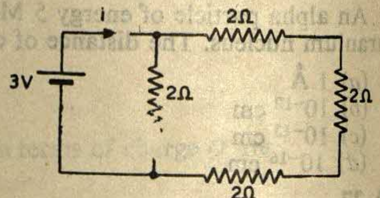



**Fig. 2**

**A-34**

The letter A is constructed with a uniform wire of resistance 1 $\Omega$ per cm. The sides of the letter are 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. The resistance of the letter between the sides of the legs is:

(*a*) $\frac{80}{3}\,\Omega$

(*b*) 60 $\Omega$

(*c*) $\frac{40}{3}\,\Omega$

(*d*) 20 $\Omega$

**A-35**

In an earth-satellite an astronaut weighing 80 kg on earth stands on a platform, with a spring balance. His weight as indicated by the spring balance will be:

(*a*) 80 kg
(*b*) 60 kg
(*c*) 40 kg
(*d*) zero

**A-36**

A hollow metal sphere is filled with water and is hung by a long thread. It is then made to oscillate. If water slowly flows through a small hole in the bottom, how will the period of oscillation be affected?

(*a*) the period will go on increasing till the sphere is empty
(*b*) the period will go on decreasing till the sphere is empty
(*c*) the period will remain unchanged throughout
(*d*) the period will first increase, then it will decrease till it is empty and the period will be finally the same as when the sphere was full of water.

**A-37**

Two pieces of wire are made of the same material. Their lengths are in the ratio 1:2 and diameters in the ratio 2:1. If they are stretched by the same force, their respective elongations will be in the ratio of:

(*a*) 2:1
(*b*) 1:2
(*c*) 1:8
(*d*) 8:1

**A-38**

A sphere is suspended by a thread of length *l*. What is the minimum horizontal velocity which should be imparted to the ball so that it reaches the height of suspension?

(*a*) $\sqrt{gl}$
(*b*) $\sqrt{2gl}$
(*c*) $2gl$
(*d*) $gl$

**A-39**

The escape velocity for a body projected vertically upwards from the surface of the earth is 11.2 km/s. If the body is projected in a direction making an angle of 45° with the vertical, the escape velocity will be:

(*a*) $11.2 \times \frac{1}{\sqrt{2}}$ km/s
(*b*) $11.2 \times \sqrt{2}$ km/s
(*c*) $11.2 \times 2$ km/s
(*d*) 11.2 km/s

**A-40**

In van der Waals' equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, the units of $a$ and $b$ are

(*a*) $Nm^4$, $m^3$
(*b*) $Nm^6$, $m^3$
(*c*) $Nm^2$, $m^3$
(*d*) $Nm^{-2}$, $m^3$

**A-41**

If 10 g of ice at 0°C are mixed with 10 g of water at 10°C, then the final temperature $t$ is given by:

(*a*) $10 \times 80 + 10(t - 0) = 10(10 - t)$
(*b*) $10 \times 10 = 10(10 - t) + 10(t - 0)$
(*c*) $t = 5°C$
(*d*) $t = 0°C$

**A-42**

Which of the following methods will increase the volume of an ideal gas to four times its original value? (Absolute temperature is used.)

(*a*) double the temperature and double the pressure
(*b*) halve the temperature, double the pressure
(*c*) quarter the temperature at constant pressure
(*d*) quarter the pressure at constant temperature

**A-43**

Which of the following cylindrical rods will conduct maximum heat when their ends are maintained at the same steady temperatures?

(*a*) length 100 cm, radius 1 cm
(*b*) length 200 cm, radius 1 cm
(*c*) length 200 cm, radius 2 cm
(*d*) length 100 cm, radius 2 cm

**A-44**

How will an image produced by a lens change if half the lens is wrapped in black paper?

(*a*) there will be no effect
(*b*) the size of the image will be reduced to one half
(*c*) the image will disappear
(*d*) the brightness of the image will be reduced

**A-45**

How much time will light take to travel through a glass slab $10^{-2}$ m thick of $\mu$ equal to 1.3?

(*a*) $\frac{1.3}{3} \times 10^{-10}$ s
(*b*) $\frac{1.3}{3} \times 10^{-8}$ s
(*c*) $\frac{3}{1.3} \times 10^{-10}$ s
(*d*) $\frac{3}{1.3} \times 10^{-8}$ s

**A-46**

A double convex lens of refractive index 1.5 has its focal length equal to 10 cm in air. When immersed in water of refractive index 4/3, its focal length will be:

(*a*) 20 cm
(*b*) 30 cm
(*c*) 40 cm
(*d*) infinite

**A-47**

The graph between the object distance along $x$-axis and image distance along $y$-axis for a convex lens is:

(*a*) a straight line
(*b*) a circle
(*c*) a parabola
(*d*) a rectangular hyperbola

**A-48**

A lens forms a sharp image on a screen. On inserting a parallel-sided slab of glass between the lens and the screen, it is found necessary to move the screen a distance $d$ away from the lens in order to focus the image sharply. If the refractive index of glass relative to air is $\mu$, then the thickness of the glass slab is given by:

(*a*) $d/\mu$
(*b*) $\mu d$
(*c*) $\frac{\mu d}{(\mu-1)}$
(*d*) $\frac{(\mu-1)d}{\mu}$

**A-49**

The photoelectric effect proves that:

(*a*) light travels in the form of quanta
(*b*) light travels in the form of transverse waves
(*c*) light travels in the form of waves
(*d*) velocity of light is infinite

**A-50**

When a monochromatic light wave enters from one medium into another which one of the following properties of the incident light remains unchanged?

(*a*) amplitude (*b*) velocity
(*c*) wavelength (*d*) frequency

**A-51**

A double convex air bubble in water will act like a:

(*a*) convex lens (*b*) concave lens
(*c*) plane slab (*d*) concave mirror

**A-52**

If there were no atmosphere, the duration of the day on the earth will:

(*a*) decrease (*b*) increase
(*c*) remain the same (*d*) depend upon the weather

**A-53**

A 4.0 $\mu$F condenser is charged to 400 V and then its plates are joined through a resistance. Heat produced in the resistance is:

(*a*) 0.64 J (*b*) 0.32 J
(*c*) 0.16 J (*d*) 1.28 J

**A-54**

*P* and *Q* are two concentric metal spheres. *P* is positively charged and *Q* earthed as shown in Fig. 3. Then

(*a*) the charge density on *Q* is same as on *P*
(*b*) the electric field between *P* and *Q* is uniform
(*c*) the electric potential inside *P* is zero
(*d*) the electric field outside *Q* is zero

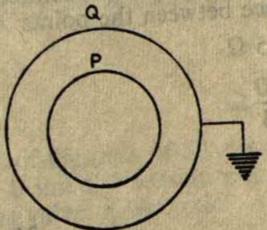


Fig. 3

**A-55**

If one of the two slits of a standard Young's double slit interference experiment is painted so that it transmits half the light intensity as the second slit,

(*a*) the fringe system will disappear altogether
(*b*) the bright fringes will become brighter and dark fringes will become darker
(*c*) both the dark and bright fringes will become darker
(*d*) the dark fringes will be brighter and the bright fringes darker.

**A-56**

The distance between the object and its real image formed by a convex lens of focal length *f* cannot be:

(*a*) greater than 2*f* (*b*) greater than 4*f*
(*c*) less than 4*f* (*d*) less than 6*f*

**A-57**

If the cover of a book looks red when seen through a piece of red glass, then the cover must be:

(*a*) red colour (*b*) red or white
(*c*) white (*d*) green

**A-58**

The wavelength of light from a given source in air is found to be $6 \times 10^{-5}$ cm. What will be wavelength of this light in water of refractive index 4/3?

(*a*) $6 \times 10^{-5}$ cm (*b*) $8.0 \times 10^{-5}$ cm
(*c*) $4.5 \times 10^{-5}$ cm (*d*) $2.25 \times 10^{-5}$ cm

**A-59**

Five resistances are connected as shown in Fig. 4. The equivalent resistance between the points *A* and *B* is:

(*a*) 10 $\Omega$
(*b*) 15 $\Omega$
(*c*) 5 $\Omega$
(*d*) 1.5 $\Omega$

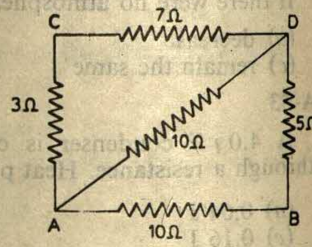



Fig. 4

**A-60**

Five resistances are connected as shown in Fig. 5. What is the effective resistance between the points *A* and *B*?

(*a*) 15 $\Omega$ (*b*) 10 $\Omega$

(*c*) $\frac{10}{3}\,\Omega$ (*d*) $\frac{10}{7}\,\Omega$

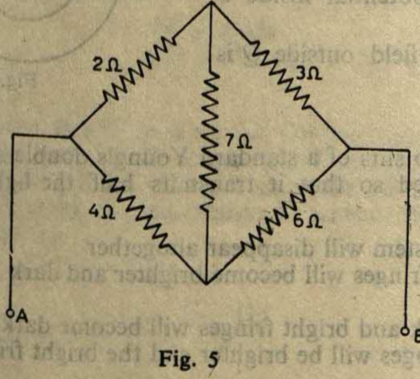



Fig. 5

**A-61**

Two waves, each of amplitude 1.5 mm and frequency 10 Hz, are travelling in opposite directions with velocity 20 mm s$^{-1}$. The distance in mm between adjacent needs is

(*a*) 1.0 (*b*) 1.2
(*c*) 1.5 (*d*) 2.0

**A-62**

A steel piano wire 0.5 m long has a total mass of 0.01 kg and is stretched with a tension of 800 N. The frequency when it vibrates in its fundamental mode is:

(*a*) 100 Hz (*b*) 200 Hz
(*c*) 400 Hz (*c*) 800 Hz

**A-63**

In a Young's double-slit experiment using green light, the fringe width was observed to be 0.20 mm. If red light replaces green light, the fringe width becomes

(*a*) 0.25 mm (*b*) 0.16 mm
(*c*) 0.13 mm (*d*) 0.10 mm

**A-64**

The value of absolute zero on the Fahrenheit scale is:

(*a*) 273° F (*b*) −423.2° F
(*c*) −459.4° F (*d*) −602° F

**A-65**

At pressure $P$ and absolute temperature $T$, the mass $M$ of an ideal gas fills a closed container of volume $V$. An additional mass $2M$ of the same gas is introduced into the container and the volume is then reduced to $V/3$ and the temperature to $T/3$. The pressure of the gas will now be

(*a*) $3P$ (*b*) $2P$
(*c*) $P/3$ (*d*) $9P$

**A-66**

Two radioactive elements X and Y have half-lifetimes of 50 minutes and 100 minutes respectively. Samples of A and B initially contain equal number of atoms. After 200 minutes, what is the value of the ratio

$$\left(\frac{\text{number of atoms of X unchanged}}{\text{number of atoms of Y unchanged}}\right),$$

(*a*) 4 (*b*) 2
(*c*) 1/2 (*d*) 1/4

**A-67**

When a lithium nucleus ($_3Li^7$) is bombarded with certain particles, only two alpha particles are produced. The bombarding particles are:

(*a*) electrons (*b*) protons
(*c*) deuterons (*d*) photons

**A-68**

Which one of the following phenomena cannot be explained by the wave theory of light?

(*a*) refraction (*b*) interference
(*c*) photoelectric effect (*d*) polarization

**A-69**

The value of $X$ in ohms which gives zero deflection on the galvanometer in Fig. 6 is

(*a*) 3 Ω (*b*) 6 Ω
(*c*) 15 Ω (*d*) 18 Ω

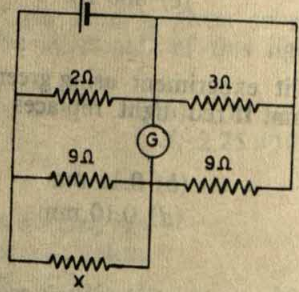



Fig. 6

**A-70**

If each resistor in Fig. 7 is 2 Ω, the effective resistance in ohms between $X$ and $Y$ is:

(*a*) 1 Ω (*b*) 2 Ω
(*c*) 4 Ω (*d*) 6 Ω

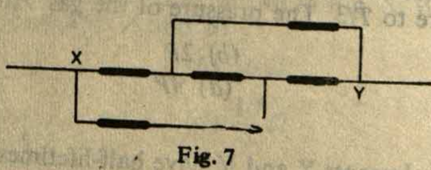



Fig. 7

**A-71**

Wires $X$ and $Y$ are made from the same material. $X$ has twice the diameter and three times the length of $Y$. If the elastic limits are not reached when each is stretched by the same tension, the ratio of the energy stored in $X$ to that in $Y$ is:

(*a*) 2:3 (*b*) 3:4
(*c*) 3:2 (*d*) 6:1

**A-72**

If $X$, $Y$ and $Z$ in Fig. 8 are identical lamps, which of the following changes in the brightness of the lamps occur when switch $S$ is closed?

(*a*) $X$ increases, $Y$ decreases
(*b*) $X$ stays the same, $Y$ decreases
(*c*) $X$ decreases, $Y$ decreases
(*d*) $X$ increases, $Y$ stays the same

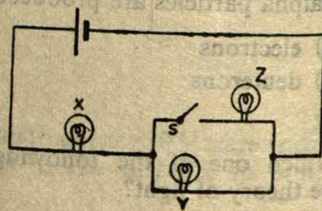



Fig. 8

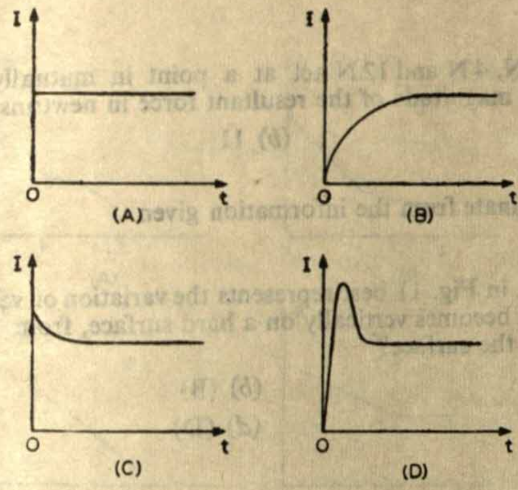



Fig. 9

**A-73**

Which of the graphs of Fig. 9 shows the variation of current with time in a tungsten filament lamp, from the moment current flows?

(*a*) (A) (*b*) (B)
(*c*) (C) (*d*) (D)

**A-74**

When the prong of a tuning fork is struck against some object, both the prongs vibrate.

(*a*) with a phase difference of $\pi$
(*b*) with a phase difference of $2\pi$
(*c*) with a phase difference of $\pi/2$
(*d*) in the same phase

**A-75**

When a lens is inserted between an object and a screen which are fixed a distance apart, the size of the image is either 6 cm or 2/3 cm. The size of the object in cm is

(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 9

**A-76**

A pendulum bob suspended by a string from the point *P* as shown in Fig. 10. It is in equilibrium under the action of three forces: *W* the weight of the bob, *T* the tension in the string and *F* a horizontally applied force. Which one of the following statements is untrue?

(*a*) $F^2+W^2=T^2$
(*b*) $W=T\cos\theta$
(*c*) $W=W\tan\theta$
(*d*) *F* and *W* are the components of *T*

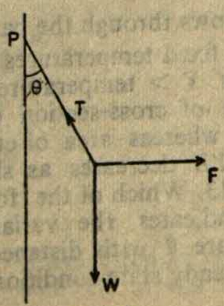



Fig. 10

**A-77**

Forces of 3 N, 4 N and 12 N act at a point in mutually perpendicular directions. The magnitude of the resultant force in newtons is:

(*a*) 5
(*b*) 11
(*c*) 13
(*d*) indeterminate from the information given.

**A-78**

Which graph in Fig. 11 best represents the variation of velocity with time of a ball which becomes vertically on a hard surface, from the moment it rebounds from the surface?

(*a*) (A)
(*b*) (B)
(*c*) (C)
(*d*) (D)

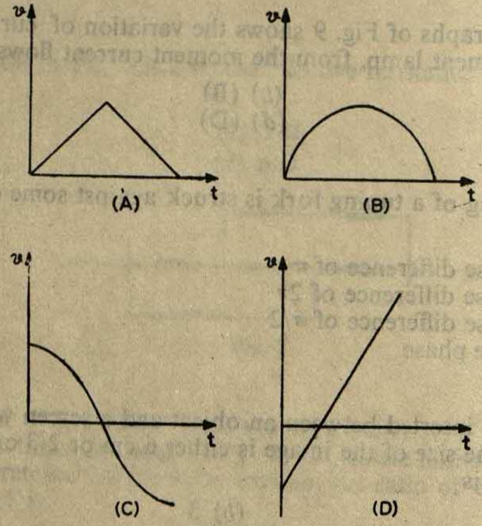



Fig. 11

**A-79**

Heat flows through the bar $XYZ$ in Fig. 12. The ends $X$ and $Z$ are maintained at fixed temperatures (temperature at $X >$ temperature at $Z$). The area of cross-section of $XY$ is uniform whereas area of cross-section of $YZ$ decreases as shown in the Fig. 13. Which of the following graphs indicates the variation of temperature $\theta$ with distance along $XZ$ for steady state conditions?

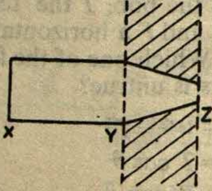



Fig. 12

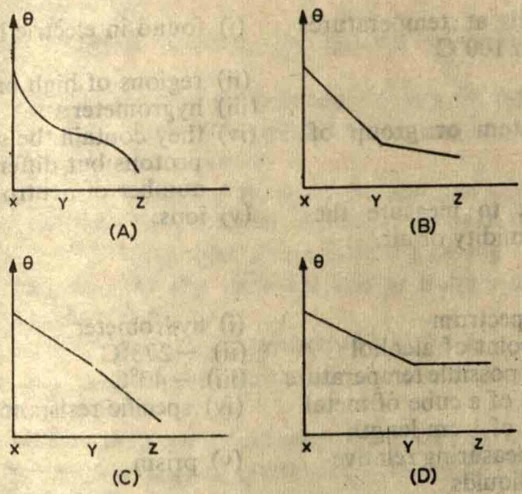



Fig. 13

(*a*) (A) (*b*) (B)
(*c*) (C) (*d*) (D)

**A-80**

An electrostatic field $E$ and a magnetic flux density $B$ act over the same region and an electron enters the region. Which one of the combinations of $E$ and $B$ in Fig. 14 can be made to cause the electron to pass undeflected?

(*a*) (A) (*b*) (B)
(*c*) (C) (*d*) (D)

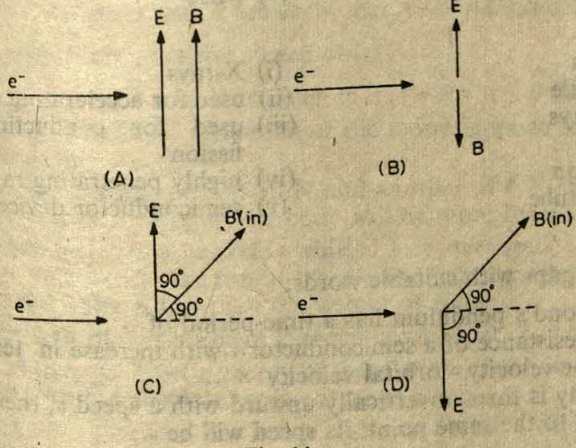



Fig. 14

Given five items listed as *a*, *b*, *c*, *d* and *e* and five others listed as (i), (ii) (iii), (iv) and (v), pair each item of the first category with one of the items of the second list.

**A-81**

| | |
|---|---|
| (*a*) Water boils at temperatures higher than 100°C | (i) found in electric lamps |
| (*b*) Isotopes | (ii) regions of high pressure |
| (*c*) Tungsten | (iii) hygrometer |
| (*d*) Charged atom or group of atoms | (iv) they contain the same number of protons but different number of neutrons |
| (*e*) Instrument to measure the relative humidity of air. | (v) ions. |

**A-82**

| | |
|---|---|
| (*a*) Produces spectrum | (i) hydrometer |
| (*b*) Freezing point of alcohol | (ii) $-273°C$ |
| (*c*) The lowest possible temperature | (iii) $-40°C$ |
| (*d*) Resistance of a cube of metal with a side of 1 cm length | (iv) specific resistance |
| (*e*) Used for measuring relative density of liquids | (v) prism |

**A-83**

| | |
|---|---|
| (*a*) Energy of a photon | (i) potential difference across a cell when no current flows |
| (*b*) Electron | (ii) kilo-watt-hour |
| (*c*) Emf of a cell | (iii) elementary unit of electric charge |
| (*d*) Wavelength of X-rays | (iv) $h\nu$ |
| (*e*) $36 \times 10^5$ J | (v) 1 Angstrom |

**A-84**

| | |
|---|---|
| (*a*) Angular momentum | (i) $M^{-1}L^3T^{-2}$ |
| (*b*) Torque | (ii) $ML^2T^{-2}$ |
| (*c*) Power | (iii) $ML^2T^{-1}$ |
| (*d*) Weight | (iv) $ML^2T^{-3}$ |
| (*e*) Gravitational constant | (v) $MLT^{-2}$ |

**A-85**

| | |
|---|---|
| (*a*) Cyclotron | (i) X-rays |
| (*b*) Atomic pile | (ii) used for accelerating protons |
| (*c*) Cosmic rays | (iii) used for conducting nuclear fission |
| (*d*) *p-n* junction | (iv) highly penetrating rays |
| (*e*) Coolidge tube | (v) semiconductor device |

**A-86**

Fill up the gaps with suitable words:

(i) A second's pendulum has a time-period of...
(ii) The resistance of a semiconductor...with increase in temperature.
(iii) Escape velocity = orbital velocity × ...
(iv) A body is thrown vertically upward with a speed $v$, then on coming back to the same point, its speed will be...
(v) A wire is stretched to three times its length, then its resultant resistance will become...times.
(vi) If charge is taken as a standard unit, the dimensions of resistivity are...

(vii) The resistance of a shunted galvanometer is...than the resistance of the galvanometer.

(viii) During projectile motion,...component of velocity remains unaltered.

(ix) The time-period of a simple pendulum in a lift falling freely is....

(x) When an alpha particle is emitted by a radioactive substance, its atomic number decreases by....

(xi) If a glass plate is inserted between the plates of an insulated charged condenser, then the potential difference between the plates will....

(xii) A person with 'long sight' uses spectacles having...lenses.

(xiii) The frequency of the lines emitted in Balmer series lie in the ... region of the spectrum.

(xiv) The absorptive power of a perfectly black body is ....

(xv) When the source and the observer recede from each other, the apparent frequency....

(xvi) The unit for mutual conductance of a triode-valve is...

(xvii) The maximum energy of photo-electrons emitted depends upon the .. incident light.

(xviii) The wavelength of the first member of the Balmer series is...than the wavelength of the second member of the same series.

(xix) The phenomenon of nuclear...is used in the liberation of energy from the hydrogen bomb.

(xx) The nucleus of Uranium 235 has...nucleons.

**A-87**

Indicate which of the following statements is True or False, by using symbols $T$ for true and $F$ for false.

(i) Momentum and impulse have the same dimensions.

(ii) Planck's constant and angular momentum have the same dimensions.

(iii) Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}+\vec{B}=\vec{C}$ and $A+B=C$. Then $\vec{A}$ and $\vec{B}$ are parallel to each other.

(iv) Two vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ then $\vec{A}=0$

(v) The gravitational attraction of the moon is much less than that of the earth.

(vi) If two forces, one of 10 N and another of 6 N, act upon a body, the resultant force on the body will be more than 10 N.

(vii) When water in a bucket is whirled fast overhead, the water does not fall out at the top of the motion because the centripetal force on the water is less than the weight of water.

(viii) The unit of coefficient of thermal conductivity is J $m^{-1}$ $s^{-1}$ $°C^{-1}$.

(ix) The average kinetic energy of translation of a molecule is proportional to the absolute temperature of the gas.

(x) In a diverging lens, the image is always virtual, erect and diminished.

(xi) When a spring of force constant $k$ is cut into two equal parts, the force constant of each half becomes $k/2$.

(xii) Kinetic energy is a vector quantity.

(xiii) Equipotential surfaces are always at right angles to the lines of force.

(xiv) The weight of a body at the centre of the earth is maximum.

(xv) Escape velocity of a body is independent of its mass.

**A-88**

Answer the following questions in two or three sentences.

(i) Why are roads and railway tracks banked on curves?

(ii) Can an object be accelerated without speeding up or slowing down? Give an example.

(iii) Can an object move westward with an acceleration that is neither eastward nor westward nor northward nor southward? Give an example.

(iv) Why does a balloon stop rising when it has attained a certain height in the sky?

(v) Why does a siphon fail to work in vacuum?

(vi) Does a ball you drop in a speeding train fall as fast as one released from a tower? Explain.

(vii) When some air is pumped into a balloon, both the pressure and volume increase for the some temperature of air. Does it violate Boyle's law?

(viii) One cannot expect centripetal force to perform any work, explain.

(ix) Some passengers sitting in a stationary carriage push it from the inside, will the carriage move?

(x) What is the effect of air resistance on (i) the total time of flight and on (ii) the horizontal range for a projectile?

(xi) Explain why steam at 100°C produces more severe burns than water at 100°C.

(xii) A beaker is completely filled with water at 1°C. Whether the temperature of this water is raised or lowered, in either case there is an outflow of water. Explain.

(xiii) Why does the setting sun look red?

(xiv) Why does an air bubble in a jar of water shine brightly?

(xv) In day-light why does a red tie seen through a green filter look black?

(xvi) Why are the filaments in the electric bulb made of tungsten?

(xvii) Is it possible to have the terminal voltage across a cell greater than its emf? Explain.

(xviii) A man in an insulated metal cage does not receive a shock when the cage is connected to a high voltage source. Explain why.

(xix) Do magnetic lines of force have a beginning and ending?

(xx) Do bends in a wire affect its electrical resistance?

## PART B

**B-1**

If the velocity of light $C$, the constant of gravitation $G$ and Planck's constant $h$ be chosen as fundamental units, find the dimensions of mass, length and time in the new system.

**B-2**

If instead of mass, length and time as fundamental quantities, we choose velocity, acceleration and force as fundamental quantities and express their dimension by $V$, $A$ and $F$ respectively, how would you express linear momentum, angular momentum and Young's modulus in terms of $V$, $A$ and $F$?

**B-3**

A mass of 2.9 kg is suspended from a string of length 50 cm and is at rest. Another body of mass 100 g, moving horizontally with a velocity of 150 m/s, strikes and sticks to it.

(i) What is the tension in the string when it makes an angle of 60° with the vertical?

(ii) Will it complete a vertical circle?

**B-4**

The length of a simple pendulum is 1 m. The bob of the pendulum of mass 10 g is released when the string is horizontal so that it is at the lowest point of its path.

(i) What is its kinetic energy?

(ii) What is the tension in the string?

**B-5**

If $p$, $q$ and $r$ are the distances covered by a body moving with a uniform acceleration during the $l$th, $m$th and $n$th seconds respectively, prove that

$$p(m-n)+q(n-l)+r(l-m)=0$$

**B-6**

A particle of mass 10 g is describing simple harmonic motion along a straight line with a speed of 2 cm/s and an amplitude of 10 cm. What is its kinetic energy when it is

(*a*) 2 cm from its equilibrium position?

(*b*) 5 cm from its equilibrium position?

How would you account for the difference between these two values of kinetic energy?

**B-7**

A particle of mass $M$ is attached to the mid-point of a wire stretched between two fixed points. If $l$ be the length of the wire and $T$ the tension in the wire, find the frequency of the lateral oscillations.

**B-8**

If the earth were a homogeneous sphere and a frictionless tunnel were bored between two places on the surface, show that the time to travel from one place to the other through the tunnel is independent of the distance between these two places. Find the time required to cover this distance. Take $g=9.8$ m s$^{-2}$ and radius of earth $=6.38\times10^6$ m.

**B-9**

Show that if a tunnel is dug along the diameter of the earth and a body is dropped into it, it will execute simple harmonic motion with a period $\sqrt{\frac{3\pi}{G\rho}}$, where the $G$ is gravitational constant and $\rho$ the average density of earth.

**B-10**

Two bodies of masses $M_1$ and $M_2$ are placed a distance $d$ apart. Show that at a point where the gravitational field due to both the masses is zero, the gravitational potential is given by

$$V=\left(\frac{G}{d}\right)(M_1+M_2-2M_1^{1/2}M_2^{1/2})$$

**B-11**

A projectile of mass 50 kg is shot vertically upwards with an initial velocity of 100 m/s. After 5 s, it explodes into two fragments, one of which having a mass of 20 kg travels vertically up with a velocity of 150 m/s.

(i) What is the velocity of the other fragment at that instant?

(ii) Calculate the sum of the moments of the two fragments 3 s after the collision. What would have been the momentum of the projectile at this instant if there had been no explosion?

**B-12**

A block P of mass $m_1$, rests on another block Q of mass $m_2$. Q rests on a fixed surface $R$ (Fig. 15). The coefficient of friction between any two surfaces is $\mu$. P and Q are connected by a massless string passing around a frictionless pulley S fixed to a wall. With what force should P be dragged so as to keep both P and Q moving with uniform speed?

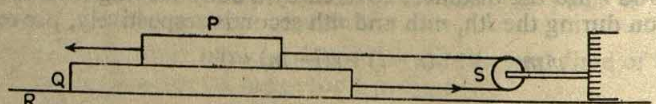



Fig. 15

**B-13**

A block of ice slides down from the top of an inclined roof of a house (angle of inclination of roof is equal to 30°C to the horizontal). The highest and lowest points of the roof are at heights of 8.1 m and 5.6 m respectively from the ground. At what horizontal distance from the starting point will the block hit the ground? (Neglect friction)

**B-14**

A gun, kept on a straight horizontal road, is used to hit a car travelling along the same road away from the gun with a uniform speed of 72 km/hr. The car is at a distance of 500 m from the gun when the gun is fired at an angle of 45° with the horizontal. Find (i) the distance of the car from the gun when the shell hits it, and (ii) the speed of projection of the shell from the gun.

**B-15**

A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 N kg$^{-1}$. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached?

**B-16**

The length of a simple pendulum is 1 m. The bob of the pendulum of mass 10 g is released when the string is horizontal. When it reaches at the lowest point of its path,

(i) What is its kinetic energy?

(ii) What is the tension in the string?

**B-17**

A light rod of length 200 cm is suspended horizontally from the ceiling by means of two vertical wires of equal length tied to its ends. Both the wires are made of steel and are of cross-section $10^{-3}$ m$^2$. Find out the position along the rod at which a weight may be suspended to produce:

(i) equal stresses in both wires, and

(ii) equal strains in both wires.
[Young's modulus of brass $= 10^{11}$ N/m$^2$,
Young's modulus of steel $= 2 \times 10^{11}$ N/m$^2$]

**B-18**

A sphere of radius 10 cm and mass 25 kg is attached to the lower end of a steel wire which is suspended from the ceiling of a room. The point of support is 5.21 m above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grazes the floor. Calculate the velocity of the ball at its lowest position. Young's modulus of steel $= 2 \times 10^{11}$ N/m$^2$, unstretched length of the wire $= 5$ m and radius of the steel wire $= 0.05$ m.

**B-19**

A sonometer wire fixed at one end has a solid mass $M$ hanging from its other end to produce tension in it. It is found that a 70 cm length of the wire produces a certain fundamental frequency when plucked. When the same mass $M$ is hanging in water and completely submerged, it is found that the length of the wire has to be changed by 5 cm in order to produce the same fundamental frequency. Calculate the density of the material of the mass $M$.

**B-20**

A rifle shot is fired in a valley formed between two parallel mountains. The echo from one mountain is heard after 2 s and the echo from the other 2 seconds later.

(i) What is the width of the valley?

(ii) Is it possible to hear the subsequent echoes from the two mountains simultaneously at the same point? If so, after what time?

**B-21**

The length of the sonometer wire between two fixed ends is 1 m. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3 ?

**B-22**

The specific gravity of oxygen and nitrogen are in the respective ratio of 16 : 14. At what temperature will the velocity of sound in oxygen be same as that in nitrogen at 15°C?

**B-23**

A policeman on duty at a crossing challenges a motor driver for crossing the speed limit of 72 km/h by detecting a change of 20 vibrations in the note of frequency 128 Hz as the car passes him. Calculate the speed of the car. Is the policeman correct? Velocity of sound = 330 m/s.

**B-24**

The planet Jupiter has an atmosphere composed of methane ($CH_4$) at a temperature of $-130$°C. Estimate the velocity of sound on that planet, assuming the ratio of principal specific heats of this gas to be 1.3. Given $R = 8.3$ joules per degree per gram molecule.

**B-25**

A certain tuning fork is found to give 2 beats per second in conjunction with a stretched string vibrating transversely under a tension of either 10.2 or 9.9 kg weight. Calculate the frequency of the fork.

**B-26**

Calculate the number of heats if there are three sources of equal intensity with frequencies 400, 401 and 402 vibrations per second.

**B-27**

A vibrating tuning fork tied to the end of a string $6/\pi$ m long, is whirled around in a circle. It makes 2 rev/s. Calculate the difference of the frequency between the highest and the lowest notes heard by an observer situated in the plane of rotation. Velocity of sound = 340 m/s.

**B-28**

A projector lens a focal length of 10 cm. It throws the image of a 2 cm × 2 cm slide on a screen 5 m from the lens. Find the size of the picture on the screen.

**B-29**

Photographs of the ground are taken from an air craft flying at an altitude of 2000 m, by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any time?

**B-30**

A glass lens has a focal length 5 cm in air. What will be its focal length in water? Refractive index of glass is 1.51 and that of water 1.33.

**B-31**

An object of height 4 cm is kept to the left of, and on the axis of, a converging lens of focal length 10 cm as shown in Fig. 16.

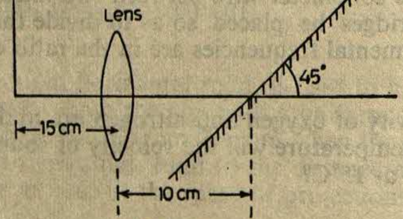



Fig. 16

A plane mirror is placed inclined at 45° to the axis of the lens at a distance of 10 cm to the right of the lens. Find the position and size of the image formed by the lens and mirror combination.

**B-32**

A rectangular loop of wire of width $D$ and resistance $R$, is placed in a uniform magnetic field $B$ acting normally to the plane of the loop. If we attempt to put it out of the field with velocity $V$, what is the power required for the purpose?

**B-33**

Two metal strips, each of length $l_0$ and thickness $d$ at temperature $T_0$ are riveted together so that their ends coincide. One strip is made of a metal having a coefficient of linear expansion $\alpha_1$ and other of metal with a coefficient of linear expansion $\alpha_2$, where $\alpha_1 > \alpha_2$. When this bimetallic strip is heated to a temperature $(T_0 + \Delta T)$, one strip becomes longer than the other and the bimetallic strip bends into an arc of a circle. Determine the radius of curvature, $R$ of the strip.

**B-34**

Two rods, each of length $l_2$ and coefficient of linear expansion $\alpha_2$, are connected freely to a third rod $l_1$ of coefficient of linear expansion $\alpha_1$ to form an isosceles triangle. The arrangement is supported on a knife-edge at the mid-point of $l_1$ which is horizontal. What relation must exist between $l_1$ and $l_2$ so that the apex of the isosceles triangle remains at a constant distance from the knife-edge as the temperature changes?

**B-35**

A lead bullet at 100°C strikes a steel plate and melts. What was its minimum speed? Sp. heat of lead = 0.03, latent heat = 5 cal/g and melting point = 327°C. The heat produced is shared equally between the plate and the bullet.

**B-36**

A reversible engine converts one sixth of the heat input into work. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. Find the temperature of the source and the sink.

**B-37**

Three slabs of thickness $d_1$, $d_2$ and $d_3$ and thermal conductivities $k_1$, $k_2$ and $k_3$ respectively are placed in contact in this order. Show that in the steady state when there is no radiation loss, the combination behaves as a single material of conductivity $k$, given by

$$\frac{d_1+d_2+d_3}{K} = \frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3}$$

**B-38**

The temperature gradient in the earth's crust is 42°C per km and the mean conductivity of the rocks is 0.008 cgs units. Taking the radius of the earth as 6000 km, calculate the daily loss of heat by the earth.

**B-39**

A metallic bob weighs 50 g in air. If it is immersed in a liquid at a temperature of 25°C, it weighs 45 g. When the temperature of the liquid is raised to 100°C, it weighs 45.1 g. Calculate the coefficient of cubical expansion of the liquid assuming the linear expansion of the metal to be

$$12 \times 10^{-6}/°C.$$

**B-40**

A glass flask of one litre capacity contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in the flask? Linear coefficient for glass = $9 \times 10^{-6}$/°C and volume coefficient for mercury = $1.8 \times 10^{-4}$/°C.

**B-41**

A steel wire of cross-sectional area 0.5 mm$^2$ is held between two fixed supports. If the tension in this wire is negligible and it is just taut at a temperature of 27°C, determine the tension when the temperature falls to 0°C (assuming the distance between two supports remains constant).

$$Y \text{ of steel} = 2.1 - 10^{12} \text{ dyne/cm}^2$$
$$\alpha \text{ for steel} = 12 \times 10^{-6}/°\text{C}.$$

**B-42**

The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. When A and B are mixed the temperature is 16°C whereas when B and C are mixed, the temperature is 23°C. What would be the temperature when A and C are mixed?

**B-43**

(*a*) Find the resistance between the points *A* and *B* in Fig. 17.

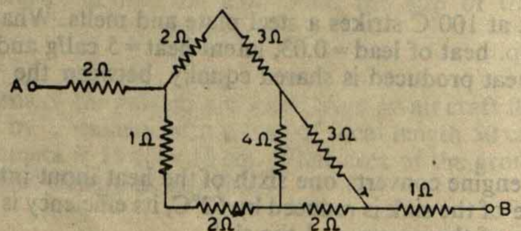



**Fig. 17**

(*b*) Two cells of emf 1.5 and 2.0 V and internal resistance 2Ω and 1Ω respectively have their negative terminals joined by a wire of 6Ω and positive terminal by a wire of 4Ω resistance. A resistance of 3Ω connects the mid-points of these wires. Calculate the potential difference across its two ends.

**B-44**

Figure 18 shows an infinitely long series-parallel chain. Each resistance of this chain is 100Ω. What is the total resistance between *A* and *B*. The part of this long chain to the right of *A′* and *B′* is the same as the whole chain.

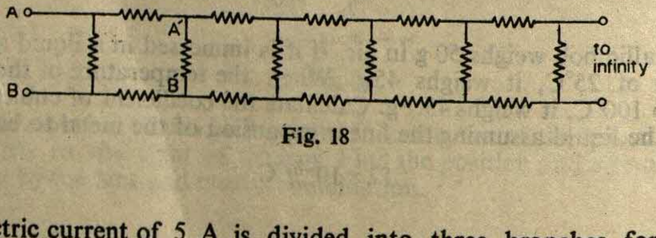



**Fig. 18**

**B-45**

An electric current of 5 A is divided into three branches forming a parallel combination. The lengths of the wires in the three branches are in the ratio 2, 3 and 4 and their diameters are in the raiio 3, 4 and 5. Find the current in each branch if the wires are of the same material.

**B-46**

A galvanometer together with an unknown resistance in series is connected across two identical batteries each of emf 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are connected in parallel the current is 0.6 A. What is the internal resistance of the batteries?

**B-47**

An electric kettle has two heating coils. When one of the coils is switched on, the kettle begins to boil in 6 minutes, and when the other coil is switched on the boiling begins in 8 minutes. In what time will the boiling begin if both coils are switched on simultaneously when connected (i) in series (ii) in parallel?

**B-48**

A potentiometer wire of length 100 cm has a resistance of $10\Omega$. It is connected in series with a resistance and a cell of emf 2 V of negligible internal resistance. A source of emf 10 mV is balanced by a length of 40 cm of the potentiometer wire. What is the value of the external resistance?

**B-49**

A fuse, made of lead wire, has an area of cross-section $0.2\ mm^2$. On short-circuiting, the current in the fuse wire reaches 30 A. How long after the short-circuiting will the fuse begin to melt? For lead, sp. heat = $0.032\ cal/g°C$, melting point = 327°C, density = $11.34\ g/cm^3$ and resistivity = $22 \times 10^{-6}\ \Omega$ cm, initial temperature of wire = 20°C.

**B-50**

Two conductors of copper and iron are connected in parallel and carry equal currents. What proportions of current will pass through each if the temperature is raised to 100°C? For copper $\alpha = 0.0043/°C$ and for iron $\alpha = 0.0063/°C$.

**B-51**

A length of a uniform wire of resistance 10 $\Omega$ is bent into a circle. Two points situated at a quarter of the circumference apart are connected with a battery of internal resistance 1 $\Omega$ and emf 3 V. Find the current in the different parts of the circuit.

**B-52**

A copper rod of length $L$ rotates at angular velocity $w$ in a uniform magnetic field $B$. Evaluate the emf developed between the two ends of the rod.

**B-53**

A fan blade of length $2a$ rotates with frequency $f$ cycles per second perpendicular to a magnetic field $B$. Find the p.d between the centre and end of the blade.

**B-54**

A spherical liquid drop has a diameter of 2 cm and is given a charge of 1 $\mu$C.

(i) What is the potential at the surface of the drop?

(ii) If two such drops coalesce to form a single drop, what is the potential at the surface of the drop so formed?

**B-55**

Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cm$^3$, the angle remains the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1.6 g/cm$^3$.

**B-56**

Find the value of the capacitance $C$ in Fig. 19. If the equivalent capacitance between the points A and B is to be 1 $\mu$F. All the capacitances are microfarads.

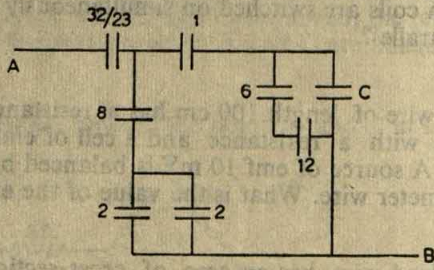



Fig. 19

**B-57**

An accident occurred in a laboratory in which a large amount of radioactive material with a known half-life of 20 days became embedded on floors, walls, etc. Tests showed that the level of radiation is 32 times the permissible level of normal occupancy of the room. Assuming that the last statement is correct, after how many days will the room become safe for occupation?

**B-58**

The equivalent wavelength of a moving electron is $0.24 \times 10^{-10}$ m. What voltage applied between two grids will bring it to rest?

**B-59**

What would be the approximate quantum number for a circular orbit of hydrogen 0.0001 mm in diameter?

**B-60**

An alpha particle and a proton are accelerated by a potential difference of 1 kV. Find the ratio of de Broglie wavelength of alpha particle and proton ($\lambda_\alpha : \lambda_p$).

**B-61**

Electrons *bombarding the anode of a Coolidge* tube produce X-rays of wavelength 1 AU. Find the energy of each electron at the moment of impact.

$$h = 6.662 \times 10^{-27} \text{ erg-s}$$
$$e = 4.80 \times 10^{-10} \text{ e.s.u.}$$

**B-62**

Calculate the wavelengths of the first two lines of the Balmer series and the excitation energy of these lines in eV. $R = 109678$ cm$^{-1}$.

**B-63**

Calculate the radius of the first orbit of hydrogen atom. Show that the velocity of the electron in the first orbit is $1/137\ c$, where $c$ is the velocity of light.

**B-64**

When 400 V electrons are diffracted by a crystal, the angular diffraction pattern is identical with that produced by X-rays of wavelength 0.61 Å. Calculate Planck's constant.

## ANSWERS

### PART A

**A-1** (*c*)
**A-2** (*d*)
**A-3** (*a*)
**A-4** (*d*)
**A-5** (*c*)
**A-6** (*b*)
**A-7** (*c*)
**A-8** (*a*)
**A-9** (*c*)
**A-10** (*a*)
**A-11** (*a*)
**A-12** (*a*)
**A-13** (*a*)
**A-14** (*a*)
**A-15** (*d*)
**A-16** (*d*)
**A-17** (*a*)
**A-18** (*a*)
**A-19** (*b*)
**A-20** (*a*)
**A-21** (*a*)
**A-22** (*b*)
**A-23** (*a*)
**A-24** (*d*)
**A-25** (*d*)
**A-26** (*c*)
**A-27** (*c*)
**A-28** (*a*)
**A-29** (*c*)
**A-30** (*b*)
**A-31** (*a*)
**A-32** (*a*)
**A-33** (*a*)
**A-34** (*a*)
**A-35** (*d*)
**A-36** (*d*)
**A-37** (*c*)
**A-38** (*b*)
**A-39** (*d*)
**A-40** (*a*)
**A-41** (*d*)
**A-42** (*d*)
**A-43** (*d*)
**A-44** (*d*)
**A-45** (*a*)
**A-46** (*c*)
**A-47** (*d*)
**A-48** (*c*)
**A-49** (*a*)
**A-50** (*d*)
**A-51** (*b*)
**A-52** (*a*)
**A-53** (*b*)
**A-54** (*d*)
**A-55** (*d*)
**A-56** (*c*)
**A-57** (*b*)
**A-58** (*c*)
**A-59** (*c*)
**A-60** (*c*)
**A-61** (*a*)
**A-62** (*b*)
**A-63** (*a*)
**A-64** (*c*)
**A-65** (*a*)
**A-66** (*d*)
**A-67** (*c*)
**A-68** (*d*)
**A-69** (*d*)
**A-70** (*b*)
**A-71** (*b*)
**A-72** (*a*)
**A-73** (*d*)
**A-74** (*a*)
**A-75** (*b*)
**A-76** (*d*)
**A-77** (*c*)
**A-78** (*d*)
**A-79** (*a*)
**A-80** (*c*)
**A-81** (*a*) (ii)
(*b*) (iv)
(*c*) (i)
(*d*) (v)
(*e*) (ii)
**A-82** (*a*) (v)
(*b*) (iii)
(*c*) (ii)
(*d*) (iv)
(*e*) (i)
**A-83** (*a*) (iv)
(*b*) (iii)
(*c*) (i)
(*d*) (v)
(*e*) (ii)

**A-84** (*a*) (iii)
(*b*) (ii)
(*c*) (iv)
(*d*) (v)
(*e*) (i)

**A-85** (*a*) (ii)
(*b*) (iii)
(*c*) (iv)
(*d*) (v)
(*e*) (i)

**A-86**
(i) 2
(ii) decreases
(iii) $\sqrt{2}$
(iv) $v$
(v) 9
(vi) $ML^3T^{-1}Q^{-2}$
(vii) less
(viii) horizontal
(ix) infinite
(x) 2
(xi) decrease
(xii) convex
(xiii) visible
(xiv) unity
(xv) decreases
(xvi) mho, Siemen, $\Omega^{-1}m^{-1}$
(xvii) frequency
(xviii) longer
(xix) fusion
(xx) 235

**A-87**
(i) T
(ii) T
(iii) T
(iv) F
(v) T
(vi) F
(vii) T
(viii) T
(ix) F
(x) T
(xi) F
(xii) F
(xiii) T
(xiv) F
(xv) T

**A-88**

(i) A centripetal force is required to make a body move along a curved path. If the track is banked inward, the normal $N$ is also inclined at angle $\theta$ to vertical and $N \sin \theta$ provides the necessary centripetal force.

(ii) When a body describes a circle with uniform speed, it possesses centripetal acceleration $v^2/r$, where $v$ is speed and $r$ is radius of circle. Here $v$ remains constant throughout and direction continuously changes.

(iii) Yes. If a body is moving along a vertical circle in east-west plane, then at the highest point, the body going from east to west will have a west-ward velocity and centripetal acceleration will be vertically downward.

(iv) At ground level, the upthrust on the balloon exceeds its weight and it goes up. As it goes up, the density of air goes on decreasing and at the highest point, the buoyant force equals the weight and it remains floating there.

(v) Because it works on account of atmospheric pressure.

(vi) Yes. The downward acceleration will be same in both the cases.

(vii) Mass of air is continuously changing, therefore, Boyle's law does not apply.

(viii) The angle between force and displacement is always 90°, therefore, work done is zero.

(ix) When the passengers apply a force $F$ on the wall of the carriage, an equal and opposite force will act on the passengers. So the net force on the system (carriage + passengers) will be zero and the carriage will not move.

(x) Time of flight will increase but horizontal range decreases.

(xi) Steam at 100°C requires more heat to condense (540 cal/g) besides the heat required to come down from 100°C to the body temperature.

(xii) Water has maximum density at 4°C. When the temperature changes, the volume increases.

(xiii) Due to the phenomenon of scattering of light,

$$\text{Scattering} \propto \frac{1}{(\text{wavelength})^4}$$

(xiv) Rays going from water into the air bubble suffer total internal reflection, so get reflected back as if from a mirror, hence the air bubble shines like a mirror.

(xv) Green filter allows only green light to pass through. Red tie sends only red rays which fail to pass through filter, hence appears black.

(xvi) High melting point of tungsten.

(xvii) By having an additional source of higher emf so connected that the current enters at positive pole and leaves at the negative terminal. Thus p.d of cell will be greater than its emf.

(xviii) Since the charge always resides on the outer surface of a sphere.

(xix) No. They are closed loops of various shapes.

(xx) No. Because electrons have negligible inertia.

## PART B

**B-1** $M=[h^{1/2}\,C^{1/2}\,G^{-1/2}]$

$L=[h^{1/2}\,C^{3/2}\,G^{1/2}]$

$T=[h^{1/2}\,G^{1/2}\,C^{-5/2}]$

**B-2** Linear momentum $=[\mathrm{FA^{-1}\,V}]$

Angular momentum $=[\mathrm{FA^{-2}\,V^{3}}]$

Young's modulus $=[\mathrm{FA^{2}\,V^{-4}}]$

**B-3** (i) 135.3 N

(ii) Yes, it completes a vertical circle

**B-4** (i) 0.098 J

(ii) 0.294 N

**B-6** (i) $480\,\pi^2\times10^{-7}$ J

(ii) $375\,\pi^2\times10^{-7}$ J

Decrease in KE $=105\,\pi^2\times10^{-7}$ J is accompanied by an increase in potential energy.

**B-7** $\dfrac{1}{2\pi}\sqrt{\dfrac{T}{Ml}}$

**B-8** 42 minutes

**B-11** (i) $-15$ m/s

(ii) 1080 kg m/s$^{-1}$, 1080 kg m/s$^{-1}$

| | |
|---|---|
| **B-12** | $\mu(3\,m_1+m_2)g$ |
| **B-13** | 8.93 m |
| **B-14** | (i) 85.6 m/s (ii) 746.8 m/s |
| **B-15** | 39.15 km |
| **B-16** | (i) 0.098 J (ii) 0.294 N |
| **B-17** | 1 m |
| **B-18** | 3.76 m/s |
| **B-19** | 7.26 g/cc |
| **B-20** | (i) 1050 m<br>(ii) 6s, 12s, 18s, etc. |
| **B-21** | 54.54 cm and 81.81 cm |
| **B-22** | 56.14°C |
| **B-23** | He is justified as the velocity of car is 33 m/s |
| **B-24** | 311.2 m/s |
| **B-25** | 268 vib/s |
| **B-26** | 1 |
| **B-27** | 0.14 times the original frequency |
| **B-28** | 98 cm × 98 cm |
| **B-29** | 158400 $m^2$ |
| **B-30** | 18.85 cm |
| **B-31** | 8 cm real image at a distance of 20 cm from the plane mirror |
| **B-32** | $\left\|\frac{B^2D^2V^2}{R}\right\|$ |
| **B-33** | $R=\frac{[2+(\alpha_1+\alpha_2)\Delta T]d}{(\alpha_1-\alpha_2)\Delta T}$ |
| **B-34** | $\frac{l_2^2}{l_1^2}=\frac{\alpha_1}{4\alpha_2}$ |
| **B-35** | 440 m/s |
| **B-36** | 1737°C, 1402°C |
| **B-38** | $1.009\times10^{20}$ cal |
| **B-39** | $3.09\times10^{-4}$/°C |
| **B-40** | 0.15 litre |
| **B-41** | 25.2 N |
| **B-42** | 20.26°C |
| **B-43** | (*a*) 6.08 Ω (*b*) 1.26 V |
| **B-44** | 73.2 Ω |
| **B-45** | 1.399 A, 1.658 A, 1.943 A, |
| **B-46** | $\frac{1}{3}\,\Omega$ |
| **B-47** | 3.43 minutes |
| **B-48** | 790 Ω |
| **B-49** | 0.095 s |
| **B-50** | 46.79%, 53.21% |
| **B-51** | $\frac{6}{23}$ A, $\frac{18}{23}$ A |

B-52 $\frac{1}{2}\omega BL^2$
B-53 $-\pi Ba^2f$
B-54 $14.75 \times 10^5$ V
B-55 2
B-56 4
B-57 100 days
B-58 2597 eV
B-59 31
B-60 2.82 : 1
B-61 $1.24 \times 10^4$ eV
B-62 6563 Å, 4870 Å; 1.99 eV, 2.55 eV
B-63 $6.59 \times 10^{-34}$ J-s

# CHEMISTRY

## PART A

Choose the correct answer in each of the following:

**A-1**

The octet rule is not valid for the molecule:

(*a*) $CO_2$ (*b*) $H_2O$
(*c*) $O_2$ (*d*) CO

**A-2**

A gaseous mixture contains oxygen and nitrogen in the ratio of 1 : 4 by weight. The ratio of their number of molecules is:

(*a*) 1 : 4 (*b*) 1 : 8
(*c*) 7 : 32 (*d*) 3 : 16

**A-3**

The number of neutrons in dipositive zinc ion with mass number 70 is:

(*a*) 34 (*b*) 36
(*c*) 38 (*d*) 40

**A-4**

The reddish-brown gas formed when nitric oxide is oxidized by air is:

(*a*) $NO_2$ (*c*) $N_2O_4$
(*b*) $N_2O_5$ (*d*) $N_2O_3$

**A-5**

The least number of molecule is in:

(*a*) 36 g of water (*c*) 28 g of carbon monoxide
(*b*) 46 g of ethyl alcohol (*d*) 54 g of nitrogen pentoxide

**A-6**

Element X is strongly electropositive and element Y strongly electronegative. Both are univalent. The compound formed would be:

(*a*) $X^+Y^-$
(*b*) $X^-Y^+$
(*c*) $X-Y$
(*d*) $X \rightarrow Y$

**A-7**

which of the following is most stable to heat?

(*a*) HCl
(*b*) HOCl
(*c*) HBr
(*d*) HI

**A-8**

Which of the following is soluble in water?

(*a*) $CS_2$
(*b*) $C_2H_5OH$
(*c*) $CCl_4$
(*d*) $CHCl_3$

**A-9**

Marsh gas mainly contains:

(*a*) $C_2H_2$
(*b*) $CH_4$
(*c*) CO
(*d*) $H_2S$

**A-10**

A solution of KBr is treated with each of the following gases. Which one would liberate bromine?

(*a*) $Cl_2$
(*b*) $I_2$
(*c*) HI
(*d*) $SO_2$

**A-11**

Which of the following decolorize alkaline $KMnO_4$ solution?

(*a*) $C_3H_8$
(*b*) $C_2H_4$
(*c*) $CH_4$
(*d*) $CCl_4$

**A-12**

Which of the following is basic?

(*a*) $CH_3—CH_2—OH$
(*b*) $HO—CH_2—CH_2—OH$
(*c*) H—O—O—H
(*d*) none of these

**A-13**

One of the constituents of German silver is:

(*a*) Ag
(*b*) Cu
(*c*) Mg
(*d*) Al

**A-14**

"Lead pencil" contains:

(*a*) Pb
(*b*) FeS
(*c*) Graphite
(*d*) PbS

**A-15**

Which of the following statements about anhydrous aluminium chroride is correct?

(*a*) it exists as $AlCl_3$ molecules
(*b*) it is not easily hydrolysed
(*c*) it sublimes at 100°C under vacuum
(*d*) it is a strong Lewis base

**A-16**

How many unpaired electrons does $Ni^{2+}$ have:

(*a*) 0 (*c*) 4
(*b*) 2 (*d*) ·8

**A-17**

The ratio of the root-mean-square velocity to the average velocity of a gas molecule at a particular temperature is:

(*a*) 1.086 : 1 (*c*) 2 : 1.086
(*b*) 1 : 1.086 (*d*) 1.086 : 2

**A-18**

Sodium thiosulphate is used in photography because of its:

(*a*) reducing behaviour (*c*) complex forming behaviour
(*b*) oxidizing behaviour (*d*) reaction with light

**A-19**

At 90°C, pure water has $[H_3^+O] = 10^{-6}$ mole litre$^{-1}$. What is the value of $K_w$ at 90°C?

(*a*) $10^{-6}$ (*c*) $10^{-14}$
(*b*) $10^{-12}$ (*d*) $10^{-8}$

**A-20**

The correct order of second ionization potential of carbon, nitrogen, oxygen and fluorine is:

(*a*) $C > N > O > F$ (*c*) $F > O > N > C$
(*b*) $O > N > F > C$ (*d*) $O > F > N > C$

**A-21**

Equal weights of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is:

(*a*) 1/3 (*c*) 1/2
(*b*) 2/3 (*d*) $\frac{1}{3} \times \frac{273}{298}$

**A-22**

Of the given anions, the strongest Brönsted base is:

(*a*) $ClO^-$ (*c*) $ClO_2^-$
(*b*) $ClO_3^-$ (*d*) $ClO_4^-$

**A-23**

A solution of sodium metal in liquid ammonia is strongly reducing due to the presence of:

(*a*) sodium atoms (*c*) sodium hydride
(*b*) sodium amide (*d*) solvated electrons

**A-24**

The pH of a $10^{-8}$ molar solution of HCl in water is:

(*a*) 8 (*c*) between 7 and 8
(*b*) $-8$ (*d*) between 6 and 7

A-25

The molecule in which the distance between the two adjacent carbon atoms is largest is:

(*a*) ethane
(*b*) ethene
(*c*) ethyne
(*d*) benzene

A-26

The compound that is not isomeric with diethyl ether is:

(*a*) *n*-propyl methyl ether
(*b*) 2-methylpropan-2-ol
(*c*) butan-l-ol
(*d*) butanone

A-27

The reagent with which both acetaldehyde and acetone react easily is:

(*a*) Fehling's reagent
(*b*) Schiff's reagent
(*c*) Grignard reagent
(*d*) Tollen's reagent

A-28

If a molecule $MX_3$ has zero dipole moment, the $\sigma$ bonding orbitals used by M (atomic number $< 21$) are:

(*a*) pure *p*
(*b*) *sp* hybrid
(*c*) $sp^2$ hybrid
(*d*) $sp^3$ hybrid

A-29

A compound that gives a positive iodoform test is:

(*a*) 1-pentanol
(*b*) 2-pentanone
(*c*) pentanal
(*d*) 3-pentanone

A-30

The compound with the highest boiling point is:

(*a*) *n*-hexane
(*b*) *n*-pentane
(*c*) 2, 2-dimethyl propane
(*d*) 2-methyl butane

A-31

Among the following, the compound that can be most readily sulphonated is:

(*a*) benzene
(*b*) toluene
(*c*) nitrobenzene
(*d*) chlorobenzene

A-32

Moderate electrical conductivity is shown by:

(*a*) silica
(*b*) graphite
(*c*) diamond
(*d*) carborundum

A-33

Among the following, the molecule that is linear is:

(*a*) $CO_2$
(*b*) $SO_2$
(*c*) $NO_2$
(*d*) $ClO_2$

A-34

Heavy water is:

(*a*) $H_2^{18}O$
(*b*) $D_2O$
(*c*) water obtained by repeated distillation
(*d*) water at 4°C

**A-35**

Chloride acts as a bleaching agent only in presence of:

(*a*) dry air (*c*) sunlight
(*b*) moisture (*d*) pure oxygen

**A-36**

Which among the following does not have the hydrogen bond?

(*a*) phenol (*c*) water
(*b*) liquid ammonia (*d*) liquid HCl

**A-37**

The principal quantum number of an atom is related to the:

(*a*) size of the orbital (*c*) orbital angular momentum
(*b*) spin angular momentum (*d*) orientation of the orbital in space

**A-38**

Which of the following will exhibit *cis-trams* (geometrical) isomerism?

(*a*) 2-butene (*c*) 2-butanol
(*b*) 2-butyne (*d*) butanal

**A-39**

The Cannizzaro reaction is not given by:

(*a*) trimethylacetaldehyde (*c*) formaldehyde
(*b*) benzaldehyde (*d*) acetaldehyde

**A-40**

In the nitration of benzene using a mixture of conc. $HNO_3$ and conc. $H_2SO_4$ the active species involved is:

(*a*) nitrate ion (*c*) nitronium ion
(*b*) nitrite (*d*) nitrogen peroxide

**A-41**

Which of the following compounds does not dissolve in conc. $H_2SO_4$ even on warming?

(*a*) ethylene (*c*) hexane
(*b*) benzene (*d*) aniline

**A-42**

Select the compound in which the oxidation number of chlorine is +5:

(*a*) $HClO$ (*b*) $HClO_2$
(*c*) $HClO_3$ (*d*) $HClO_4$

**A-43**

Crystalline substances that change into liquid on exposure to moist air are:

(*a*) non-hygroscopic (*b*) deliquescent
(*c*) highly volatile (*d*) non-volatile

**A-44**

Which of the following contains multiple covalent bonds?

(*a*) $Br_2$ (*b*) $H_2$
(*c*) $N_2$ (*d*) $Cl_2$

**A-45**

Which one of the following gases is most soluble in water?

(*a*) $CO_2$ (*b*) $NH_3$
(*c*) $O_2$ (*d*) $O_3$

**A-46**

Which one of the following gases has the greatest rate of diffusion?

(*a*) $H_2$ (*b*) $CO_2$ (*c*) $NH_3$ (*d*) $O_2$

**A-47**

Which one of the following substances conduct electricity even in solid state?

(*a*) iodine (*b*) diamond
(*c*) sodium chloride (*d*) $CO_2$

**A-48**

How many grams of $H_2O_2$ are contained in 0.6 moles of it?

(*a*) 0.204 g (*b*) 2.04 g
(*c*) 20.4 g (*d*) 204 g

**A-49**

Which one of the following has the highest pH value?

(*a*) M $H_2SO_4$ (*b*) 1 M KOH
(*c*) chlorine water (*d*) water containing carbon dioxide

**A-50**

Bell metal is an alloy of copper and:

(*a*) tin (*b*) aluminium
(*c*) zinc (*d*) nickel

**A-51**

The best conductor of heat and electricity is:

(*a*) iron (*c*) aluminium
(*b*) silver (*d*) copper

**A-52**

Which one of the following sulphates is insoluble in water?

(*a*) $Na_2SO_4$ (*b*) $CuSO_4$
(*c*) $BaSO_4$ (*d*) $K_2SO_4$

**A-53**

The metal that reacts with water at room temperature is:

(*a*) Bi (*b*) Mg
(*c*) Ca (*d*) Pb

**A-54**

The indicator used in the titration of oxalic acid and sodium hydroxide is:

(*a*) methyl orange (*b*) phenolphthalein
(*c*) methyl red (*d*) $NH_4OH$

**A-55**

Aqua regia is:

(*a*) 1 part HCl and 3 parts $HNO_3$
(*b*) 3 parts HCl and 1 part $HNO_3$

(*c*) 2 parts HCl and 1 part $NNO_3$
(*d*) 3 parts HCl and 2 parts $HNO_3$

**A-56**

Crimson colour imparted to flame is due to a salt of:
(*a*) bromine (*b*) strontium (*c*) copper (*d*) calcium

**A-57**

Which one of the following compounds produces $SO_3$?
(*a*) $CaSO_4$ (*b*) $Na_2SO_4$
(*c*) $CuSO_4$ (*d*) $Fe_2(SO_4)_3$

**A-58**

The chemical formula of the 'laughing gas' is:
(*a*) NO (*b*) $N_2O$
(*c*) $N_2O_4$ (*d*) $N_2O_5$

**A-59**

Which one of the following is not an alloy of Al?
(*a*) alclad (*b*) magnalium
(*c*) duralumin (*d*) brass

**A-60**

The commercial name for calium hydride is:
(*a*) lime (*b*) hydrolyth
(*c*) slaked lime (*d*) calgon

**A-61**

The noble gas used in radiotherapy is:
(*a*) Kr (*b*) Ar
(*c*) Ra (*d*) Xe

**A-62**

Which one of the following is colourless?
(*a*) $Mn^{2+}$ (*b*) $Cu^{+}$
(*c*) $Cr^{3+}$ (*d*) $Fe^{2+}$

**A-63**

Which one of the following is the most electropositive element?
(*a*) sodium (*b*) lithium
(*c*) chlorine (*d*) potassium

**A-64**

Which one of the following halogens can displace the remaining three from their halides?
(*a*) $F_2$ (*b*) $Cl_2$
(*c*) $Br_2$ (*d*) $I_2$

**A-65**

Which of the following is used as an antiseptic?
(*a*) chloroform (*b*) acetone
(*c*) iodoform (*d*) methyl alcohol

**A-66**

The general formula for aliphatic eithers is:

(*a*) $C_nH_{2n}O$
(*b*) $C_nH_{2n+1}O$
(*c*) $C_nH_{2n+2}O$
(*d*) $C_nH_{2n-2}O$

**A-67**

A paste of bleaching powder on heating with ethyl alcohol gives:

(*a*) chloroethane
(*b*) acetyl chloride
(*c*) ethylene
(*d*) chloroform

**A-68**

Which one of the following is used as an anaesthetic?

(*a*) $CH_3OCH_3$
(*b*) $CH_3OC_2H_5$
(*c*) $C_2H_5OC_2H_5$
(*d*) $C_3H_7OC_2H_5$

**A-69**

Which one of the following does not give silver mirror test on heating with ammoniacal silver nitrate?

(*a*) acetaldehyde
(*b*) formaldehyde
(*c*) acetic acid
(*d*) formic acid

**A-70**

Which of the following is used for artificial ripening of green fruit?

(*a*) methane
(*b*) propane
(*c*) ethylene
(*d*) acetylene

**A-71**

Which one of the following does not give a red precipitate with Fehling's solution?

(*a*) formaldehyde
(*b*) acetaldehyde
(*c*) propionaldehyde
(*d*) acetone

**A-72**

Which one of the following has a smell of bitter almonds?

(*a*) phenol
(*b*) benzoic acid
(*c*) aniline
(*d*) benzaldehyde

**A-73**

Phenol, on distillation with Zn dust, gives:

(*a*) benzaldehyde
(*b*) benzoic acid
(*c*) benzene
(*d*) toluene

**A-74**

In the compound $Na_2S_2O_3$, the oxidation state of sulphur is:

(*a*) $-2$
(*b*) $+2$
(*c*) $+4$
(*d*) $+6$

**A-75**

Which one of the following does not yield oxide on heating?

(*a*) $ZnCO_3$
(*b*) $Na_2CO_3$
(*c*) $CaCO_3$
(*d*) $Ag_2CO_3$

**A-76**

The mass of one Avogadro number of helium atoms is:

(*a*) 1.00 g (*b*) 4.00 g
(*c*) 8.00 (*d*) $4.0 \times 6.02 \times 10^{23}$ g

**A-77**

One gram equivalent of a substance is the weight of that amount of substance which is equivalent to:

(*a*) 0.25 mole of oxygen
(*b*) 0.50 mole of oxygen
(*c*) 1.00 mole of oxygen
(*d*) 8.00 mole of oxygen

**A-78**

10 ml of 1 M sodium hydroxide solution will neutralize:

(*a*) 2.5 ml (*b*) 5 ml
(*c*) 10 ml (*d*) 20 ml

1 M of sulphuric acid solution

**A-79**

Which of the following oxides reacts with both HCl and NaOH?

(*a*) CaO (*b*) $CO_2$
(*c*) $N_2O_3$ (*d*) ZnO

**A-80**

The compound that contains both ionic and covalent bonds is

(*a*) $CH_4$ (*b*) $H_2$
(*c*) KCN (*d*)KCl

**A-81**

Arrange the following according to given instructions:

(i) HI, HCl, HBr (increasing order of acid strength)
(ii) Na, Li, K (increasing order of atomic size)
(iii) Hg, Na, Cu (increasing order of density)
(iv) Na, Cu, Zn (increasing electropositive character)
(v) $Br_2$, $Cl_2$, $H_2$ (increasing boiling point)
(vi) PbO, MgO, ZnO (increasing order of basic nature)
(vii) Graphite, Cu, S (increasing order of electrical conductivity)
(viii) $HNO_3$, $H_2S$, $H_2SO_3$ (decreasing order of reducing power)
(ix) Zn, Al, Cu (increasing order of reactivity with HCl)
(x) $N_2$, $O_2$, $Cl_2$ (increasing order of boiling point)
(xi) $NH_4NO_3$, $Pb(NO_3)_2$, $KNO_3$ (increasing order of stability on heating)
(xii) HClO, $HClO_4$, $HClO_3$ (increasing order of acid strength)
(xiii) $NO_2$, $K_2O$, ZnO (increasing order of basicity)
(xiv) Al, Na, Fe (increasing order of reactivity with moist air)
(xv) $CO_2$, $CH_4$, $C_2H_5OH$ (decreasing order of solubility in water)
(xvi) $Na_2O$, MgO, $Al_2O_3$ (increasing order of basicity)
(xvii) $HNO_3$, $HNO_2$, $NO_2$ (increasing order of valency of nitrogen)
(xviii) $KClO_4$, HCl, $KClO_3$ (increasing order of valency of chlorine)

(xix) Al, Pb, Fe (increasing order of density)
(xx) Br, I, F, Cl (increasing order of atomic size)
(xxi) $NaHCO_3$, $Na_2CO_3$, $CaCO_3$ (increasing order of thermal stability)
(xxii) AgCl, $PbCl_2$, NaCl (increasing order of solubility in water)

**A-82**

Indicate the nature of bonding in the following substances:

(i) $CaCl_2$
(ii) $H_2$
(iii) $H_2S$
(iv) $CO_2$
(v) Diamond
(vi) NaCl
(vii) $SnCl_4$
(viii) HCl (g)
(ix) Metallic copper
(x) $CH_4$
(xi) $NH_4^+$
(xii) $ClO_4^-$
(xiii) $MgCl_2$
(xiv) Calcium carbide
(xv) Sodium hydride
(xvi) Sodium sulphide
(xvii) Ethylene
(xviii) $CCl_4$
(xix) $C_2H_2$
(xx) $K_2O$

**A-83**

State the valency for the following:

(i) Nitrogen in $N_2O$
(ii) Nitrogen in NO
(iii) Nitrogen in $N_2O_5$
(iv) Nitrogen in $NO_2$
(v) Nitrogen in $N_2O_3$
(vi) Nitrogen in $N_2$
(vii) Nitrogen in $Ca_3N_2$
(viii) Oxygen in $H_2O_2$
(ix) Carbon in diamond
(x) Mn in $KMnO_4$

**A-84**

Indicate the oxidation state of the italicized elements:

(i) $\mathit{N}_2H_2$
(ii) $\mathit{Cl}_2O_7$
(iii) $\mathit{H}_2$
(iv) $Na\mathit{N}O_2$
(v) $K\mathit{Mn}O_4$
(vi) $K\mathit{Cr}O_3Cl$
(vii) $\mathit{Ba}Cl_2$

**A-85**

Indicate the substances, if any, which are oxidised/reduced in the following reactions:

(i) $SO_2+Cl_2+2H_2O \rightarrow 2HCl+H_2SO_4$
(ii) $N+N \rightarrow N_2$
(iii) $3Hg+2I_2 \rightarrow Hg_2I_2+HgI_2$
(iv) $Ca+H_2 \rightarrow CaH_2$
(v) $4KClO_3 \rightarrow 3KClO_4+KCl$
(vi) $AlCl_3+3K \rightarrow Al+3KCl$
(vii) $SO_2+2H_2S \rightarrow 3S+2H_2O$
(viii) $BaCl_2+Na_2SO_4 \rightarrow BaSO_4+2NaCl$
(ix) $3I_2+6NaOH \rightarrow NaIO_3+5NaI+3H_2O$
(x) $NaH+H_2O \rightarrow NaOH+H_2$

(xi) $CdS + I_2 \rightarrow CdI_2 + S$
(xii) $Si + 2KOH + H_2O \rightarrow K_2SiO_3 + 2H_2$
(xiii) $CaCO_3 \rightarrow CaO + CO_2$
(xiv) $3HNO_2 \rightarrow HNO_3 + 2NO + H_2O$
(xv) $2Pb(NO_3)_2 \rightarrow 2PbO + 4NO_2 + O_2$

**A-86**

Complete and balance the following equations:

(i) $Al + NaOH + H_2O \rightarrow \ldots + H_2$
(ii) $KMnO_4 + KCl + H_2SO_4 \rightarrow K_2SO_4 + MnSO_4 + H_2O + \ldots$
(iii) $NaOH + CO_2 \rightarrow Na_2CO_3 + \ldots$
(iv) $NaNO_3 \rightarrow NaNO_2 + \ldots$
(v) $Cu_2S + O_2 \rightarrow \ldots + \ldots$
(vi) $\ldots + \ldots \rightarrow Cu + SO_2$
(vii) $Br_2 + OH^- \rightarrow BrO^- + \ldots + H_2O$
(viii) $Cr_2O_7^{2-} + SO_2 + H^+ \rightarrow \ldots + HSO_4^- + H_2O$
(ix) $MnO_4^- + C_2O_4^{2-} + H^+ \rightarrow Mn^{++} + \ldots + H_2O$
(x) $NH_4NO_2$ (heat) $\rightarrow \ldots + \ldots$
(xi) $AlCl_3 + NaOH$ (excess) $\rightarrow NaCl + \ldots + H_2O$
(xii) $CuSO_4 + KI \rightarrow Cu_2I_2 + \ldots + K_2SO_4$
(xiii) $MnO_4^- + Fe^{2+} + H^+ \rightarrow Mn^{++} + \ldots + H_2O$
(xiv) $KClO_3$ (heat) $\rightarrow \ldots + \ldots$
(xv) $Pb(NO_3)_2 \xrightarrow{\text{heat}} PbO + O_2 + \ldots$

**A-87**

Write the chemical formula of each of the following:

| | |
|---|---|
| (i) quick lime | (ii) talc |
| (iii) brimstone | (iv) lunar caustic |
| (v) laughing gas | (vi) baryta water |
| (vii) alumina | (viii) microcosmic salt |
| (ix) calomel | (x) phosgene |
| (xi) nitre | (xii) chalk |
| (xiii) litharge | (xiv) chrome alum |
| (xv) oil of vitriol | (xvi) corrosive sublimate |
| (xvii) white vitriol | (xviii) Epsom salt |
| (xix) Green vitriol | (xx) Glauber's salt |
| (xxi) Calcite | (xxii) Magnetite |
| (xxiii) Plaster of paris | (xxiv) Hypo |
| (xxv) Marsh gas | (xxvi) Galena |
| (xxvii) Dolomite | (xxviii) Red lead |
| (xxix) Magnesite | (xxx) Soda ash |

**A-88**

Complete the following sentences by choosing the appropriate terms from those given within brackets:

(isomorphous, isotopes, allotropes, peroxides, cations, complex ions, metals, inert gases)

(i) $Ca^{++}$ and $Zn^{++}$ are...
(ii) Ne and He are...
(iii) $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ and $(NH_4)_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ are...
(iv) $O_3$ and $O_2$ are...
(v) $H_2O_2$ and $Na_2O$ are...
(vi) Sodium and magnesium are...
(vii) $Cl^{35}$ and $Cl^{37}$ are...
(viii) $Ag(NH_3)^+$ and $Ag(CN)_2^-$ are...

**A-89**

Fill up the blanks:

(i) Two atoms of similar electronegativity would be expected to form a ... compound.
(ii) The oxidation number of chlorine in $NaClO_3$ is ... .
(iii) 5.6 litres of a monoatomic gas at S.T.P. contains ... atoms.
(iv) For the isotope $^{13}C_6$, the number of nucleons is ... .
(v) The shape of the methane molecule is ... .
(vii) Two compounds that have the same molecular formula but different properties are called ... .
(vii) The reaction of calcium carbide with water produces ... .
(viii) Oxidation is the ... of electrons.

**A-90**

Match the compounds with the colour:

| | |
|---|---|
| (i) $CHCl_3$ | (*a*) Colourless |
| (ii) $Pb_3O_4$ | (*b*) Brown |
| (iii) $NO_2$ | (*c*) Yellow |
| (iv) $CuSO_4 \cdot 5H_2O$ | (*d*) Blue |
| (v) $Cl_2$ | (*e*) Greenish yellow |
| (vi) $PbI_2$ | (*f*) Red |
| (vii) Strontium salt | (*g*) Pale yellow |
| (viii) Hydrogen burns in air | (*h*) Crimson colour |

**A-91**

Match the following:

| | |
|---|---|
| (*a*) Nitric acid | (i) Rocket fuel |
| (*b*) Liquid $NH_3$ | (ii) Conductor of electricity |
| (*c*) Liquid $O_2$ | (iii) Solvent |
| (*d*) Graphite | (iv) Photographic processing |
| (*e*) $CHCl_3$ | (v) Highly covalent material |
| (*f*) Sodium thiosulphate | (vi) Fertilizer |
| (*g*) Diamond | (vii) Bleaching agent |
| (*h*) $(NH_4)_2SO_4$ | (viii) Reducing agent |
| (*i*) Carbon | (ix) Refrigerant |
| (*j*) $H_2O_2$ | (x) Explosive |

**A-92**

Account for the following. Limit your answer to one sentence for each:

(i) Bleaching of flowers by $Cl_2$ is permanent, while bleaching with $SO_2$ is temporary.

(ii) Nitric oxide becomes brown when exposed to air.

(iii) $H_2O$ is a liquid while $H_2S$ is a gas at ordinary temperature.

(iv) Although the molecules in a gas are moving about rapidly, a closed vessel of negligible weight, containing the gas, still does not move.

(v) A few drops of conc. $HNO_3$ is added precipitating the hydroxides of the third group, in qualitative analysis.

(vi) Aqueous solution of $AlCl_3$ behaves acidic towards litmus while that of NaCl does not.

(vii) Sodium is kept under kerosene.

(viii) A small amount of acid or alkali is added to the electrolyte before electrolysis.

(ix) $CO_2$ does not burn in air and does not support combustion, but a burning Mg wire continues to burn in it.

(x) Chloroform is stored in dark-coloured bottles.

## PART B

**B-1**

50 ml of a gaseous mixture of hydrogen and hydrogen chloride was exposed to sodium amalgam. The volume decreased to 42.5 ml. If 100 ml of this mixture is mixed with 50 ml of gaseous ammonia and then exposed to water, what will be the final volume? Given that all the volumes are measured at a same temperature and pressure.

**B-2**

0.44 g of a colourless oxide of nitrogen occupies 224 ml at 1520 mm of Hg and 273°C. Identify the compound. How much in grams does one molecule of this compound weigh?

**B-3**

A certain gas occupies 0.418 litre at 27°C and 740 mm of Hg.

(i) What is its volume at S.T.P.?

(ii) If the same gas weighs 3.0 g, what is its molecular weight?

(iii) If we increase the weight of the gas to 7.5 g in the same vessel i.e. 0.418 litre capacity and the temperature is reduced to 280 K, what would be its pressure now?

**B-4**

A 2 litre glass vessel contains nitrogen and water vapour in equimolecular proportions at a pressure of $10^{-3}$ mm mercury at 303 K. calculate (i) the number of moles of nitrogen and water vapour present, and (ii) the total mass of the gaseous mixture. Find the mass of the gas when the mixture is cooled to $-50$°C.

**B-5**

A compound exists in the gaseous state both as a monomer (A) and a dimer ($A_2$). The molecular weight of the monomer is 48. In an experiment,

96 g of the compound was confined in a vessel of volume 33.6 litres and heated to 273°C. Calculate the pressure developed, if the compound exists as a dimer to the extent of 50 per cent by weight under these conditions.

**B-6**

It was found that 300 ml of a gas at 27°C and 800 mm of Hg weighed 0.455 g. Find the molecular weight of the gas.

**B-7**

180 ml of a hydrocarbon diffuses through a porous membrane in 15 minutes while 120 ml of $SO_2$, under identical conditions, diffuses in 20 minutes. What is the molecular weight of the hydrocarbon?

**B-8**

A mixture, containing 1.12 litres of $D_2$ at N.T.P. is taken inside a bulb connected to another bulb by a stop cock with a small opening. The second bulb is fully evacuated, the stop cock opened for a certain time and then closed. The first bulb is now found to contain 0.05 g of $H_2$. Determine the percentage composition by weight in the second bulb. (At. wt. : H = 1 D = 2)

**B-9**

One litre each of $O_2$ and $H_2$ are taken in a vessel of 2 litre capacity at N.T.P. The gases are made to combine by aplying an electric spark. Assuming that water is formed quantitatively, how many grams of water are formed? What is the other component present in the vessel and of what weight? If the vessel is now heated to 100°C, what will be the total pressure inside the vessel in mm of Hg?

At. wt. : H = 1

O = 16

$R = 0.082$ litre atm/deg/mol

**B-10**

Calculate the relative rates of diffusion of $^{235}UF_6$ and $^{238}UF_6$ in gaseous form. (At. wt. of F = 19)

**B-11**

How many millilitres of concentrated sulphuric acid of specific gravity 1.84 containing 94% $H_2SO_4$ by weight are required to prepare 200 ml of 0.5 N sulphuric acid?

**B-12**

1.00 g sample of $H_2O_2$ solution containing $X$ per cent of $H_2O_2$ by weight requires $X$ ml of $KMnO_4$ solution for complete oxidation under acidic conditions. Calculate the normality of the $KMnO_4$ solution.

**B-13**

A bottle of commercial sulphuric acid (density = 1.787 g/ml) is labelled as 86 per cent by weight. What is the molarity of the acid? What volume of the acid has to be used to make 1 litre of 0.2 M $H_2SO_4$?

**B-14**

(*a*) A zinc rod weighing 25 g was kept in 100 ml of 1 M $CuSO_4$ solution. After a certain time, the molarity of $Cu^{++}$ in the solution was 0.8. What was the molarity of $SO_4^{--}$? What was the weight of Zn rod after cleaning?

(*b*) If the above experiment is done with a copper rod of weight 25 g and 50 ml of 2 M $ZnSO_4$ solution, what would be the molarity of $Zn^{++}$ at the end of same interval of time?

**B-15**

What is the molarity and molality of a 13% solution (by weight) of $H_2SO_4$ of density 1.090 g/ml. To what volume should 100 ml of this acid be diluted in order to prepare a 1.5 N solution?

**B-16**

Calculate the total number of electrons present in 1.6 g of methane. Take Avogadro's number $= 6.02 \times 10^{23}$.

**B-17**

10 ml of a gaseous hydrocarbon was burnt completely in 80 ml of $O_2$ at N.P.T. The remaining gas occupied 70 ml at N.T.P. This volume became 50 ml on treatment with KOH solution. What is the empirical formula of hydrocarbon?

**B-18**

What volume of oxygen gas at S.T.P is necessary for complete combustion of 20 litres of propane ($CH_3—CH_2—CH_3$) measured at 27° and 760 mm of Hg?

**B-19**

One litre of an acidified solution of $KMnO_4$ containing 15.8 g of $KMnO_4$ is decolorised by passing sufficient amount of $SO_2$. If $SO_2$ is produced by roasting of iron pyrites ($FeS_2$), what will be the amount of pyrites required to produce the necessary amount of $SO_2$?

**B-20**

Nine volumes of a gaseous mixture consisting of a gaseous organic compound A, and just sufficient amount of oxygen required for complete combustion yield, on burning, 4 volumes of $CO_2$, 6 volumes of the water vapour and 2 volumes of $N_2$, all volumes measured at the same temperature and pressure. If the compound A contained only carbon, hydrogen and nitrogen (i) how many volumes of oxygen are required for complete combustion? (ii) what is the molecular formula of compound A?

**B-21**

500 cc of a gaseous hydrocarbon burnt in excess of oxygen yield 2500 cc of carbon dioxide and 3.0 litres of water, all the volumes being measured at the same temperature and pressure. What is the formula of the hydrocarbon?

**B-22**

One gram of an alloy of aluminium and magnesium reacts with excess of hydrochloric acid to form aluminium chloride, magnesium chloride and hydrogen. The hydrogen collected over mercury at 0°C, occupied 1200 ml at 699 mm of Hg. What is the composition of the alloy?

**B-23**

25.4 g of iodine and 14.2 g of chlorine are made to react completely to yield a mixture of ICl and $ICl_3$. Calculate the ratio of the moles of ICl and $ICl_3$ formed.

**B-24**

Brass is an alloy of copper and zinc. A sample of brass weighing 5.793 g when treated with an excess of dilute sulphuric acid gives 324 ml of dry hydrogen at 20°C and 750 mm of Hg pressure. What is the percentage by weight of copper in the alloy?

**B-25**

Compound A is a greenish crystalline salt which gives the following results when tested:

(*a*) Addition of $BaCl_2$ solution to a solution of A results in the formation of a white precipitate B, which is insoluble in dil HCl.

(*b*) On heating, water vapour and two oxides of sulphur, C and D are liberated, leaving a red brown residue E.

(*c*) E dissolves in warm conc. HCl to give a yellow solution F.

(*d*) With $H_2S$, the solution F yields a yellowish-white precipitate G, which when filtered leaves a greenish filtrate H.
Identify the substances A to H.

**B-26**

An organic liquid (A), containing C, H and O with boiling point 78°C and possessing a rather pleasant odour, on heating with concentrated sulphuric acid, gives a gaseous product (B) with the empirical formula, $CH_2$. B decolourizes bromine water as well as alkaline $KMnO_4$ solution and takes up one mole of $H_2$ (per mole of B) in the presence of finely-divided nickel at high temperature. Identify A and B.

**B-27**

Compound A is a green, crystalline solid. It gives the following tests:

(*a*) It dissolves in dil. $H_2SO_4$. No gas is produced.

(*b*) A drop of $KMnO_4$ is added to the above solution. The pink colour disappears.

(*c*) A is heated strongly. Gases B and C with pungent smell come out. A brown residue D is left behind.

(*d*) The gas mixture (B and C) is passed into a dichromate solution which turns green.

(*e*) The green solution of step (d) gives a white precipitate E with a solution of barium nitrate.

(*f*) Residue D from step (c) is heated on charcoal in a reducing flame. It gives a magnetic substance E.

Name the compound A to E.

**B-28**

One mole of a hydrocarbon (A) reacts with one mole of bromine giving a dibromocompound $C_5H_{10}Br_2$. 'A' on treatment with cold, dilute alkaline $KMnO_4$ forms $C_5H_{12}O_2$. On ozonolysis A gives equimolar quantities of propanone and ethanal. Deduce the structural formula of A.

**B-29**

A mixture of FeO and $Fe_3O_4$ when heated in air to a constant weight, gains 5% in its weight. Find the composition of the intial mixture.

**B-30**

A sample of gaseous hydrocarbon occupying 1.12 litres at N.T.P. when completely burnt in air produced 2.2 g of $CO_2$ and 1.8 g of $H_2O$. Calculate

the weight of the compound taken and the volume of $O_2$ at N.T.P required for its burning. Find the molecular formula of the hydrocarbon.

**B-31**

From 200 mg of $CO_2$, $10^{21}$ molecules are removed. How many moles of $CO_2$ are left?

**B-32**

Naturally occurring boron consists of the isotopes whose atomic weights are 10.01 and 11.01. The atomic weight of natural boron is 10.81. Calculate the percentage of each isotope in natural boron.

**B-33**

One gram of an alloy of aluminium and magnesium when treated with excess of dilute HCl forms magnesium chloride, aluminium choride and hydrogen. The evolved hydrogen, collected over mercury at 0°C, has a volume of 1.20 litres at 0.92 atmospheric pressure. Calculate the composition of the alloy.

**B-34**

A sample of a volatile compound, weighing 0.22 g, containing carbon, hydrogen and chlorine only, yielded on combustion in oxygen, 0.195 g $CO_2$ and 0.0804 g $H_2O$. 0.120 g of compound occupied a volume of 37.24 ml at 105°C and 768 mm of Hg pressure. Calculate the molecular formula of the compound.

**B-35**

What weight of AgCl will be precipitated when a solution containing 4.77 g of NaCl is added to a solution of $AgNO_3$?

**B-36**

Which of the following has the highest mass?

(i) 20 g of phosphorous
(ii) 5 moles of water
(iii) 2 equivalent weights of $Na_2CO_3$
(iv) $12 \times 10^{24}$ atoms of hydrogen

**B-37**

A current of 3.7 A is passed for 6 hours between nickel electrode in 0.5 litres of a 2 M solution of $Ni(NO_3)_2$. What will be the molarity of the solution at the end of the electrolysis?

**B-38**

Igniting $MnO_2$ in air converts it quantitatively to $Mn_3O_4$. A sample of pyrolusite is of the following composition:
$MnO_2$, 80% $SiO_2$ and other inert constituents, 15%, rest being water. The sample is ignited in air to constant weight. What is the percentage of Mn in the ignited sample?

**B-39**

How many moles of sodium propionate should be added to one litre of an aqueous solution containing 0.020 mole of propionic acid to obtain a buffer solution of pH 4.75? What will be the pH if 0.010 mole of HCl is dissolved in the above buffer solution? Compare the last pH value with the pH of a 0.010 molar HCl solution. $K_a$ at 25°C for propionic acid $= 1.34 \times 10^{-5}$

**B-40**

The enthalpy for the following reactions ($\Delta H^\circ$) at 25°C are given:

$$\tfrac{1}{2}H_2(g) + \tfrac{1}{2}O_2(g) \rightarrow OH(g);\ 10.06\ \text{kcal}$$
$$H_2(g) \rightarrow 2H(g);\ 104.18\ \text{kcal}$$
$$O_2(g) \rightarrow 2O(g);\ 118.32\ \text{kcal}$$

Calculate the O—H bond energy in the hydroxyl radical.

# ANSWERS

## PART A

**A-1** (d)
**A-2** (*c*)
**A-3** (*d*)
**A-4** (*a*)
**A-5** (*a*)
**A-6** (*a*)
**A-7** (*a*)
**A-8** (*b*)
**A-9** (*b*)
**A-10** (*a*)
**A-11** (*b*)
**A-12** (*d*)
**A-13** (*b*)
**A-14** (*c*)
**A-15** (*c*)
**A-16** (*b*)
**A-17** (*a*)
**A-18** (*c*)
**A-19** (*b*)
**A-20** (*d*)
**A-21** (*a*)
**A-22** (*a*)
**A-23** (*d*)
**A-24** (*d*)
**A-25** (*a*)
**A-26** (*d*)
**A-27** (*c*)
**A-28** (*c*)
**A-29** (*b*)
**A-30** (*a*)
**A-31** (*b*)
**A-32** (*b*)
**A-33** (*a*)
**A-34** (*b*)
**A-35** (*b*)
**A-36** (*d*)
**A-37** (*a*)
**A-38** (*a*)
**A-39** (*d*)
**A-40** (*c*)
**A-41** (*c*)
**A-42** (*c*)
**A-43** (*b*)
**A-44** (*c*)
**A-45** (*b*)
**A-46** (*a*)
**A-47** (*c*)
**A-48** (*c*)
**A-49** (*b*)
**A-50** (*a*)
**A-51** (*b*)
**A-52** (*c*)
**A-53** (*c*)
**A-54** (*b*)
**A-55** (*b*)
**A-56** (*b*)
**A-57** (*d*)
**A-58** (*b*)
**A-59** (*d*)
**A-60** (*b*)
**A-61** (*c*)
**A-62** (*b*)
**A-63** (*d*)
**A-64** (*a*)
**A-65** (*c*)
**A-66** (*a*)
**A-67** (*d*)
**A-68** (*c*)
**A-69** (*c*)
**A-70** (*d*)
**A-71** (*d*)
**A-72** (*d*)
**A-73** (*c*)
**A-74** (*b*)
**A-75** (*b*)
**A-76** (*b*)
**A-77** (*a*)
**A-78** (*b*)
**A-79** (*d*)
**A-80** (*c*)

**A-81**

(i) $HT < HBr < HCl$
(ii) $Li < Na < K$
(iii) Na, Cu, Hg
(iv) $Cu < Zn < Na$
(v) $H_2 < Cl_2 < Br_2$
(vi) $ZnO < PbO < MgO$
(vii) S < graphite < Cu
(viii) $H_2S < H_2SO_3 < HNO_3$

(ix) $Cu < Zn < Al$

(x) $N_2 < O_2 < Cl_2$

(xi) $KNO_3 < Pb(NO_3)_2 < NH_4NO_3$

(xii) $HClO < HClO_3 < HClO_4$

(xiii) $NO_2 < ZnO < K_2O$

(xiv) $Al < Fe < Na$

(xv) $C_2H_5OH < CO_2 < CH_4$

(xvi) $Al_2O_3 < MgO < Na_2O$

(xvii) $HNO_2 < NO_2 < HNO_3$

(xviii) $HCl < KClO_3 < KClO_4$

(xix) $Al < Fe < Pb$

(xx) $F < Cl < Br < I$

(xxi) $NaHCO_3 < CaCO_3 < Na_2CO_3$

(xxii) $AgCl < PbCl_2 < NaCl$

**A-82**

(i) ionic
(ii) covalent
(iii) ionic
(iv) covalent
(v) covalent
(vi) ionic
(vii) covalent
(viii) covalent
(ix) metallic
(x) Covalent
(xi) covalent
(vii) covalent
(xiii) ionic
(xiv) ionic
(xv) ionic
(xvi) ionic
(xvii) covalent
(xviii) covalent
(xix) covalent
(xx) ionic

**A-83**

(i) +1
(ii) +2
(iii) +5
(iv) +4
(v) +3
(vi) 0
(vii) −3
(viii) −1
(ix) 0
(x) +7

**A-84**

(i) −2
(ii) +7
(iii) 0
(iv) +3
(v) +7
(vi) +6
(viii) +2

**A-85**

(i) $SO_2$ is oxidized to $H_2SO_4$ and $Cl_2$ is reduced to HCl.
(ii) There is neither oxidation nor reduction.
(iii) Hg is oxidised to $HgI_2$ and $Hg_2I_2$.
(iv) Ca is reduced to $CaH_2$.
(v) $KClO_3$ is oxidised to $KClO_4$.
(vi) $K^{\circ}$ is oxidised to $K^{+1}Cl$ and $Al^{+3}Cl_3$ is reduced to $Al^{\circ}$.
(vii) $H_2S^{-2}$ is oxidised to $S^{\circ}$ and $S^{+4}O_2$ is reduced to $S^{\circ}$.
(viii) Since there is no change in the oxidation number of any atom, therefore, none is oxidised or reduced.
(ix) $I^0$ is oxidised to $NaI^{5+}O_2$ and $I_2^0$ is reduced to $NaI^{-1}$.
(x) $NaH^{-1}$ is oxidized to $H_2^0$ and $H_2^{+1}O$ is reduced to $H_2^0$
(xi) $CdS^{-2}$ is oxidised to $S^0$ and $I_2^0$ is reduced to $CdI_2^{1-}$.

(xii) $Si^0$ is oxidised to $K_3Si^{4+}O_3$ and $H_2^{+1}O$ is reduced to $H_2^0$.
(xiii) Since there is no change in the oxidation number of any atom, therefore, none is oxidised or reduced.
(xiv) $HN^{+3}O_2$ is oxidised to $HN^{+5}O_2$ and $H^{3+}NO_2$ is reduced to $N^{-2}O$
(xv) $[NO_3^{-2}]^-$ is oxidised to $O_2^0$ and $N^{+5}O_3^-$ is reduced to $N^{+4}O_2$.

**A-86**

(i) $2Al + 2NaOH + 2H_2O \rightarrow 2NaAlO_2 + 3H_2$
(ii) $2KMnO_4 + 10KCl + 8K_2SO_4 \rightarrow 6K_2SO_4 + 2MnSO_4 + 8H_2O + 5Cl_2$
(iii) $2NaOH + CO_2 \rightarrow Na_2CO_3 + H_2O$
(iv) $2NaNO_3 \xrightarrow{heat} NaNO_2 + O_2$
(v) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$
(vi) $2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$
(vii) $Br_2 + 2OH^- \rightarrow BrO^- + Br^- + H_2O$
(viii) $Cr_2O_7^{2-} + 3SO_2 + 5H^+ \rightarrow 2Cr^{3+} + 3HSO_4^- + H_2O$
(ix) $2MnO_4^- + 5C_2O_4^{2-} + 5H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$
(x) $NH_4NO_2$ (heat) $\rightarrow N_2 + 2H_2O$
(xi) $AlCl_3 + 4NaOH \rightarrow 3NaCl + NaAlO_2 + 2H_2O$
(xii) $2CuSO_4 + 4KI \rightarrow Cu_2I_2 + I_2 + K_2SO_4$
(xiii) $MnO_4^- + 5Fe^{++} + 8H^+ \rightarrow Mn^{++} + 5Fe^{+++} + 4H_2O$
(xiv) $2KClO_3$ (heat) $\rightarrow 2KCl + 3O_2$
(xv) $2Pb(NO_3)_2 \xrightarrow{heat} 2PbO + O_2 + 4NO_2$

**A-87**

(i) $CaO$
(ii) $3MgO \cdot 4SiO_2 \cdot H_2O$
(iii) sulphur
(iv) $AgNO_3$
(v) $N_2O$
(vi) $Ba(OH)_2$
(vii) $Al_2O_3$
(viii) $Na(NH_4) \cdot HPO_4 \cdot 4H_2O$
(ix) $Hg_2Cl_2$
(x) $COCl_2$
(xi) $KNO_3$
(xii) $CaCO_3$
(xiii) $PbO$
(xiv) $K_2SO_4 \cdot Cr_2(SO_4)_3 \cdot 24H_2O$
(xv) $H_2SO_4$
(xvi) $HgCl_2$
(xvii) $ZnSO_4 \cdot 7H_2O$
(xviii) $MgSO_4 \cdot 7H_2O$
(xix) $FeSO_4 \cdot 7H_2O$
(xx) $Na_2SO_4 \cdot 10H_2O$
(xxi) $CaCO_3$
(xxii) $Fe_2O_3$
(xxiii) $2CaSO_4 \cdot H_2O$
(xxiv) $Na_2S_2O_3 \cdot 5H_2O$
(xxv) $CH_4$
(xxvi) $PbS$
(xxvii) $CaCO_3 \cdot MgCO_3$
(xxviii) $Pb_3O_4$
(xxix) $MgCO_3$
(xxx) $Na_2CO_3$

**A-88**

(i). Cations
(ii) Inert gases
(iii) Isomorphous
(iv) Allotropes
(v) Peroxides
(vi) Metals
(viii) Isotopes
(viii) Complex ions

**A-89**

(i) Covalent
(ii) +5
(iii) $1/4 \times 6.023 \times 10^{23}$
(iv) 13
(v) Tetrahedral
(vi) Isomer
(vii) Acetylene and calcium hydroxide
(viii) loss

**A-90**

(i) (*a*)
(ii) (*f*)
(iii) (*b*)
(iv) (*d*)
(v) (*e*)
(vi) (*c*)
(vii) (*h*)
(viii) (*g*)

**A-91**

(*a*) (x)
(*b*) (ix)
(*c*) (i)
(*d*) (ii)
(*f*) (iv)
(*g*) (v)
(*h*) (vi)
(*i*) (viii)
(*j*) (vii)

**A-92**

(i) $Cl_2$ bleaches the colour by oxidation but $SO_2$ bleaches by reaction
(ii) Because of $NO_2$ formation which is brown in colour
(iii) Because of hydrogen bending in $H_2O$
(iv) Motion of molecules is at random, therefore, no net force in any particular direction
(v) $HNO_3$ oxidizes ferrous to ferric
(vi) $AlCl_3$ hydrolyses giving HCl
(vii) Because sodium reacts with air and moisture of the atmosphere
(viii) To make water electrically conducting
(ix) Mg reduces $CO_2$ to C, hence the wire continues to burn

$$2Mg + CO_2 \rightarrow 2MgO + C$$

(x) $CHCl_3$ reacts wiih moisture in sunlight.

$$CHCl_3 + H_2O \xrightarrow{\text{Sunlight}} COCl_2 + 2HCl$$

## PART B

**B-1** 70 ml
**B-2** $N_2O$, $7.30 \times 10^{-23}$ g
**B-3** (i) 0.3704 litre
(ii) 181.14
(iii) 2.27 atm.
**B-4** (i) $0.529 \times 10^{-7}$, $0.529 \times 10^{-7}$
(ii) $24.33 \times 10^{-7}$ g, $14.81 \times 10^{-7}$ g
**B-5** 2 atm
**B-6** 28
**B-7** 16

**B-8** $H_2$ : 41.7%
$D_2$ : 58.3%
**B-9** 0.8 g, 778.3 mm of Hg
**B-10** 1.0043 : 1.00
**B-11** 2.72 ml
**B-12** 0.059 N
**B-13** 12.73 ml
**B-14** (*a*) 1M, 23.692 gm (*b*) 2 M
**B-15** Molarity = 1.44
Molarity = 1.52
Volume = 192 ml
**B-16** $6.02 \times 10^{23}$
**B-17** $C_2H_4$
**B-18** 91 litres
**B-19** 15 g
**B-20** (i) 7 volumes (ii) $C_2H_6N_2$
**B-21** $C_5H_{12}$
**B-22** % of Al : 54.77
% of Mg : 45.23
**B-23** 1 : 1
**B-24** 85.03%
**B-25** A is $FeSO_4$
B is $BaSO_4$
C is $SO_2$
D is $SO_3$
E is $Fe_2O_3$
F is $FeCl_3$
G is S
H is $FeCl_2$
**B-26** A is $C_2H_5OH$
B is $C_2H_4$
**B-27** A is $FeSO_4 \cdot 7H_2O$
B is $SO_2$
C is $SO_3$
D is $Fe_2O_3$
E is Fe
**B-28** $\begin{array}{l} CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-CH_3 \end{array}$
**B-29** FeO : 20.2%, $Fe_3O_4$ : 79.8%
**B-30** 16 gm; 22.4 litres, $CH_4$
**B-31** $2.88 \times 10^{-3}$ moles
**B-32** Isotope of mass 10.01 is 20% and the other is 80%
**B-33** Weight of Al = 0.566 gm
Weight of Mg = 0.434 gm
**B-34** $C_2H_4Cl_2$
**B-35** 4.87 g
**B-36** equivalents of $Na_2CO_3$
**B-37** 1.172
**B-38** 59.4 %
**B-39** 4.09, For HCl in $H_2O$, pH = 2
**B-40** $-101.19$ kCal mol$^{-1}$

# MATHEMATICS

## PART A

Select the correct alternative in each of the following.

**A-1**

The area of the triangle with vertices at $(-4, -1)$, $(1, 2)$, $(4, -3)$ is:

(a) 17 sq. units (b) 18 sq. units

(c) $\frac{15}{2}$ sq. units (d) none of these

**A-2**

The area of the quadrilateral with vertices at $(2, -1)$, $(4, 3)$, $(-1, 2)$, $(-3, -2)$ is:

(a) 24 sq. units (b) 18 sq. units

(c) 14 sq. units (d) none of these

**A-3**

The vertices of a triangle are at $(0, 0)$, $(3, 0)$ and $(0, 4)$. Its orthocentre is:

(a) $\left(\frac{3}{2}, 4\right)$ (b) $(4, 2)$

(c) $(0, 3)$ (d) $(0, 0)$

**A-4**

The lines $x^2-3y^2=0$ and $x=a$ enclose a triangle that is:

(a) equilateral (b) right angled

(c) isosceles with base angles equal to 30°

(d) none of these

**A-5**

The length of the tangent from $(5, 1)$ to the circle

$$x^2+y^2+6x-4y-3=0 \text{ is:}$$

(a) 21 (b) 14

(c) 7 (d) 2

**A-6**

The straight line passing through the point of intersection of the straight lines $x-3y+1=0$ and $2x+5y-9=0$ having an infinite slope and at a distance of 2 units from the origin is:

(a) $x=2$ (b) $3x-4y=5$

(c) $y=2$ (d) $x+y=12$

**A-7**

The distance between $6x+8y-15=0$ and $3x+4y-9=0$ is:

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$

(c) $\frac{3}{10}$ (d) $\frac{2}{7}$

**A-8**

$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$ is equal to:

(*a*) 4 (*b*) 2
(*c*) 1 (*d*) none of these

**A-9**

The greatest value of $\sin\theta + \cos\theta$ is:

(*a*) 1 (*b*) 2
(*c*) $\sqrt{2}$ (*d*) 0

**A-10**

Given $x\cos\theta = y\cos(\theta + 120) = z\cos(\theta + 240°)$, then the value of $xy + yz + zx$ is:

(*a*) 0 (*b*) 1
(*c*) $-1$ (*d*) 2

**A-11**

The equation $(a+b)^2 \sec^2 x = 4ab$ is possible only for:

(*a*) $a = 0$ (*b*) $b = 0$
(*c*) $a \neq b$ (*d*) $a = b$

**A-12**

The equation $y = 3\sin x + 4\cos x$ has a real solution if:

(*a*) $|y| \leqslant 5$ (*b*) $y < 5$
(*c*) $y < 5$ (*d*) $y \geqslant 5$

**A-13**

If $\sin 2\theta = \cos 3\theta$ where $\theta$ is an acute angle then $\sin\theta$ equals:

(*a*) $\frac{\sqrt{5}-1}{4}$ (*b*) $\frac{-\sqrt{5}-1}{4}$

(*c*) $\frac{\sqrt{5}+1}{4}$ (*d*) $\frac{-\sqrt{5}+1}{4}$

**A-14**

In $\triangle ABC$, $\operatorname{cosec} A(\sin B\cos C + \cos B\sin C)$ is equal to:

(*a*) 1 (*b*) 0
(*c*) $-1$ (*d*) none of these

**A-15**

$\tan 5x - \tan 3x - \tan 2x$ is equal to:

(*a*) $\tan 5x \tan 3x \tan 2x$
(*b*) $-\tan 5x \tan 3x \tan 2x$
(*c*) $\tan 5x \tan 3x - \tan 3x \tan 2x - \tan 2x \tan 5x$
(*d*) none of these

**A-16**

$\sin(180 + \phi)\sin(180 - \phi)\operatorname{cosec}^2$ is equal to

(*a*) 1 (*b*) $-1$
(*c*) 0 (*d*) $\infty$

**A-17**

If $a \neq b \neq c$, one value of $x$ that satisfies the equation:.

$$\begin{vmatrix} 0 & x-a & x-a \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0, \text{ is given by}$$

(*a*) $x=0$ (*b*) $x=a$
(*c*) $x=b$ (*d*) $x=c$

**A-18**

In $\triangle ABC$, $AB=5$ units, $AC=12$ units and $BC=13$ units. The distance of $A$ from $BC$ is

(*a*) 60/13 units (*b*) 25/14 units
(*c*) 60/17 units (*d*) none of these

**A-19**

The term with the highest numerical value in the expansion of $(1+4x)^8$ when $x=1/3$, is the

(*a*) sixth (*b*) fifth
(*c*) fourth (*d*) third

**A-20**

$$\text{If } \triangle = \begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix}, \text{ then the roots of } \triangle = 0 \text{ are given by:}$$

(*a*) $x=a, x=m$ (*b*) $x=a, x=b$
(*b*) $x=-a, x=-b$ (*d*) $x=0, x=m$

**A-21**

The value of deteriminant

$$\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix} \text{ is equal to}$$

(*a*) 0 (*b*) 1 (*c*) $-1$ (*d*) 4

**A-22**

If $\omega$ is a complex cube root of unity, then the value of

$(3+\omega^2+\omega^4)^6$ is equal to:

(*a*) 0 (*b*) 49
(*c*) 64 (*d*) 124

**A-23**

If $\omega^m+\omega^{2m}+1=0$, $m$ being an integer, then $m$ must be a multiple of:

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 7

**A-24**

If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3+8=0$ are

(*a*) $-1, -1, -1$ (*b*) $-1, \omega, \omega^2$
(*c*) $-1, 1-2\omega, 1-2\omega^2$ (*d*) $1, \omega, 1-\omega^2$

**A-25**

If $\omega$ is a complex cube root of unity, then the value of

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} \text{ is equal to:}$$

(*a*) 0 (*b*) 1
(*c*) $-1$ (*d*) 2

**A-26**

$\left(\dfrac{1+\sqrt{-1}}{1-\sqrt{-1}}\right)^m$ gives the smallest integral value for:

(*a*) $n=4$ (*b*) $n=6$
(*c*) $n=8$ (*d*) $n=10$

**A-27**

If $p=(i+\sqrt{3})/2$, then $p^3$ is equal to:

(*a*) $i$ (*b*) $-i$
(*c*) 1 (*d*) $-1$

**A-28**

In the complex plane, the set of points defined by the equation $|Z| = 1$ is a/an:

(*a*) circle (*b*) straight line
(*c*) ellipse (*d*) parabola

**A-29**

If $\alpha, \beta$ are the roots of $ax^2+bx+c=0$, then the equation whose roots are $\alpha^{-1}$ and $\beta^{-1}$ is:

(*a*) $ax^2+2bx+c=0$ (*b*) $ax^2-bx-c=0$
(*c*) $cx^2+bx+a=0$ (*d*) $bx^2-ax+c=0$

**A-30**

If $x=(1+(1+(1+\ldots)^{1/2})^{1/2})^{1/2}$, then the value of $x$ is equal to

(*a*) $\dfrac{1-\sqrt{5}}{2}$ (*b*) $\dfrac{1+\sqrt{5}}{2}$

(*c*) $\dfrac{\sqrt{5}}{2}$ (*d*) 1

**A-31**

The $m^{\text{th}}$ terms of the series $3+10+17+\ldots$ and $63+65+67+\ldots$ are equal for:

(*a*) $m=5$ (*b*) $m=7$
(*c*) $m=9$ (*d*) $m=13$

**A-32**

If $y=1+x+x^2+x^3+\dots$, then

(*a*) $x=y$ (*b*) $x=\frac{y}{1-y}$

(*c*) $x=\frac{y-1}{y}$ (*d*) none of these

**A-33**

If $A$, $G$ and $H$ are respectively the arithmetic, geometric and harmonic means of two numbers $x$ and $y$, then

(*a*) $A=GH$ (*b*) $G=AH$

(*c*) $A=\sqrt{AH}$ (*d*) $H=AG$

**A-34**

If $H$ is the harmonic mean of $p$ and $q$, then the value of

$$\frac{1}{H-p}+\frac{1}{H-q} \text{ is:}$$

(*a*) 0 (*b*) 1

(*c*) $pq$ (*d*) $\frac{p+q}{pq}$

**A-35**

If $a$, $b$ and $c$ are in G.P., then $\log_a p$, $\log_b p$, $\log_c p$ are in:

(*a*) A.P. (*b*) G.P.

(*c*) H.P. (*d*) none of these

**A-36**

The number of terms in the expansion $(1+5\sqrt{2}\,x)^9+(1-5\sqrt{2}\,x)^9$ are:

(*a*) 5 (*b*) 7

(*c*) 9 (*d*) 10

**A-37**

A polygon has 44 diagonals, the number of its sides is given by:

(*a*) 7 (*b*) 9

(*c*) 11 (*d*) 13

**A-38**

The value of ${}^{24}C_4+\sum_{j=1}^{5} {}^{29-j}C_3$ is equal to:

(*a*) ${}^{24}C_5$ (*b*) ${}^{29}C_5$

(*c*) ${}^{29}C_4$ (*d*) ${}^{29}C_6$

**A-39**

If 7 points out of 12 lie on the same straight line, the number of triangles formed by them is equal to:

(*a*) 144 (*b*) 185

(*c*) 213 (*d*) 246

**A-40**

$\lim\limits_{x\to\infty} \dfrac{\sin x}{x}$ is equal to:

(a) 1 (b) 0

(c) $\infty$ (d) $\pi/2$

**A-41**

If $f(x)=\dfrac{|x-a|}{x-a}$, then $f(x)$ is discontinuous when:

(a) $x=a$ (b) $x<a$

(c) $x>a$ (d) $x=1$

**A-42**

All the points on the curve $y=\sqrt{x+\sin x}$, at which the tangent is parallel to $x$-axis lie on a / an :

(a) straight line (b) circle

(c) parabola (d) ellipse

**A-43**

The function $\sin^{-1}\sqrt{x}$ is defined in the interval:

(a) $(-1, 1)$ (b) $(0, 1)$

(c) $(1, 0)$ (d) $(-1, 2)$

**A-44**

If $\phi(x)=\dfrac{a^x-1}{b^x-1}$, then $\lim\limits_{x\to\infty} \phi(x)$ is equal to:

(a) $a \log b$ (b) $\log_a b$

(c) $\log_b^a$ (d) $a/b$

**A-45**

The extreme value of $x^{1/x}$ is:

(a) $e$ (b) $e^{1/e}$

(c) $\left(\dfrac{1}{e}\right)$ (d) $\left(\dfrac{1}{e}\right)^e$

**A-46**

The minimum value of $\dfrac{1}{x}\log x$ is:

(a) $\dfrac{1}{e}$ (b) $e$

(c) 1 (d) $e^2$

**A-47**

If $f(x)=\sin x-\dfrac{x}{2}$ is an increasing function, then

(a) $-\pi/3<x<\pi/3$ (b) $-\pi/3<x<0$

(c) $0<x<\pi/3$ (d) $-\pi<x<\pi$

**A-48**

If $\frac{x}{1+x} < \log(1+x) < x$, then

(*a*) $x > 0$ (*b*) $x < 0$
(*c*) $x = 0$ (*d*) $x =$ infinity

**A-49**

The value of $\lim\limits_{x \to 0} \frac{e^x - (1+x)}{x^2}$ is

(*a*) 2 (*b*) 1
(*c*) 1/2 (*d*) 0

**A-50**

$\lim\limits_{n \to \infty} 4^3(3^{1/n} - 1)$ is equal to:

(*a*) 0 (*b*) $\infty$
(*c*) 4/3 (*d*) 3/4

**A-51**

$\lim\limits_{n \to \infty} \frac{\sum n^2}{n^3}$ is equal to:

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{1}{4}$ (*d*) 1

**A-52**

Differentiation of $\log\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$ w.r.t. $x$ is:

(*a*) $\frac{1}{\sqrt{x}(1-x)}$ (*b*) $\sqrt{x}$

(*c*) $\frac{1}{1-x}$ (*d*) $\frac{\sqrt{x}}{1-x}$

**A-53**

The value of $\int_0^{\pi} \phi \sin^2 \phi \cos \phi \, d\phi$ is:

(*a*) $\frac{4}{9}$ (*b*) $\frac{-4}{9}$

(*c*) $\frac{9}{4}$ (*d*) $\frac{-9}{4}$

**A-54**

The value of $\int_1^e \frac{1+\log x}{x} dx$ is:

(*a*) $\frac{3}{2}$ (*b*) $\frac{1}{2}$

(*c*) $e$ (*d*) $\frac{1}{e}$

**A-55**

If $\int_0^1 \frac{dx}{1-x+x^2} = k\pi$, then $k$ is:

(*a*) $\frac{1}{2\sqrt{3}}$ (*b*) $\frac{2}{3\sqrt{3}}$

(*c*) $\frac{1}{\sqrt{3}}$ (*d*) $2\sqrt{3}$

**A-56**

The value of $\int_0^{\pi/2} \frac{\sin x}{\cos x+\cos x}\, dx$ is:

(*a*) 1 (*b*) $\pi/2$

(*c*) 0 (*d*) $\pi/4$

**A-57**

The area within the curve

$|x|+|y|-4=0$ is:

(*a*) 24 (*b*) 32

(*c*) 36 (*d*) 40

**A-58**

If $\int_0^{\pi} f(x)\, dx=0$, then

(*a*) $f(\pi-x)=f(x)$ (*b*) $f(\pi-x)=-f(x)$

(*c*) $f(\pi+x)=f(x)$ (*d*) $f(\pi+x)=-f(x)$

**A-59**

$\int_{\pi/6}^{\pi/3} \frac{dx}{\sin 2x}$ is equal to:

(*a*) $\frac{1}{2}\log(-1)$ (*b*) $\log(-1)$

(*c*) $\log\sqrt{3}$ (*d*) $\log 3$

**A-60**

If $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, $|\mathbf{a}|=3$, $|\mathbf{b}|=5$, $|\mathbf{c}|=7$, then the least angle between **a** and **b** is:

(*a*) $\frac{2\pi}{3}$ (*b*) $\frac{\pi}{3}$

(*c*) $\frac{\pi}{2}$ (*d*) $\frac{5\pi}{3}$

**A-61**

$\hat{\mathbf{i}}\cdot(\hat{\mathbf{j}}\times\hat{\mathbf{k}})+\hat{\mathbf{j}}\cdot(\hat{\mathbf{i}}\times\hat{\mathbf{k}})+\hat{\mathbf{k}}\cdot(\hat{\mathbf{i}}\times\hat{\mathbf{j}})$ is equal to

(*a*) 0 (*b*) 1

(*c*) $-1$ (*d*) 3

**A-62**

If **a**, **b** and **c** are any three coplanar unit vectors, then

(*a*) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=1$ (*b*) $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}=0$

(*c*) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=1$ (*d*) $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=0$

**A-63**

Two identical dice marked 1 to 6 are tossed together. The probability of the total being equal to 7 in a single throw is:

(*a*) $\frac{5}{6}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{1}{7}$

**A-64**

A person draws a card from a pack of 52 cards, replaces it and shuffles the pack. He continues doing this until he draws a spade. The chance that he will fail first two times is:

(*a*) $\frac{9}{16}$ (*b*) $\frac{1}{16}$

(*b*) $\frac{3}{16}$ (*d*) $\frac{1}{64}$

**A-65**

$\mathbf{a}=3\hat{\mathbf{i}}-\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}$, are the adjacent sides of a parallelogram. Then the area of the parallelogram is:

(*a*) $\frac{\sqrt{7}}{2}$ (*b*) $\frac{\sqrt{3}}{7}$

(*c*) $\frac{\sqrt{3}}{5}$ (*d*) $\sqrt{7}$

Fill in the blanks with suitable substitutions

**A-66**

The solution set of the equation

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0 \text{ is} \ldots$$

**A-67**

If $\begin{vmatrix} 1+y & 1-y & 1-y \\ 1-y & 1+y & 1-y \\ 1-y & 1-y & 1+y \end{vmatrix} = 0$, then the values of $y$ are ... and ...

**A-68**

The equations

$$3x-2y+z=0$$
$$\lambda x-14y+15z=0$$
$$x+2y-3y=0$$

will have non-trivial solution when $\lambda = \ldots$

**A-69**

The sum of the coefficients of the polynomial $(1+x-3x^2)^{2163}$ is ...

**A-70**

If $y=f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x)=\sin x^2$, then $\frac{dy}{dx}=\ldots$

**A-71**

If $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$, then $\cos\theta+\sin\theta=\ldots$

**A-72**

If $\sin x+\sin^2 x=1$, then $\cos^2 x+\cos^4 x=\ldots$

**A-73**

The equation $\sec^2\theta=\frac{4ab}{(a+b)^2}$ is possible only when....

**A-74**

If $7\sin^2\alpha+3\cos^2\alpha-4=0$, $\tan\alpha=\ldots$

**A-75**

If $x+y=8$, the maximum value of $xy$ is ...

**A-76**

The function $(x^3-x^2-x-4)$ is decreasing in the interval $(a, b)$. Then $a=\ldots$ and $b=\ldots$

**A-77**

$\int_0^{\pi/4} e^{2x}(\sin 2x+\cos 2x)\,dx=\ldots$

**A-78**

The area enclosed within the curve

$|x|+|y|=1$ is ...

**A-79**

The vector $\mathbf{a}\times(\mathbf{b}\times\mathbf{a})$ is prependicular to ...

**A-80**

If $\hat{n}$ is a unit vector making an angle of 45° with the $z$-axis and $(\hat{n}+\hat{i}+\hat{j})$ is also a unit vector, then $\hat{\mathbf{n}}=\ldots$

## PART B

**B-1**

(*a*) Prove that

$$\cot 7\tfrac{1}{2}^\circ=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$

(*b*) If $a\cos A-b\sin A=c$, prove that

$$a\sin A+b\cos A=\pm\sqrt{a^2+b^2-c^2}$$

(*c*) If $(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$
$=(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)(\sec\gamma-\tan\gamma)$
prove that each side equals $\pm 1$.

(*d*) If $A+B=m$ and $A-B=n$, find the value of tan $2A$ and tan $2B$ in terms of $m$ and $n$.

(*e*) If $\sin\ \theta+\operatorname{cosec}\ \theta=2$, prove that

$$\sin^n\theta+\operatorname{cosec}^n\theta=2$$

**B-2**

(*a*) If $m\tan(\theta-30°)=n$ tap $(\theta+120°)$, prove that

$$\cos 2\theta=\frac{m+n}{2(m-n)}$$

(*b*) Prove that

$$\frac{1}{\cos 80}-\frac{\sqrt{3}}{\sin 80}=40$$

(*c*) If $\alpha$ and $\beta$ be two distinct values of $\theta$ lying between 0 and $2\pi$ which satisfy $3\cos\theta+4\sin\theta-6=0$, prove that

$$\sin(\alpha+\beta)=\frac{24}{25}$$

(*d*) Prove that

$$(A-\sin A)<(B-\sin B) \text{ provided}$$
$$0<A<B<\pi/2$$

(*e*) The bisector of $\angle A$ of $\triangle ABC$ meets the base $BC$ at $L$. Show that

$$AL=\left(\frac{2bc}{b+c}\right)\cos\frac{A}{2}$$

**B-3**

(*a*) The angular elevation of a tower $CD$ at a place due south is 30° and at a place $B$ due west of $A$ is 18°. If $AB=a$, show that the height of the tower is

$$\frac{a}{\sqrt{2+2\sqrt{5}}}$$

(*b*) Draw the graphs of $\sin 2x$ and $\cos x$ for $x$ between 0 and $\pi$. Hence prove that $\pi/6$, $\pi/2$ and $5\pi/6$ are the roots of the equation

$$\sin 2x=\cos x$$

(*c*) Sketch the graph in the $xy$-plane corresponding to the truth set

$$x\geqslant 0,\ y\geqslant 0,\ x^2+y^2-2y\leqslant 0,\ y\leqslant\sec x$$

(*d*) If $\sin\alpha=\sin\beta$ and
$\cos\alpha=\cos\beta$, prove that
$\alpha=2n\pi+\beta,\qquad n\in I$

(*e*) Show that in $\triangle\ ABC$,

$$a\sin A+b\sin B+C\sin C$$
$$=\sqrt{a^2+b^2+c^2}\ \sqrt{\sin^2 A+\sin^2 B+\sin^2 C}$$

(*f*) Within what limits must $\frac{A}{2}$ lie when

$$2\sin\frac{A}{2}=\sqrt{1+\sin A}+\sqrt{1-\sin A}$$

**B-4**

(*a*) If the arithmetic mean between two number is twice as great as their geometric mean, show that the ratio of the numbers is

$$2+\sqrt{3} : 2-\sqrt{3}$$

(*b*) If the $(m+1)^{th}$, $(n+1)^{th}$ and $(r+1)^{th}$ terms or an A.P. are in G.P. and $m, n, r$ are in H.P., show that the ratio of the common difference to the first term in the A.P. is $-2/n$.

(*c*) Sum to infinite terms

$$\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}+\cdots$$

(*d*) If $\dfrac{a^{m+1}+b^{m+1}}{a^m+b^m}$ be the harmonic mean between $a$ and $b$, show that $m=-1$.

(*e*) If $x$, $y$ and $z$ are in H.P., show that

$$\log(x+z)+\log(x-2y+z)=2\log(x-z)$$

(*f*) The $p^{th}$, $q^{th}$ and $r^{th}$ terms of an H.P. as well as a G.P. are $a$, $b$ and $c$ respectively. Prove that

$$a(b-c)\log a+b(c-a)\log b+c(a-b)\log c=0$$

**B-5**

(*a*) If $1, \omega, \omega^2$ are the three cube roots of unity, prove that

$$(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)=a+b+c$$

(*b*) Show that the origin and the complex numbers represented by the roots of the equation

$$Z^2+aZ+b=0,$$

form an equilateral triangle if

$$a^2=3b$$

(*c*) If $\arg Z_1Z_2=\arg Z_1+\arg Z_2+n\pi$

Show that

$$n=0, \text{ if } -\pi \leqslant \arg Z_1+\arg Z_2<\pi$$
$$n=1, \text{ if } -2\pi \leqslant \arg Z_1+\arg Z_2<-\pi$$

(*d*) Find the real value of $\alpha$, such that

$$\frac{1-i\sin\alpha}{1+2i\sin\alpha}$$

is (i) real,

(ii) purely imaginary.

**B-6**

(*a*) If the equations

$$y^2+\alpha y+\beta y=0$$

and

$$y^2+\beta y+\gamma\alpha=0$$

have a common root, prove that their other roots are given by the equation

$$y^2+\gamma y+\alpha\beta=0$$

(*b*) If $\alpha$ and $\beta$ are the roots of the equation

$$\lambda(y^2-y)+y+5=0$$

and if $\lambda_1$, $\lambda_2$ are the two values of $\lambda$ obtained from $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}$, show that

$$\frac{\lambda_1}{\lambda_2^2}+\frac{\lambda_2}{\lambda_1^2}=4048$$

(*c*) If one root of the equation $ax^2+bx+c=0$ be the square of the other show that

$$a(c-b)^3=c(a-b)^3$$

(*d*) If $\alpha$ and $\beta$ are the roots of $x^2-p(x+1)-c=0$, show that
(i) $(\alpha+1)(\beta+1)=1-c$

and (ii) $\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c}+\frac{\beta^2+2\beta+1}{\beta^2+2\beta+c}=1$

**B-7**

(*a*) Prove by the method of mathematical induction that
(i) $x^{2n-1}+y^{2n-1}$ is divisible by $(x+y)$
(ii) $\log x^n = n\log x$ for $x>0$

(*b*) Prove that

$$\frac{1}{2}+\cos\alpha+\cos 2\alpha+\cos 3\alpha+\ldots+\cos n\alpha=\frac{\sin\left(n+\frac{1}{2}\right)\alpha}{2\sin\frac{\alpha}{2}}$$

$$\alpha\neq 0, \pm 2\pi, \pm 4\pi, \ldots$$

(*c*) Find the term independent of $x$ in

$$\left(\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right)^{10}$$

(*d*) Find the coefficient of $x^{-7}$ in $\left(ax-\frac{1}{bx^2}\right)^{11}$. Find the relation between $a$ and $b$ so that the coefficiens of $x^7$ in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and $x^{-7}$ in $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal.

(*e*) Find the greatest term in the expansion of

$$\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$$

**B-8**

(*a*) Prove that

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)(a^2+b^2+c^2)$$

(b) Prove that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2-2bv+cy^2)$$

(c) If $i=\sqrt{-1}$, prove that

$$\begin{vmatrix} 1+i & 1-i & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix} = 4+7i$$

(d) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

**B-9**

(a) Show that the lines $ax \pm by \pm c = 0$ enclose a rhombus whose area is $\frac{2c^2}{ab}$.

(b) (1, 2) and (3, 8) are a pair of opposite vertices of a square. Find the equations of the sides and the diagonals of the square passing through (1, 2).

(c) The adjacent sides of a parallelogram are $4x+5y=0$ and $7x+2y=0$. If the equation of one diagonal be $11x+7y-9=0$, show that the equation of the other diagonal is $y=x$.

(d) If $O$ to be the origin and coordinates of any two points $Q_1$ and $Q_2$ be $(x_1, y_1)$ and $(x_2, y_2)$ respectively, prove that

$$OQ_1 \times OQ_2 \cos Q_1OQ_2 = x_1x_2 + y_1y_2$$

(e) Find the area of triangle formed by $y$-axis, the straight line $L$ passing through (1, 1) and (2, 0) and the straight line perpendicular to $L$ and passing through $\left(\frac{1}{2}, 0\right)$.

**B-10**

(a) If the vertices of a triangle have integral coordinates, prove that the triangle cannot be equilateral.

(b) Find the equation of a straight line passing through the point of intersection of the lines $x-3y+1=0$ and $2x+5y-9=0$ and whose distance from the origin is $\sqrt{5}$.

(c) A point moves such that the sum of the squares of its distance from the sides of a square of side unity is equal to $P$. Show that the locus is a circle whose centre coincides with the centre of the square. Also show that its radius is two units.

(*d*) If $x \cos \alpha + y \sin \alpha - p = 0$ is the equation of a chord on the circle $x^2+y^2-a^2=0$, the equation of the circle for which this chord is a diameter, is given by

$$x^2+y^2-a^2-2p(x \cos \alpha + y \sin \alpha - p) = 0$$

(*e*) Show that the common tangents to the circles $x^2+y^2+2x=0$ and $(x-3)^2+y^2=9$, form an equilateral triangle.

**B-11**

(*a*) Differentiate $\sqrt{\sin x^2}$ w.r.t. $x$ from first principles

(*b*) Solve $\frac{d}{dx}(\tan^2 ax)$ from first principles.

(*c*) Solve *ab* initio:

$$\frac{d}{dx}[\cos(ax^2+bx+c)]$$

(*d*) If $[x]$ denotes the largest integer less than or equal to $x$, then discuss the continuity for the function $f(x) = x + [x]$, for $x \geqslant 0$, at $x = 2$. Given $f(2) = 4$.

(*e*) Show that

$$\lim_{x \to \infty} \left( \frac{x}{\sqrt{4x^2+1} - 1} \right) = \frac{1}{2}$$

(*f*) Show that

$$\lim_{x \to \pi/2} (1+\cos x)^{3 \sec x} = e^3$$

(*g*) Use the result $\lim_{x \to 0} (1+x)^{1/x} = e$, to prove that

$$\lim_{x \to e} \left( \frac{\log x - 1}{x - e} \right) = e^{-1}$$

**B-12**

(*a*) Find the equations of the tangents drawn from $(4, -2)$ to the circle $x^2+y^2=1$.

(*b*) Prove that tangent to the circle

$$x^2+y^2=5$$

at $(1, -2)$ also touches the circle

$$x^2+y^2-8x+6y+20=0$$

Also show that $(3, -1)$ is its point of contact.

(*c*) Show that the area of the triangle formed by a tangent to the hyperbola

$$2xy = a^2$$

and the coordinate axes is constant.

(*d*) Show that the angle between the tangent at any point $P$ and the line joining $P$ to the origin $O$ is the same at all points of the curve

$$\log(x^2+y^2) = k \tan^{-1} \frac{y}{x},$$

where $k$ is constant.

(*e*) If $f(x) = x(1-\log x)$, $x > o$. show that

$$(a-b)\log c = b(1-\log b) - a(1-\log a), \; o < a < b$$

**B-13**

(*a*) Show that

$$f(x)=\left(1+\frac{1}{x}\right)^{x+1}$$

is a decreasing function of $x$ for $x > 0$.

(*b*) The function $y=\dfrac{ax+b}{(x-1)x-4)}$ has a turning poin at $P(2, -1)$. Find the values of $a$ and $b$ and show that $y$ is a maximum at $P$.

(*c*) Show that the least value of the portion of the tangent to $x^2/a^2+y^2/b^2=1$ intercepted between the axes is $(a+b)$.

(*d*) If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that

$$\frac{dy}{dx}=-\frac{1}{(1+x)^2}$$

(*e*) If $y=(\sqrt{x})^{x^{x^{\cdots}}}$, find $\dfrac{dy}{dx}$.

(*g*) If $y=x\log\left(\dfrac{x}{a+bx}\right)$, show that

$$x^2\frac{d^2y}{dx^2}=\left(\frac{xdy}{dx}-y\right)^2$$

**B-14**

(*a*) Prove that

$$\int_0^{\pi}\frac{dx}{1-2a\cos x+a^2}=\frac{\pi}{1-a^2},\ a<1$$

(*b*) Find $\int(\log x)^2\sqrt{x}\,dx$

(*c*) Find $\int_0^a\sqrt{\dfrac{a-x}{x}}\,dx$

(*d*) Evaluate

(i) $\int_0^{\pi}\sin^m x\cos^n x\,dx$

(ii) $\int_0^{\pi/2}\dfrac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$

(*e*) Find the årea of the region bounded by the $x$-axis and the parabola

$$y=\frac{(2-x-2x^2)}{2}$$

(*f*) Evaluate $\int\sqrt{1+\sin\dfrac{x}{2}}\,dx$

**B-15**

(*a*) Find the vector whose length is 10 units and is inclined to the vectors $(-2\hat{i}+3\hat{j}+6\hat{k})$, $(-\hat{i}-4\hat{j}+8\hat{k})$ at angles 120° and 90° respectively.

(*b*) A constant force having components $\vec{P}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{P}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{P}_3 = \hat{j} - \hat{k}$ act on a particle at a point $A$. Determine the work done when the particle is displaced from $A(4, -3, -2)$ to $B(6, 1, -3)$. Also find the moment of the force $P_1$ about the point (1, 0, 1).

(*c*) A speaks the truth in 75% and B in 80% of the cases. In what percent of cases are they likely to contradict each other in narrating the same incident?

(*d*) A sportman's chance of shooting an animal at a distance $r$ is $a^2/r^2$. He fires when $r = 2a$ and if the misses, he reloads and fires when $r = 3a, 4a, \ldots$. If he misses at a distance $na$, the animal escapes. Show that the odds against the sportsman are $(n+1)/(n-1)$.

## ANSWERS

A-1 (*a*)
A-2 (*b*)
A-3 (*d*)
A-4 (*a*)
A-5 (*c*)
A-6 (*a*)
A-7 (*c*)
A-8 (*b*)
A-9 (*c*)
A-10 (*a*)
A-11 (*d*)
A-12 (*a*)
A-13 (*a*)
A-14 (*a*)
A-15 (*a*)
A-16 (*b*)
A-17 (*a*)
A-18 (*a*)
A-19 (*a*)
A-20 (*b*)
A-21 (*a*)
A-22 (*c*)
A-23 (*a*)
A-24 (*c*)
A-25 (*d*)
A-26 (*a*)
A-27 (*a*)
A-28 (*a*)

A-29 (*c*)
A-30 (*b*)
A-31 (*d*)
A-32 (*c*)
A-33 (*c*)
A-34 (*d*)
A-35 (*c*)
A-36 (*a*)
A-37 (*c*)
A-38 (*c*)
A-39 (*b*)
A-40 (*b*)
A-41 (*a*)
A-42 (*c*)
A-43 (*b*)
A-44 (*c*)
A-45 (*b*)
A-46 (*a*)
A-47 (*a*)
A-48 (*a*)
A-49 (*c*)
A-50 (*b*)
A-51 (*b*)
A-52 (*a*)
A-53 (*b*)
A-54 (*a*)
A-55 (*b*)
A-56 (*d*)

A-57 (*b*)
A-58 (*b*)
A-59 (*c*)
A-60 (*a*)
A-61 (*b*)
A-62 (a)
A-63 (*c*)
A-64 (*a*)
A-65 (*a*)
A-66 $\{2, -7\}$
A-67 1 and 3
A-68 5
A-69 $-1$
A-70 $\sin\left[\left(\frac{2x-1}{x^2+1}\right)^2\right] \cdot \frac{4x}{x^2+1}$
A-71 $\sqrt{2}\cos\theta$
A-72 1
A-73 $a = b$
A-74 $\pm \frac{1}{\sqrt{3}}$
A-75 16
A-76 $\frac{-1}{3}$; 1
A-77 $\frac{1}{2}e^{\pi/2}$
A-78 2
A-79 $a$
A-80 $\frac{-\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{\sqrt{2}}$

## PART B

B-3 (*f*) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

B-4 (*c*) $\frac{1}{12}$

B-5 (*d*) (i) $\alpha = n\pi$ (ii) $\alpha = n\pi + (-1)^n(\pm\pi/4)$

B-7 (*c*) 5/12

(*d*) $462 \frac{a^5}{b^6}, ab = 1$

(*e*) $t_8 = {}^{20}C_7 \times \frac{1}{27}$

B-9 (*b*) $2y - x - 3 = 0$

$y + 2x - 4 = 0, \quad y - 3x + 1 =$

(*e*) $\frac{25}{16}$

B-10 (*b*) $2x + y - 5 = 0$

B-11 (*a*) $\frac{2x \cos x^2}{\sqrt{\sin x^2}}$

(*b*) $2a \tan(ax) \sec^2(ax)$

(*c*) $-(2ax + b) \sin(ax^2 + bx + c)$

(*d*) discontinuous

B-12 (*a*) $x - 3y - 10 = 0$

$3x + y - 10 = 0$

B-13 $a = 1, \quad b = 0$

(*e*) $y\left[\frac{\alpha}{2x} + \frac{\alpha^2}{x(1 - \alpha \log x)} \log \sqrt{x}\right]$

where $\alpha = x^{x^{x\cdots}}$

B-14 (*b*) $\frac{2}{27}x^{3/2}[9(\log x)^2 - 12 \log x + 16] + C$

(*c*) $\frac{\pi a}{2}$

(*d*) (i) 0 (ii) $\pi/4$

(*e*) $\frac{125}{48}$ sq. units

(*f*) $4\sqrt{2} \sin\left(\frac{x - \pi}{4}\right) + C$

**B-15 (*a*) $a\hat{i} + b\hat{j} + c\hat{k}$, where $a = 9.2$, $b = 3.9$, $c = -0.8$**

**or $a = -6, b = -7.1$ $c = -4.3$**

**(*b*) 9 units, $6\sqrt{2}$ units**

**(*c*) 35%**

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This edition covers the physics, chemistry and mathematics papers from the Joint Entrance Exam of 1984 up to the latest exam of 1988. The English papers of 1985 to 1987 (on the new pattern) have also been provided. A 'model solution' is provided to each question.

**What is a 'Model Solution'?**

It is *the* solution that will get the student the highest score. Each step of the answer carries marks and the 'model solution' shows the steps that win those valuable marks. The model solutions show how the mind of the paper-setter works and what the evaluator expects. It will prepare the student for the tricky questions that are always asked in this exam.

**What does the 'Question Bank' contain?**

About 150 vital questions on each subject have been carefully prepared. They are of the same standard as those asked at the exam and follow the same pattern—objective and subjective types. They are meant for the student to practice on, after having studied the Model Solutions section and having understood the method by which problems are to be tackled. Answers are provided to each question.

The book is the result of the combined efforts of teachers with vast experience and success at preparing candidates for this exam.

The section on 'Some useful hints to the examinee' will be very useful to the examinees.

**Tata McGraw-Hill Publishing Company Limited**
**4/12 Asaf Ali Road, New Delhi 110 002**

**Rs. 50.00** **0-07-460419-8**